MODULE 5

COMBINATIONAL LOGIC

Course Material for Combinational

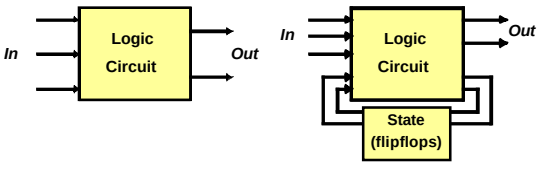
Extra: Slides about how to implement a static combinational gate with NMOS/PMOS transistors, given the Boolean function

Combinational Logic - Outline

- Conventional Static CMOS basic principles
- Complementary static CMOS
  - Complex Logic Gates
  - VTC, Delay and Sizing
- Ratioed logic
- Pass transistor logic
- Dynamic CMOS gates ➔only illustration

Complementary Static CMOS Basic Principles

Combinational vs. Sequential Logic



(a) Combinational

Output = *f* (In)

(b) Sequential

Output = *f* (In, History)

§ 6.2

Reminder

DeMorgan Transformations

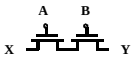
$$\overline{A+B} = \overline{A}.\overline{B}$$

$$\overline{A.B} = \overline{A}+\overline{B}$$

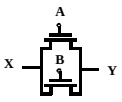
NMOS Transistors in Series/Parallel Connection

Transistors can be thought as a switch controlled by its gate signal

NMOS switch closes when switch control input is high



Y = X if A and B



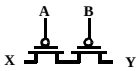
Y = X if A or B

§ 6.2.1

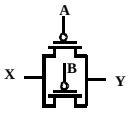
PMOS Transistors in Series/Parallel Connection

PMOS switch closes when switch control input is low

Y = X if ...

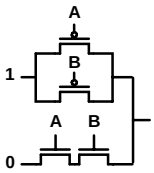


Y = X if $\overline{A}$ and $\overline{B}$



Y = X if $\overline{A}$ or $\overline{B}$

2-Input Nand



Y = 1 if $\overline{A}$ OR $\overline{B}$
Y = 1 if $\overline{A}$ AND $\overline{B}$ } DeMorgan

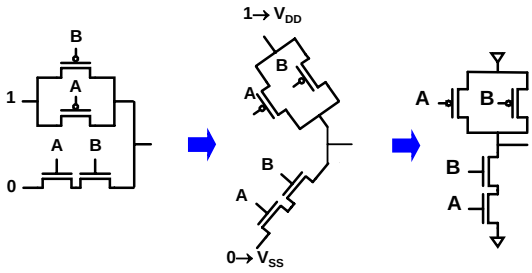
Y = $\overline{A \text{ AND } B}$

Y = 0 if A AND B

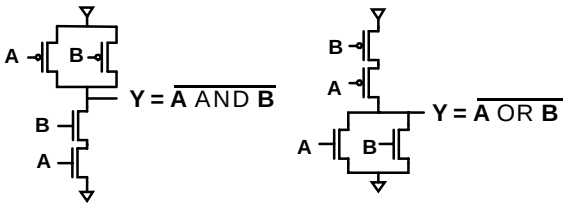
B	A	Y
0	0	1
0	1	1
1	0	1
1	1	0

2-Input Nand

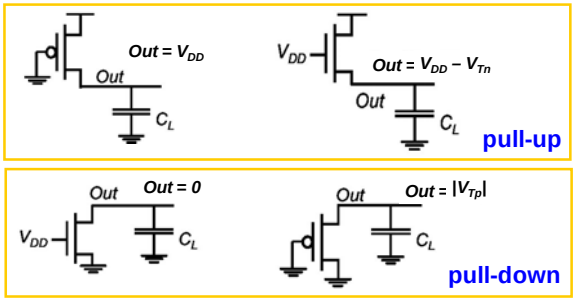
Y = $\overline{A \text{ AND } B}$



2-input Nand/Nor

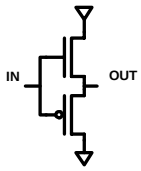


NMOS vs. PMOS, pull-down vs. pull-up



- PMOS is better pull-up
- NMOS is better pull-down

Bad Idea



Exercise: Determine logic function

Determine V_{out} for $V_{in} = V_{DD}$ and $V_{in} = V_{SS}$

Why is this a bad circuit?

CMOS Gate is Inverting.

Assume full-swing inputs (high = V_{DD}, low = V_{SS})

- Highest output voltage of NMOS is
  $V_{GS} - V_{Tn} = V_{DD} - V_{Tn}$
- An 1 on NMOS gate can produce a strong 0 at the drain, but not a strong 1
- Lowest output voltage of PMOS is
  $V_{DD} + V_{GS} - V_{Tp} = |V_{Tp}|$
  (with V_{GS}, $V_{Tp} < 0$ for PMOS)
- An 0 on PMOS gate can produce a strong 1 at the drain, but not a strong 0
- Need NMOS for pull-down, PMOS for pull-up
  A 1 at input can pull-down, 0 at input can pull-up
  A 1 can produce a 0, a 0 can produce a 1

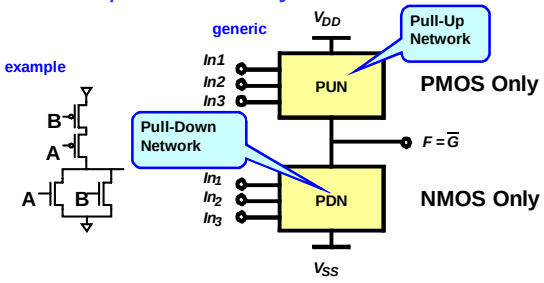
Inverting behavior

For a non-inverting Complementary CMOS Gate, you can only use 2 inverting gates

Complementary static CMOS

- Complex Logic Gates
- VTC, Delay and Sizing

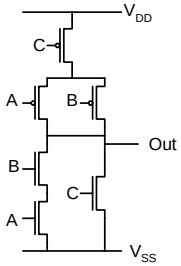
Complementary Static CMOS



- Conduction of PDN and PUN must be mutually exclusive (Why?)
- Pull-up network (PUN) and pull-down network (PDN) are dual

Mutual Exclusive PDN and PUN

Out = (AB + C)'



C	B	A	PDN	PUN	Out	
0	0	0	?	1	1	PDN Off, PUN On
0	0	1	?	1	1	
0	1	0	?	1	1	
0	1	1	0	?	0	PUN Off, PDN On
1	0	0	0	?	0	
1	0	1	0	?	0	
1	1	0	0	?	0	
1	1	1	0	?	0	

For all Complementary Static CMOS Gates, either the PUN or the PDN is conducting, but never both.

Complementary Static CMOS (2)

- Conduction of PUN and PDN must be mutually exclusive
- PUN is dual (complement) network of PDN
  series ⇔ parallel
  nmos ⇔ pmos
- Complementary gate is inverting
- No static power dissipation
- Need 2N transistors for N-input gate

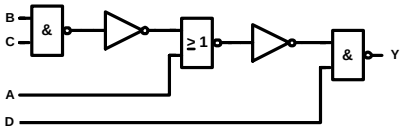
Implementation of Combinational Logic

- How van we construct an arbitrary combinational logic network in general, using NMOS and PMOS transistors (using Complementary static CMOS)?

- Example: $Y = \overline{(A + BC)D}$
- Remember: only inverting gates available

Implementation of Combinational Logic

- Example: $Y = \overline{(A + BC)D}$
- Remember: only inverting gates available
- Logic depth: number of gates in longest path ⇨ DELAY

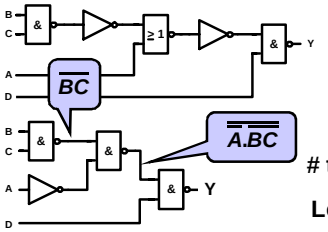


transistors ☐ logic depth ☐

- Q: Can this be improved?

Improved Gate Level Implementation

- Using DeMorgan $A + BC = \overline{\overline{A}.\overline{BC}}$



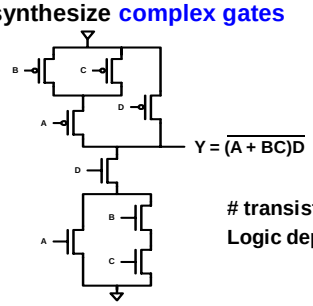
$Y = \overline{(A + BC)D}$

transistors ☐
Logic depth ☐

- Q: Can this be further improved?

Complex CMOS Logic Gates

- Restriction to basic NAND, NOR etc. not necessary
- Easy to synthesize complex gates

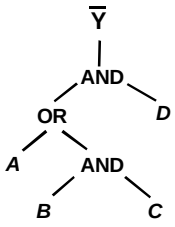


$Y = \overline{(A + BC)D}$

transistors: 8
Logic depth: 1

How to Synthesize Complex Gates

$Y = \overline{(A + BC)D}$

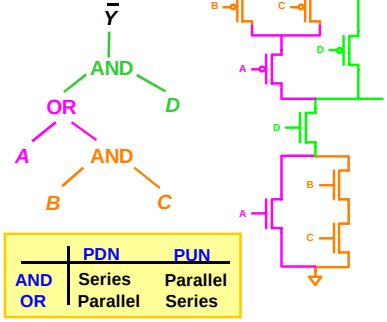


- Using tree representation of Boolean function
- *Operator* with branches for *operands*
- As a series-parallel network

	PDN	PUN
AND	Series	Parallel
OR	Parallel	Series

Complex Gate Synthesis Example

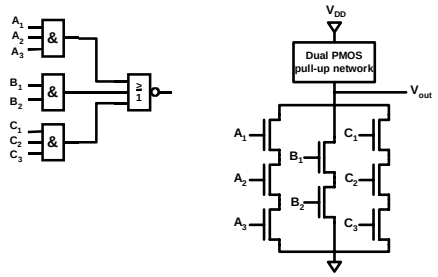
$\overline{Y} = (A + (BC))D$



	PDN	PUN
AND	Series	Parallel
OR	Parallel	Series

Recipe
- Write $\overline{Y}$ = f(inputs)
- Decompose f in tree form
- Realize tree branches according to table at bottom-left
- Use inverted inputs if necessary

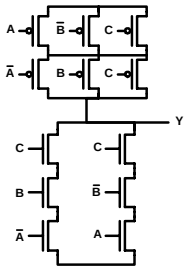
And-Or-Invert Gate



And-Or-Invert Example

- From a Truth-Table: take 0-outputs

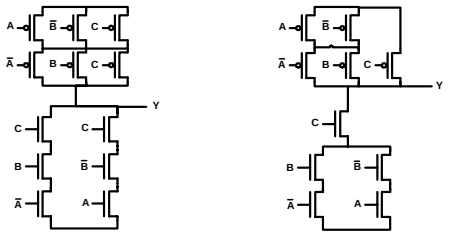
A	B	C	Y	
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	→ $\bar{A}BC$
1	0	0	1	
1	0	1	0	→ $A\bar{B}C$
1	1	0	1	
1	1	1	1	



$\bar{Y} = \bar{A}BC + A\bar{B}C$

$\bar{A}$, $\bar{B}$ to be created with extra inverters (or by restructuring previous circuits)

And-Or-Invert Improvement



$Y = \overline{\bar{A}BC + A\bar{B}C}$

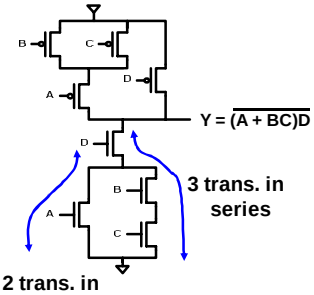
12 transistors

$Y = \overline{(\bar{A}B + A\bar{B})C}$

10 transistors

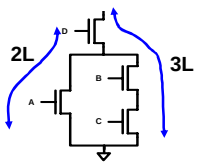
2-level logic minimization: see Brown (CS1), Ch. 4

CMOS Complex Gate Sizing



- Function of gate independent of transistor sizes: ratio-less
- But current-drive capability depends on transistor sizes
- Worst-case current-drive depends on number of transistors in series

CMOS Complex Gate Sizing



Assume all transistor lengths are equal

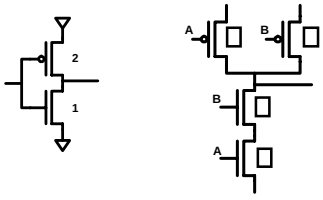
Find $(W/L)_i$ such that PDN has strength of W/L=2

$(W/L)_B=(W/L)_C=(W/L)_D = 6$

$(W/L)_A = 4$

1. For all possible single pull-down paths:
   a. Let L_{sum} be the sum of lengths of all transistors in path,
   b. Compute W to give the desired drive strength to a transistor of this length L_{sum}
2. For all transistors:
   a. Set W to be the maximum found among all the paths of which the transistor is part
3. Repeat this procedure for PUN, using W_p for PUN transistor of inverter.

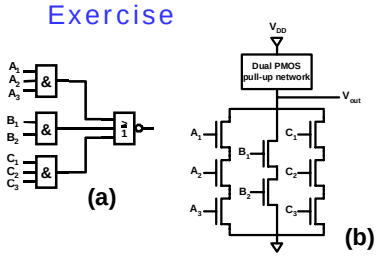
Gate Sizing



- W/L ratios
- what are the W/L of 2-input NAND for same drive strength?

0-th order calculation

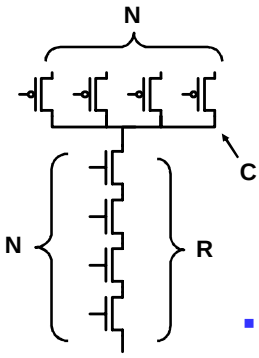
Exercise



Exercise:

- Perform gate sizing of (a) for nominal drive strength equal to that of min size inverter, assume PU/PD = 3
- Determine PUN of (b)
- Perform gate sizing of (b) for same drive strength (same PU/PD)
- Compare sum of gate areas in (a) and (b). Note: area ~ width

Avoid Large Fan-In



C of PUN linear in N

R of PDN linear in N

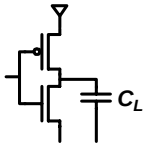
Delay ∝ RC quadratic in N

Empirical

Delay = $a_1FI + a_2FI^2 + a_3FO$

- Sizing can make R independent of N, at the (huge) cost of size
- Consider case of NOR your self

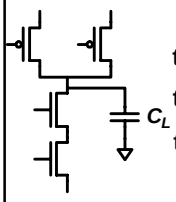
Data-Dependent Timing



$t_{PHL} = 0.69R_NC_L$

$t_{PLH} = 0.69R_PC_L$

You should be able to identify the transistor paths that charge or discharge C_L, and calculate resulting RC delay model, including effects of wires and fan-out

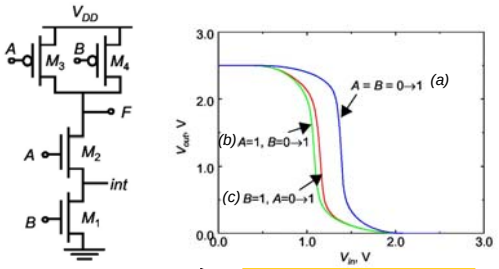


$t_{PHL} = 0.69(R_N \times 2)C_L$ Series connection

$t_{PLH} = 0.69R_pC_L$ One input goes low

$t_{PLH} = 0.69(R_p/2)C_L$ Two inputs go low, parallel connection

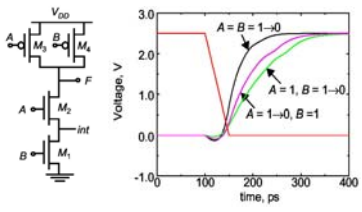
Data-dependent VTC: 2nd order effects



- Charge at 'int'
- Body effect in M_2
- Short-circuit currents

- Don't need to be able to work with these effects
- But remember: there is more going on than shown by our simple, 1st order model

Data-dependent Timing (2).

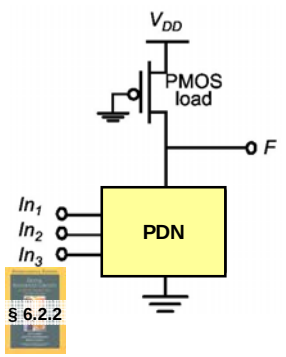


Input Data Pattern	Delay (pS)
A = B= 0→1	69
A = 1, B= 0→1	62
A= 0→1, B = 1	50
A=B=1→0	35
A=1, B = 1→0	76
A= 1→0, B = 1	57

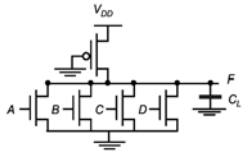
Ratioed logic

Pass transistor logic

Pseudo NMOS Ratioed Logic



- ☺ Reduced area
- ☺ Reduced capacitances
- ☹ Increased V_{OL}
- ☹ Reduced noise margins
- ☹ Static dissipation



Ratioed Logic V_{OL} Computation.

Exercise: verify these assumptions/steps

I_{Dn} (linear) = I_{Dp} (saturation)

$$k_n\left((V_{DD}-V_{Tn})V_{OL}-\frac{V_{OL}^2}{2}\right)=k_p\left((-V_{DD}-V_{Tp})V_{DSAT}-\frac{V_{DSAT}^2}{2}\right)$$

Ignore quadratic terms (they are relatively small)

$$k_n(V_{DD}-V_{Tn})V_{OL}\approx k_p(-V_{DD}-V_{Tp})V_{DSAT}$$

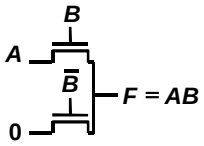
Ignore, because approximately equal

$$V_{OL}\approx\frac{k_p}{k_n}|V_{DSAT}|\approx\frac{\mu_p W_p}{\mu_n W_n}|V_{DSAT}|$$

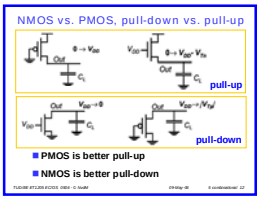
Pass-transistor and Pass-gate circuits

Pass Transistor Logic

- Save area, capacitances
- Need complementary inputs (extra inverters)

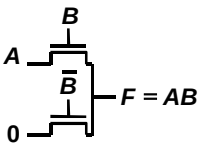


But remember:



Pass Transistor Logic

- Save area, capacitances
- Need complementary inputs (might mean extra inverters)
- Reduced V_{OH}, noise margins



- $V_{OH}=V_{DD}-\left(V_{Tno}+\gamma\left(\left(\sqrt{|2\phi_f|+V_{OH}}\right)-\sqrt{|2\phi_f|}\right)\right)$
- Static dissipation in subsequent static inverter/buffer
- Disadvantages (and advantages) may be reduced by complementary pass gates (NMOS + PMOS parallel)

Exercise: Why is there static dissipation in next conventional gate?

Pass Gates

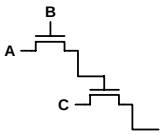
- Remedy: use an N-MOS and a P-MOS in parallel

- Pass gates eliminate some of the disadvantages of simple pass-transistors
- But also some of the advantages
- Design remains a trade-off!

Pass-gate a.k.a. Transmission-gate

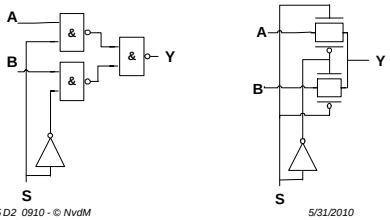
Exercise

- Discuss what happens when you connect the output of a single pass-transistor (not a pass-gate) to the input of another pass-transistor stage (i.e. the gate of another pass-transistor). Why should you never use such a circuit?



Pass Transistor Logic.

- Most typical use: for multiplexing, or path selecting
- Assume in circuit below it is required to either connect A or B to Y, under control by S
- Y = AS + BS' (S' is easier notation for S-bar = S-inverse = $\overline{S}$)
- Y = ((AS)' (BS)')' allows realization with 3 NAND-2 and 1 INV: 14 transistors
- Pass gate needs only 6 (or 8, when including restoring inverter at output) transistors



Summary

- Conventional Static CMOS basic principles
- Complementary static CMOS
  - Complex Logic Gates
  - VTC, Delay and Sizing
- Ratioed logic
- Pass transistor logic