MODULE 5

COMBINATIONAL LOGIC

Course Material for Combinational

Extra: Slides about how to implement a static combinational gate with NMOS/PMOS transistors, given the Boolean function

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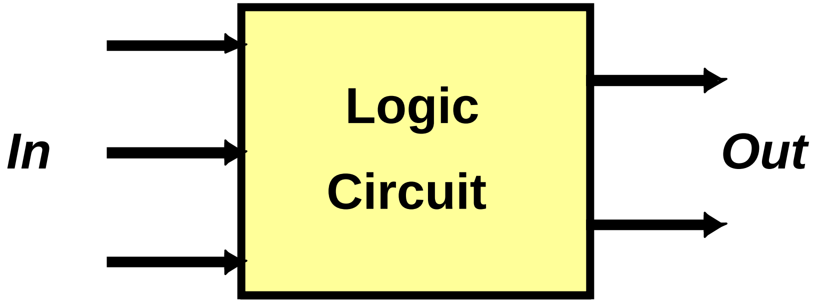
Combinational Logic - Outline

- **Conventional Static CMOS basic principles**
- **Complementary static CMOS**
  - **Complex Logic Gates**
  - **VTC, Delay and Sizing**
- **Ratioed logic**
- **Pass transistor logic**
- **Dynamic CMOS gates → only illustration**

Complementary Static CMOS Basic Principles

Combinational vs. Sequential Logic

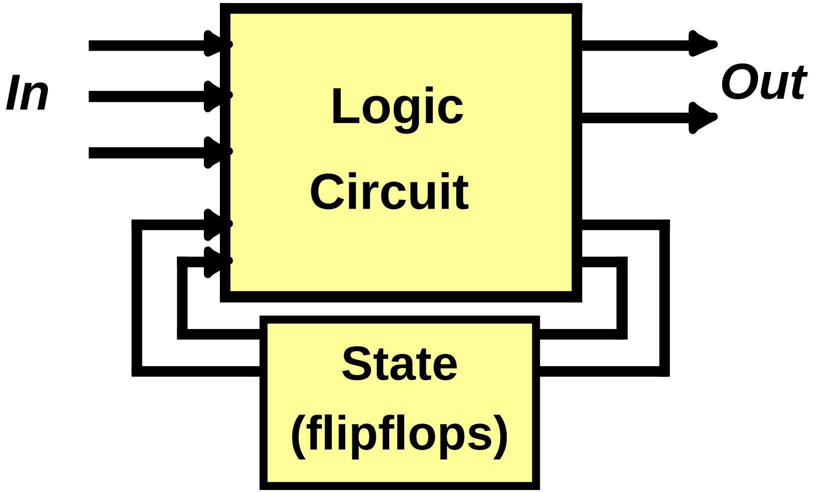


In

Logic

Circuit

Out



In

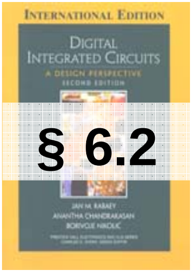
Logic

Circuit

Out

State

(flipflops)



§ 6.2

(a) Combinational

Output = *f* (In)

(b) Sequential

Output = *f* (In, History)

Reminder

DeMorgan Transformations

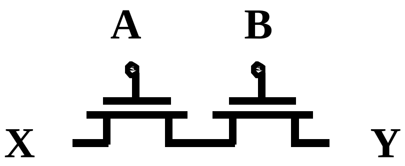
$$\overline{A+B} = \overline{A}.\overline{B}$$

$$\overline{A.B} = \overline{A}+\overline{B}$$

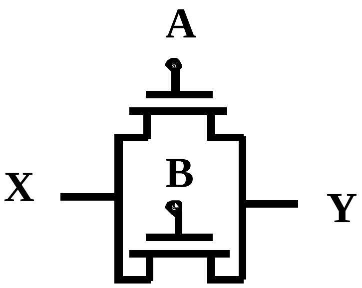
NMOS Transistors in Series/Parallel Connection

Transistors can be thought as a switch controlled by its gate signal

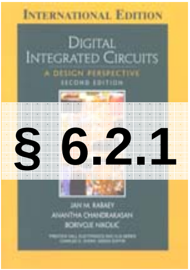
NMOS switch closes when switch control input is high



Y = X if A and B



Y = X if A or B

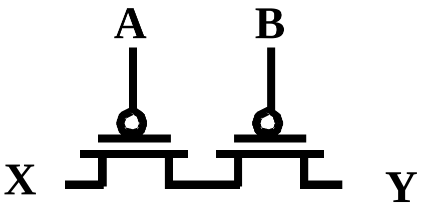


§ 6.2.1

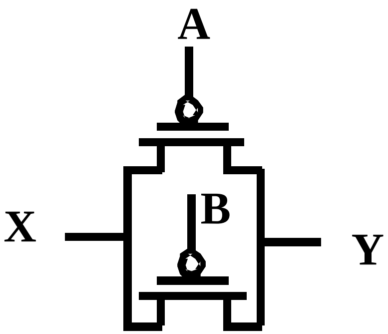
PMOS Transistors in Series/Parallel Connection

PMOS switch closes when switch control input is low

Y = X if …

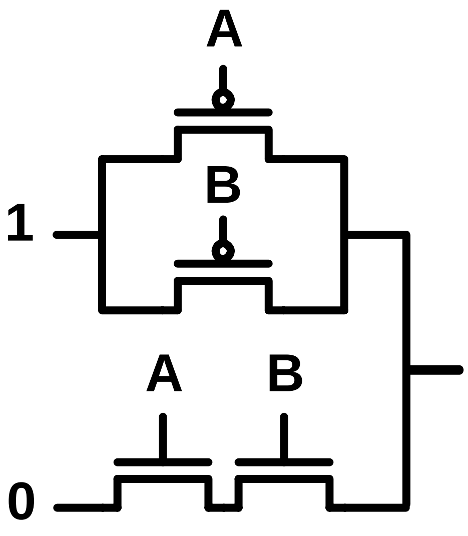


Y = X if $\overline{A}$ and $\overline{B}$



Y = X if $\overline{A}$ or $\overline{B}$

2-Input Nand



$Y = 1 \text{ if } \overline{A} \text{ OR } \overline{B}$

$Y = 1 \text{ if } \overline{A \text{ AND } B}$

DeMorgan

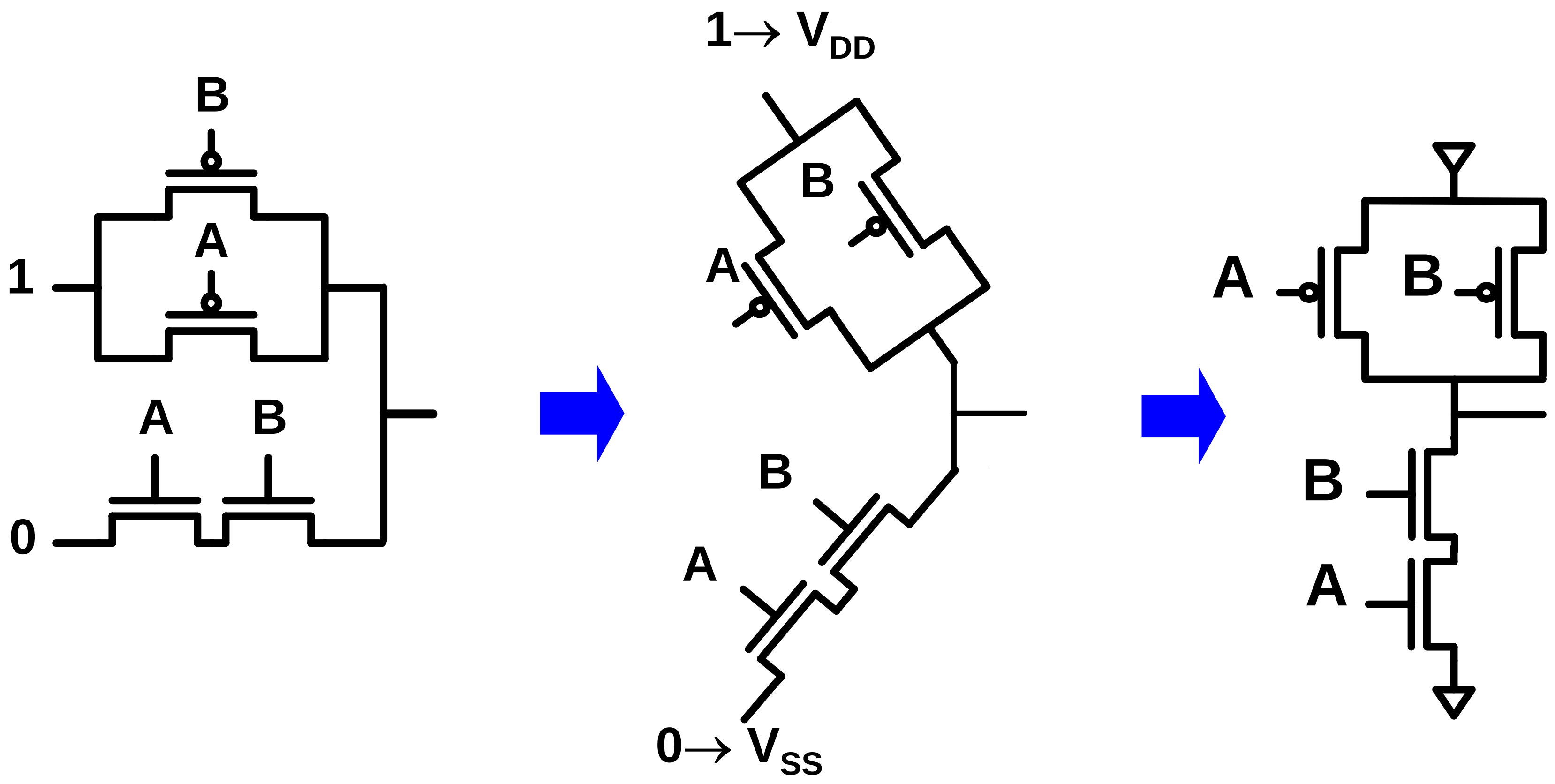
$Y = \overline{A \text{ AND } B}$

$Y = 0 \text{ if } A \text{ AND } B$

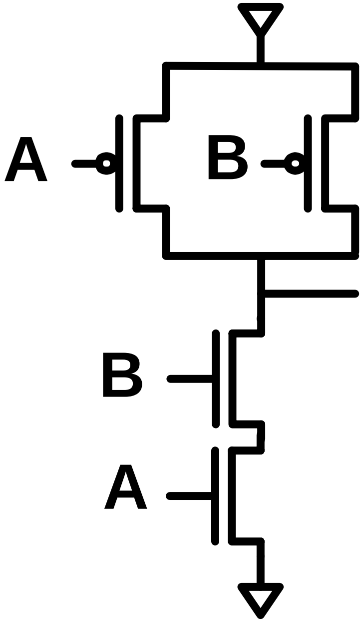
B	A	Y
0	0	1
0	1	1
1	0	1
1	1	0

2-Input Nand

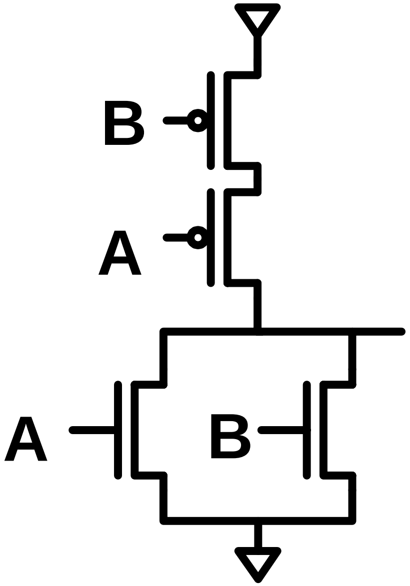
$$Y = \overline{A \text{ AND } B}$$



2-input Nand/Nor

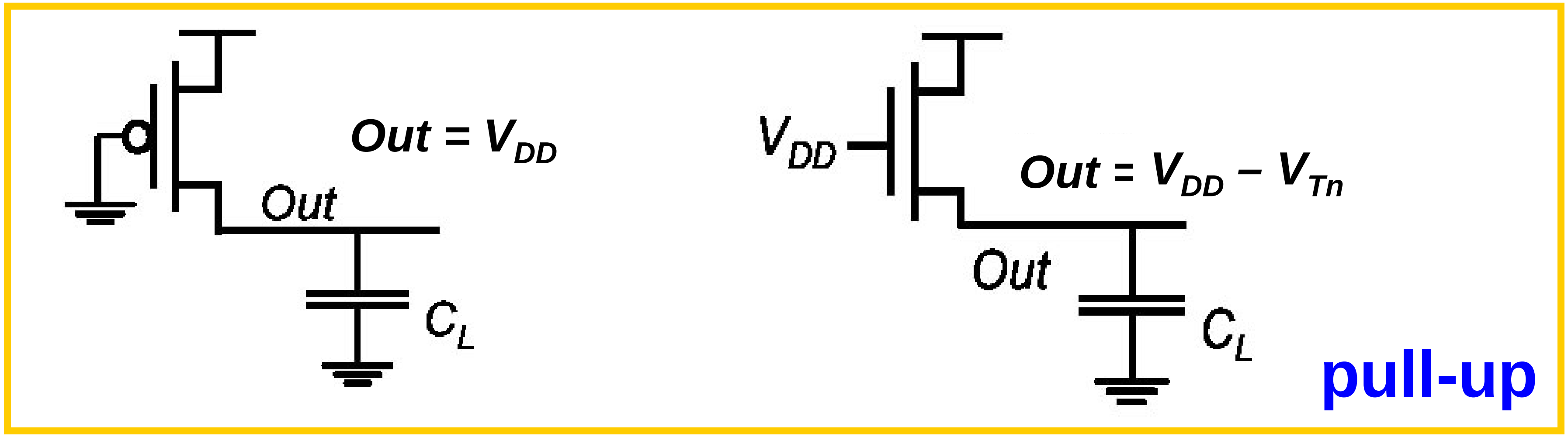


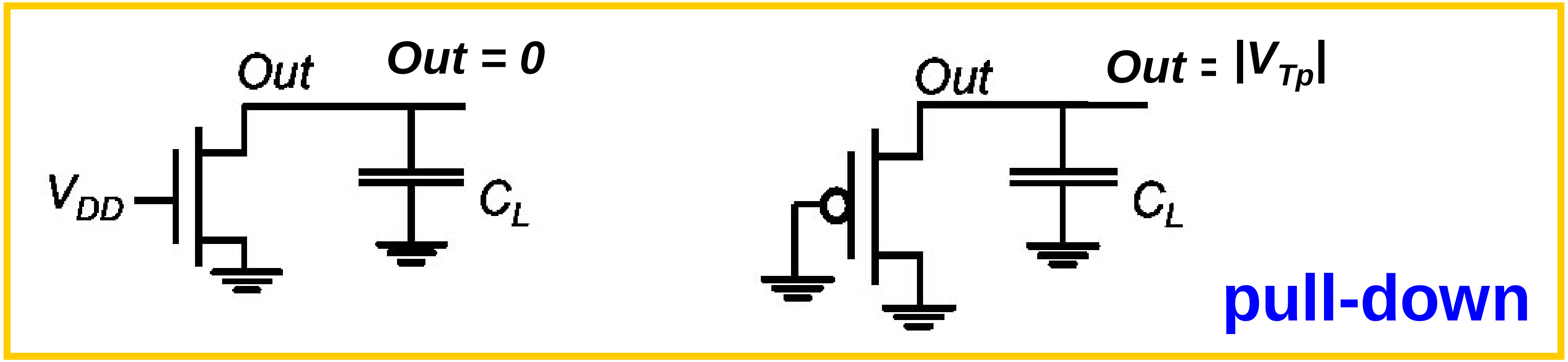
$Y = \overline{\mathbf{A} \text{ AND } \mathbf{B}}$



$Y = \overline{\mathbf{A} \text{ OR } \mathbf{B}}$

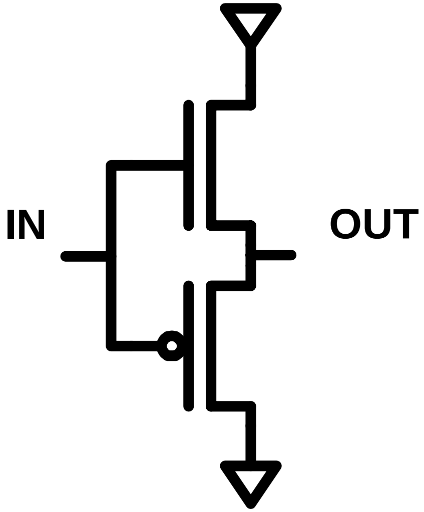
NMOS vs. PMOS, pull-down vs. pull-up





- **PMOS is better pull-up**
- **NMOS is better pull-down**

Bad Idea



Exercise: **Determine logic function**

Determine V_{out}

for $V_{in} = V_{DD}$ and $V_{in} = V_{SS}$

Why is this a bad circuit?

CMOS Gate is Inverting.

Assume full-swing inputs (high = V_{DD}, low = V_{SS})

- **Highest output voltage of NMOS is**

  $$V_{GS} - V_{Tn} = V_{DD} - V_{Tn}$$

- **An 1 on NMOS gate can produce a strong 0 at the drain, but not a strong 1**
- **Lowest output voltage of PMOS is**

  $$V_{DD} + V_{GS} - V_{Tp} = |V_{Tp}|$$

  (with V_{GS}, V_{Tp} < 0 for PMOS)

- **An 0 on PMOS gate can produce a strong 1 at the drain, but not a strong 0**
- **Need NMOS for pull-down, PMOS for pull-up**

  A 1 at input can pull-down, 0 at input can pull-up
  A 1 can produce a 0, a 0 can produce a 1

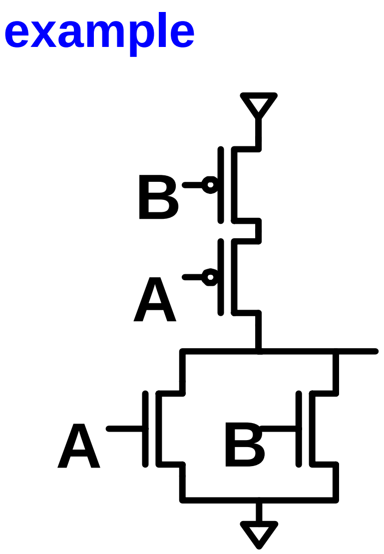
Inverting behavior

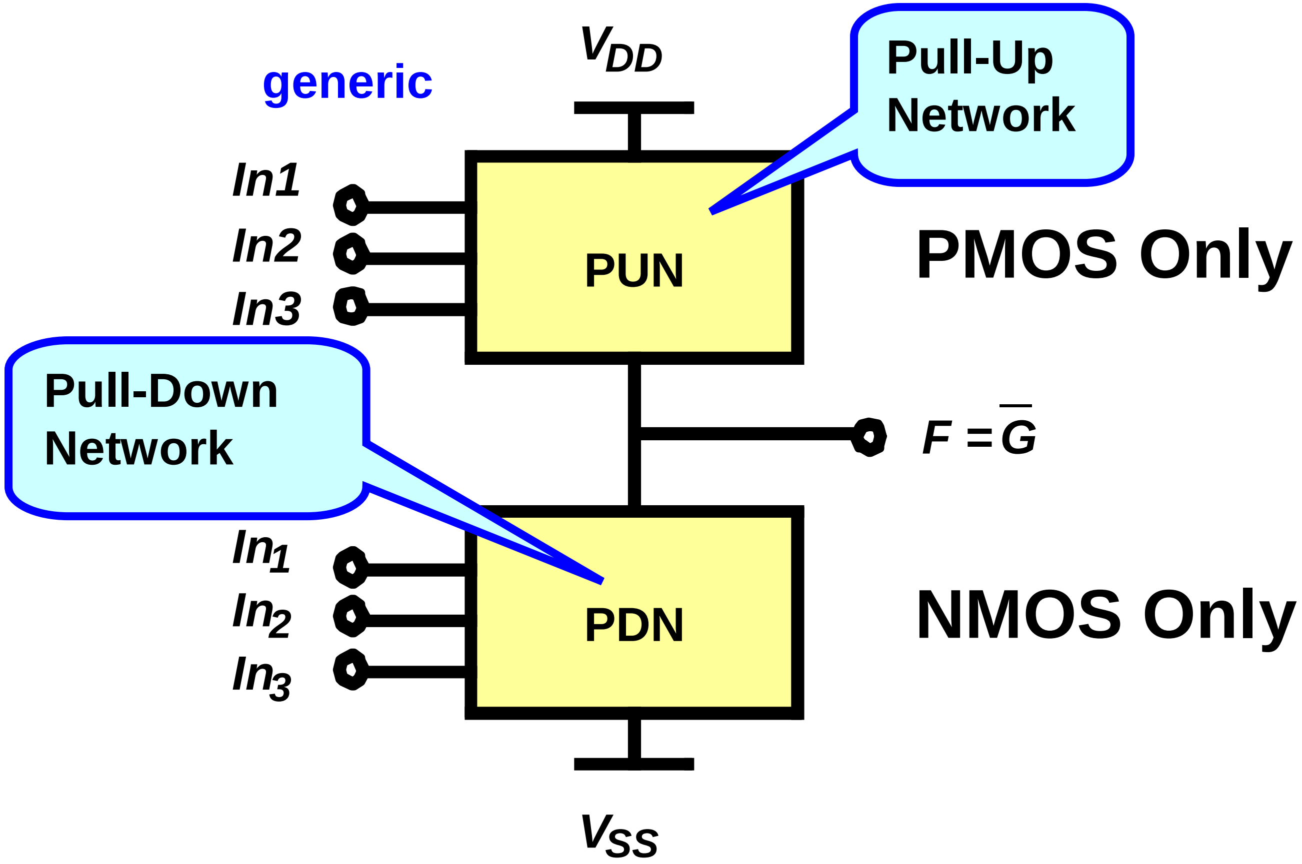
For a non-inverting Complementary CMOS Gate, you can only use 2 inverting gates

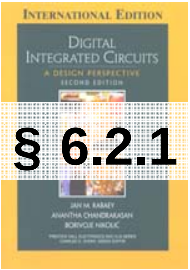
Complementary static CMOS

- Complex Logic Gates
- VTC, Delay and Sizing

Complementary Static CMOS





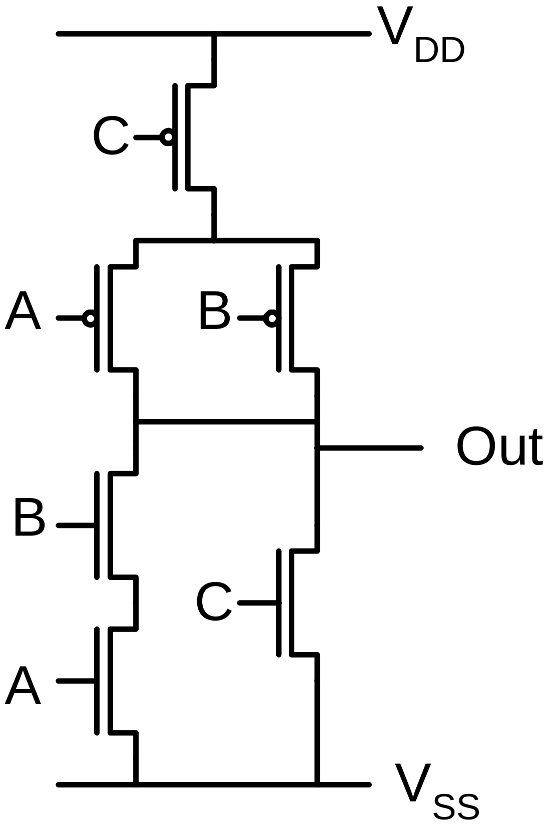


§ 6.2.1

- **Conduction of PDN and PUN must be mutually exclusive (Why?)**
- **Pull-up network (PUN) and pull-down network (PDN) are dual**

Mutual Exclusive PDN and PUN

Out = (AB + C)’



C	B	A	PDN	PUN	Out
0	0	0	?	1	1
0	0	1	?	1	1
0	1	0	?	1	1
0	1	1	0	?	0
1	0	0	0	?	0
1	0	1	0	?	0
1	1	0	0	?	0
1	1	1	0	?	0

PDN Off
PUN On

PUN Off
PDN On

For all Complementary Static CMOS Gates, either the PUN or the PDN is conducting, but never both.

Complementary Static CMOS (2)

- **Conduction of PUN and PDN must be mutually exclusive**
- **PUN is dual (complement) network of PDN**

  series ⇔ parallel

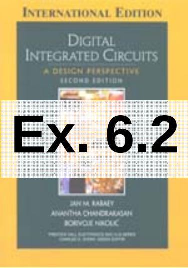
  nmos ⇔ pmos
- **Complementary gate is inverting**
- **No static power dissipation**
- **Need 2N transistors for N-input gate**

Implementation of Combinational Logic

- **How van we construct an arbitrary combinational logic network in general, using NMOS and PMOS transistors (using Complementary static CMOS)?**

- **Example:** $Y = \overline{(A + BC)D}$

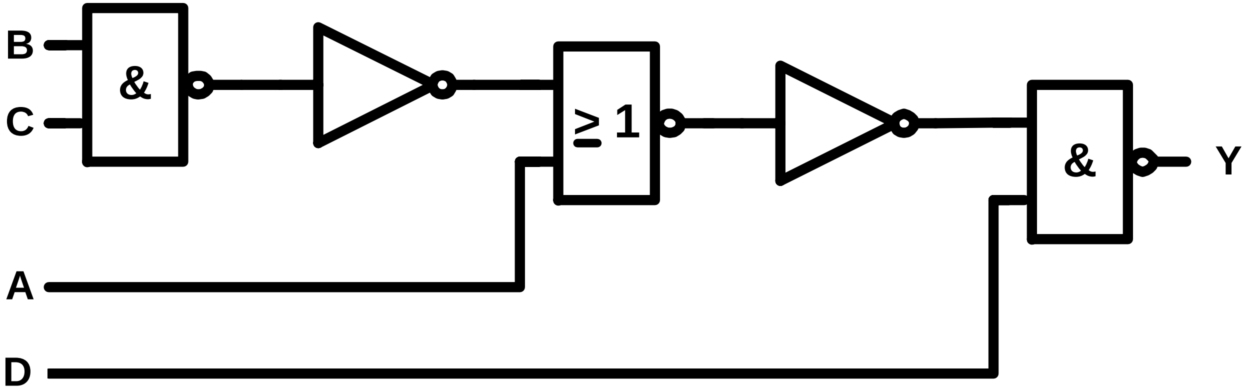
- **Remember: only inverting gates available**



Ex. 6.2

Implementation of Combinational Logic

- **Example:** $Y = \overline{(A + BC)D}$
- **Remember: only inverting gates available**
- **Logic depth:** **number of gates in longest path ⇨ DELAY**

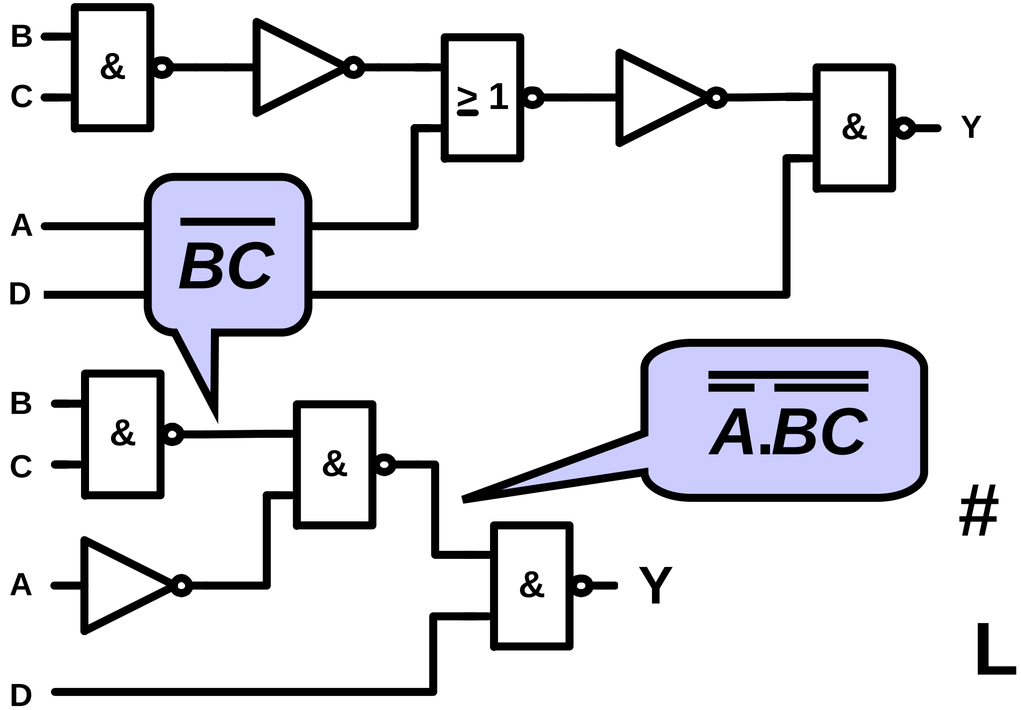


 # transistors ☐ **logic depth** ☐

- **Q: Can this be improved?**

Improved Gate Level Implementation

- **Using DeMorgan** $A + BC = \overline{\overline{A}.\overline{BC}}$



$Y = \overline{(A + BC)D}$

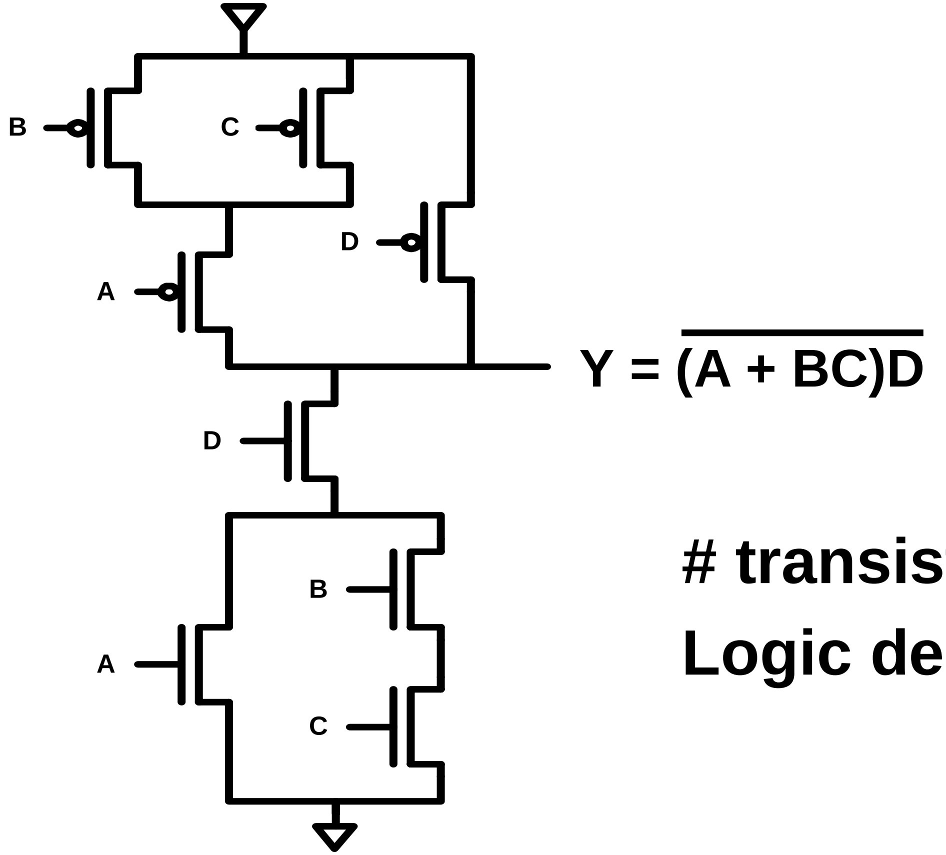
 # transistors ☐

Logic depth ☐

- **Q: Can this be further improved?**

Complex CMOS Logic Gates

- **Restriction to basic NAND, NOR etc. not necessary**
- **Easy to synthesize complex gates**



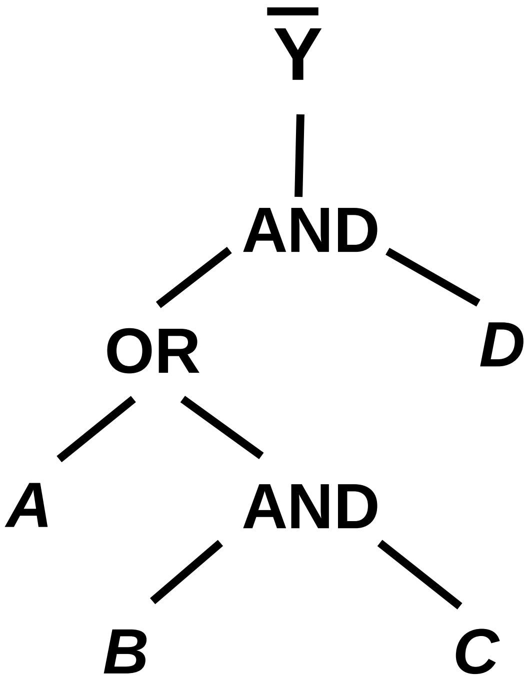
$Y = \overline{(A + BC)D}$

 # transistors: 8

Logic depth: 1

How to Synthesize Complex Gates

$$Y = \overline{(A + BC)D}$$

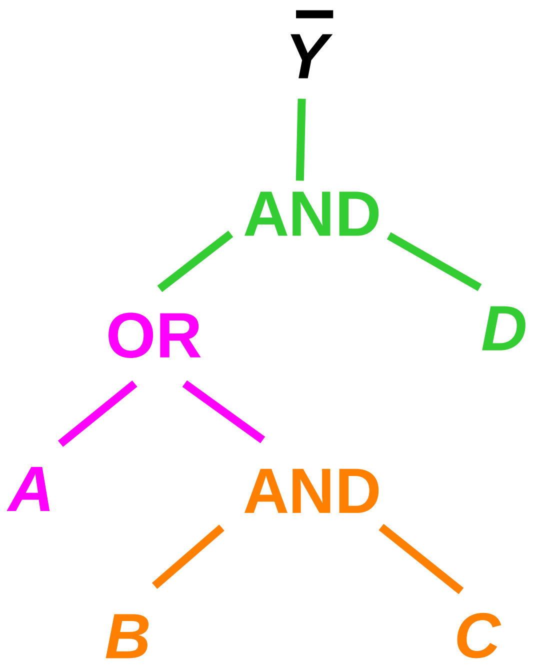


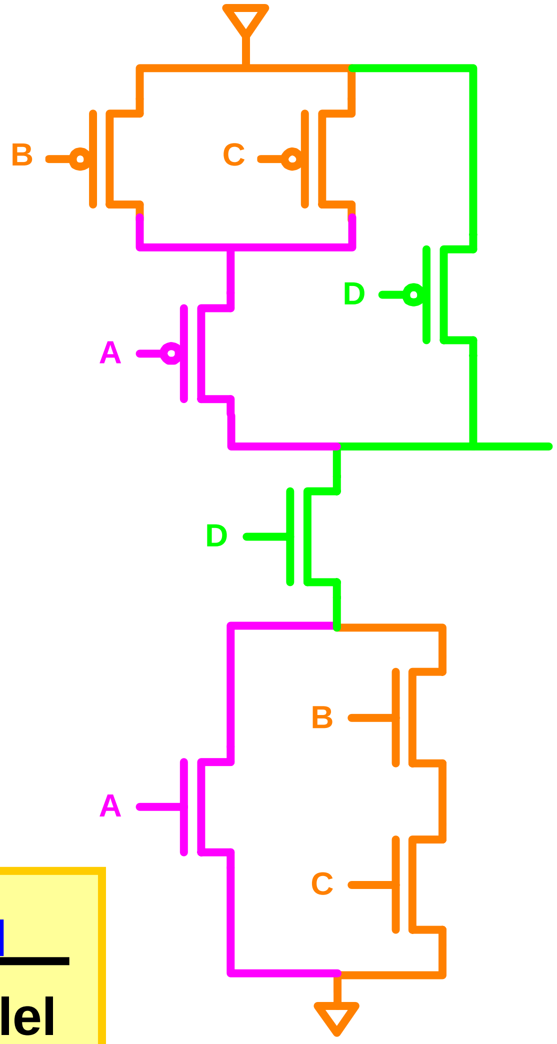
- **Using tree representation of Boolean function**
- ***Operator* with branches for *operands***
- **As a series-parallel network**

	PDN	**PUN**
AND	**Series**	**Parallel**
OR	**Parallel**	**Series**

Complex Gate Synthesis Example

$\overline{Y} = (A + (BC))D$



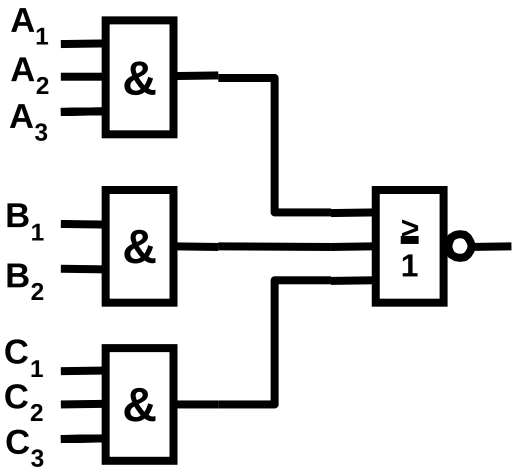


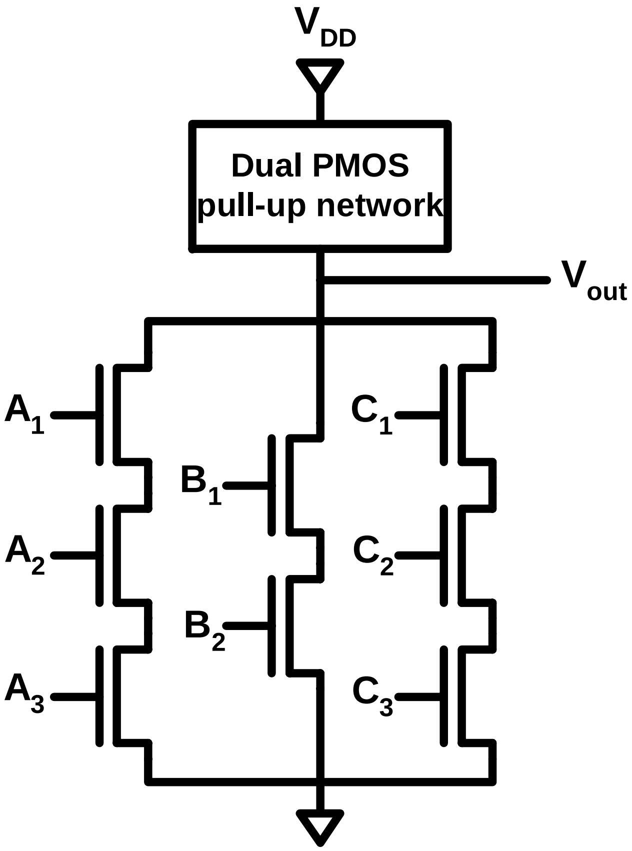
	PDN	PUN
AND	**Series**	**Parallel**
OR	**Parallel**	**Series**

Recipe

- **Write $\overline{Y}$ = f(inputs)**
- **Decompose f in tree form**
- **Realize tree branches according to table at bottom-left**
- **Use inverted inputs if necessary**

And-Or-Invert Gate

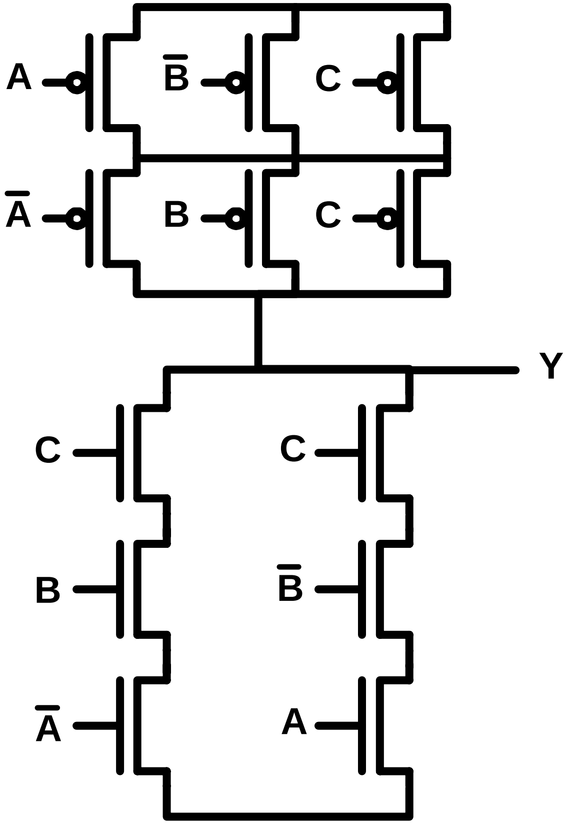




And-Or-Invert Example

- **From a Truth-Table: take 0-outputs**

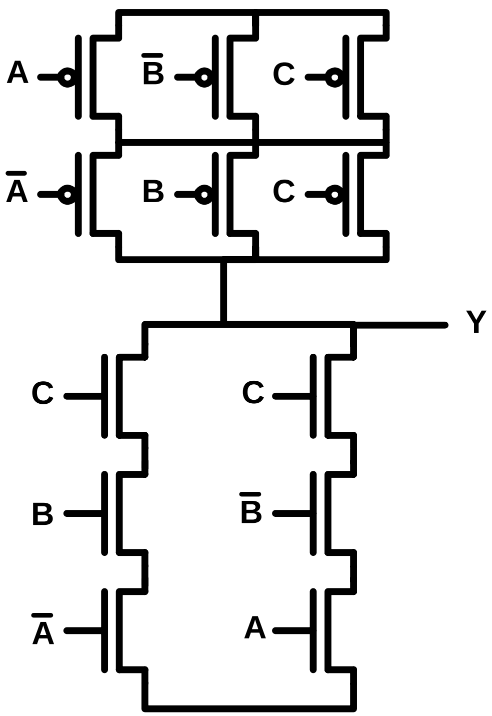
A	B	C	Y	
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	→ $\bar{A}BC$
1	0	0	1	
1	0	1	0	→ $A\bar{B}C$
1	1	0	1	
1	1	1	1	



$$\overline{Y} = \bar{A}BC + A\bar{B}C$$

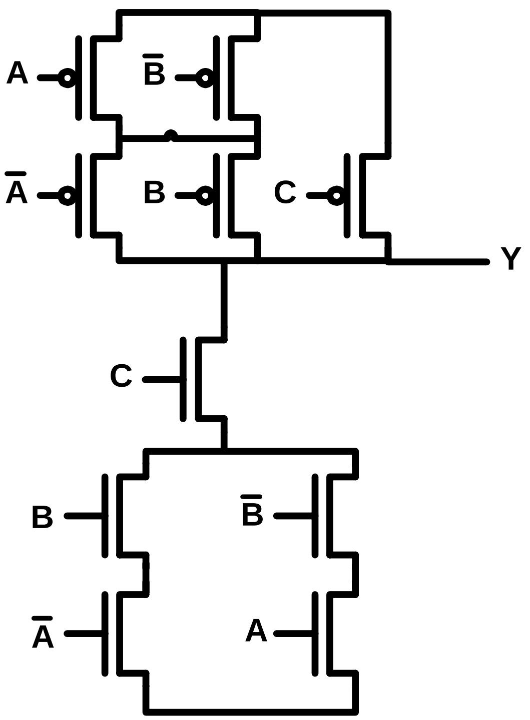
$\bar{A}$, $\bar{B}$ to be created with extra inverters (or by restructuring previous circuits)

And-Or-Invert Improvement



$$Y = \overline{\bar{A}BC + A\bar{B}C}$$

12 transistors

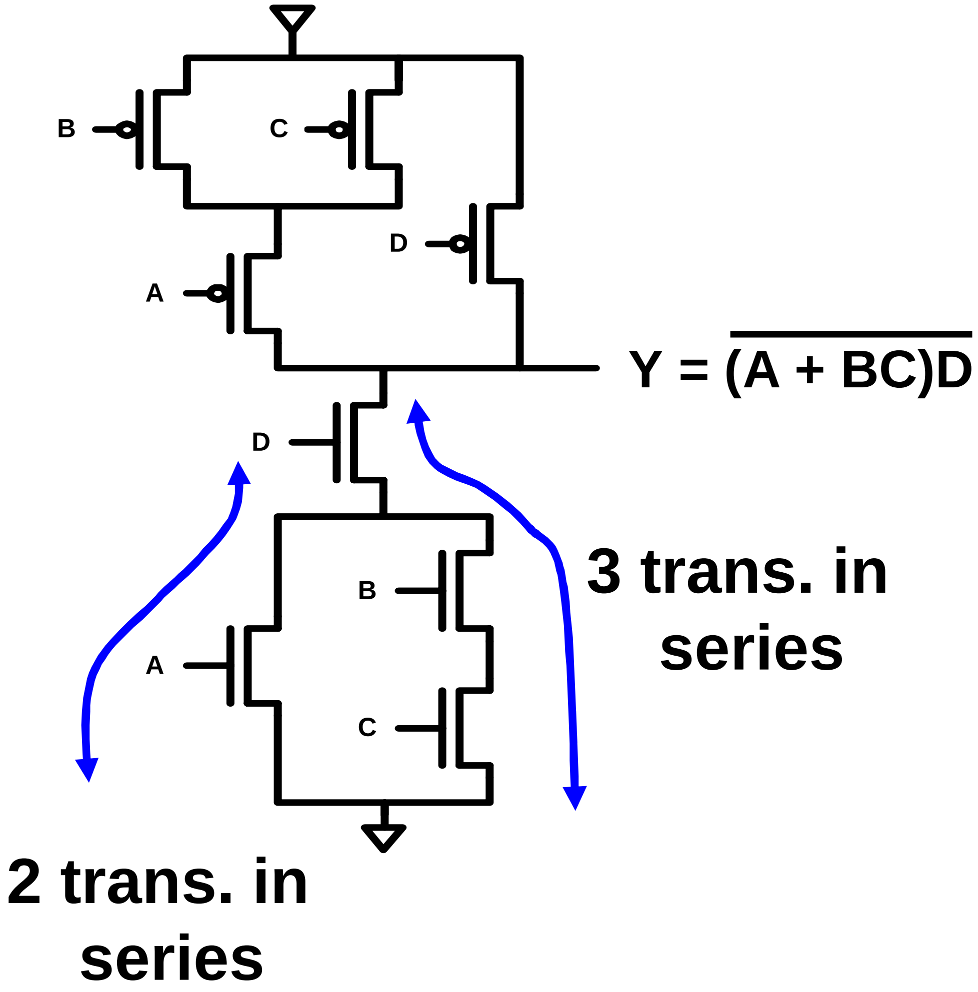


$$Y = \overline{(\bar{A}B + A\bar{B})C}$$

10 transistors

2-level logic minimization: see Katz (CS1), § 2.3

CMOS Complex Gate Sizing



$Y = \overline{(A + BC)D}$

3 trans. in series

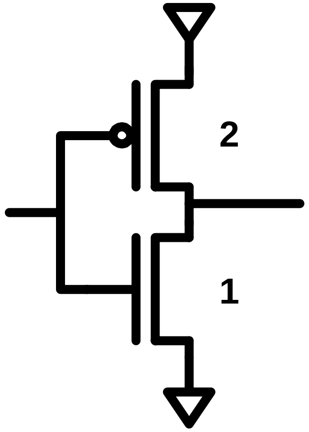
2 trans. in series

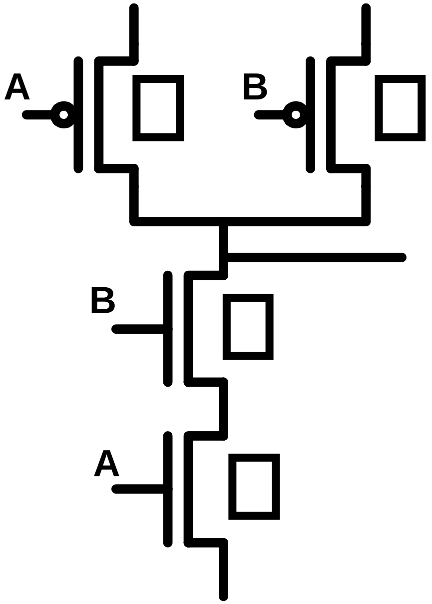
- **Function of gate independent of transistor sizes: ratio-less**
- **But current-drive capability depends on transistor sizes**
- **Worst-case current-drive depends on number of transistors in series**

CMOS Complex Gate Sizing

- **Assume all transistors will have mininum length L**
- **Determine W_n for PDN transistor of inverter that would give the desired 'drive strength'**
- **For each transistor in PDN of complex gate do the following:**
    - **Determine the length l of the longest PDN chain in which it participates**
    - **Set $W = l\,W_n$**
- **Repeat this procedure for PUN, using W_p for PUN transistor of inverter.**

Gate Sizing

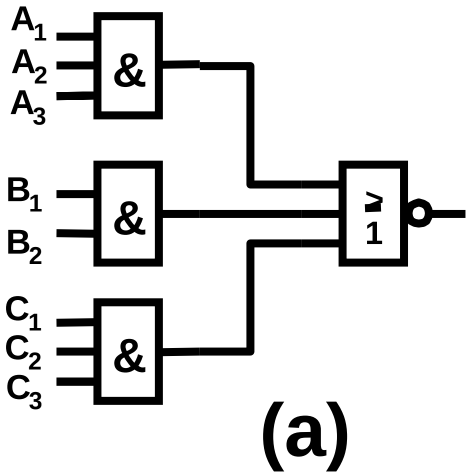


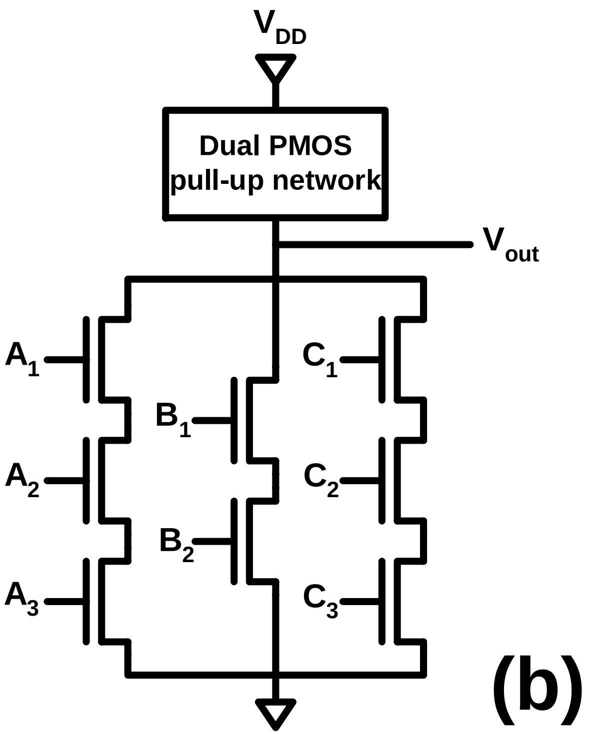


- **W/L ratios**
- **what are the W/L of 2-input NAND for same drive strength?**

0-th order calculation

Exercise

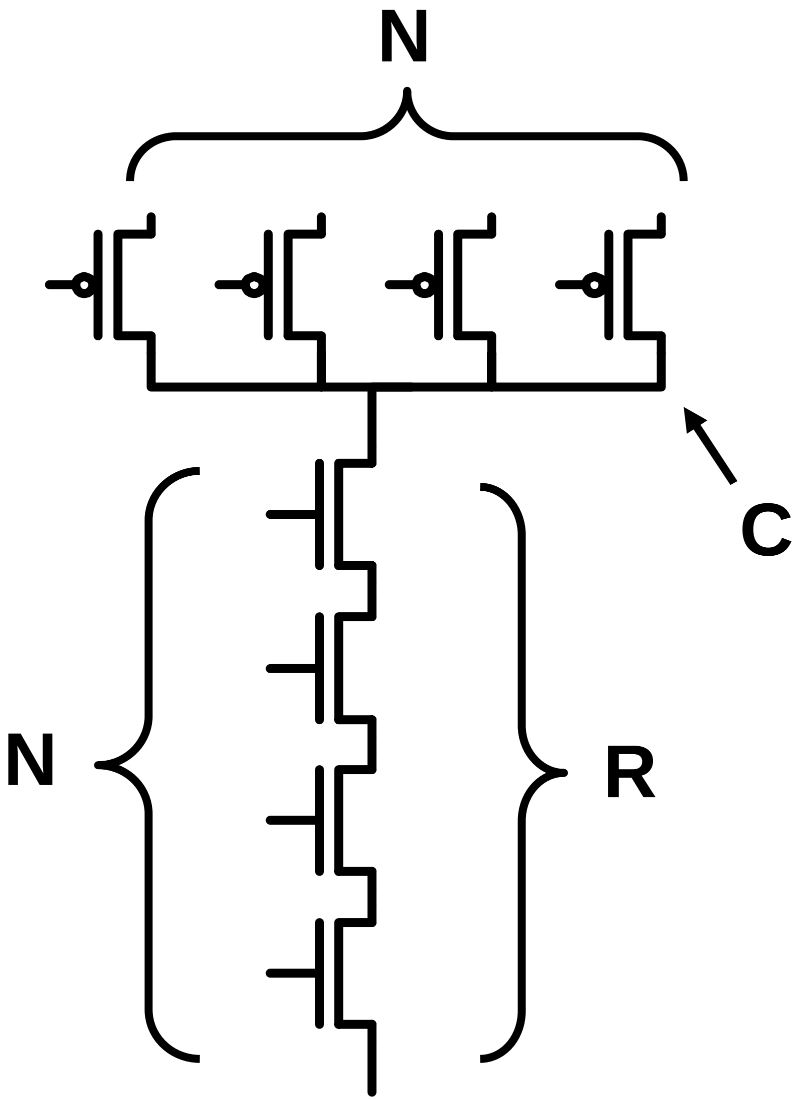




Exercise:

- Perform gate sizing of (a) for nominal drive strength equal to that of min size inverter, assume PU/PD = 3
- Determine PUN of (b)
- Perform gate sizing of (b) for same drive strength (same PU/PD)
- Compare sum of gate areas in (a) and (b). Note: area ~ width

Avoid Large Fan-In



C linear in N

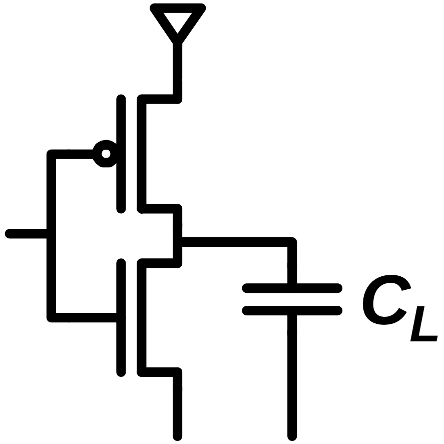
R linear in N

Delay $\propto$ RC quadratic in N

Empirical

Delay = a_1FI + a_2FI^2 + a_3FO

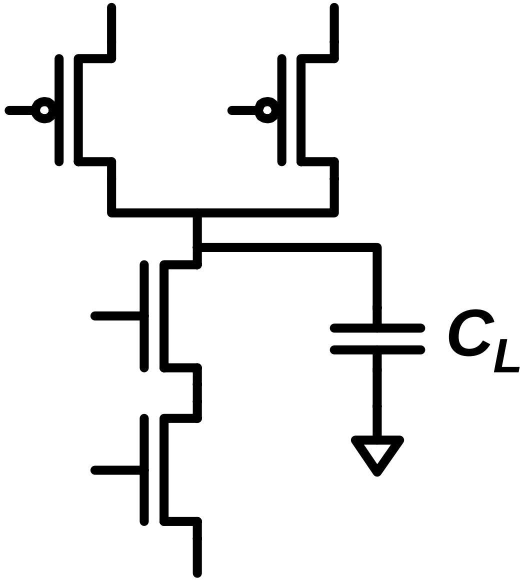
Data-Dependent Timing



$t_{PHL} = 0.69 R_N C_L$

$t_{PLH} = 0.69 R_P C_L$

You should be able to identify the transistor paths that charge or discharge C_L, and calculate resulting RC delay model, including effects of wires and fan-out

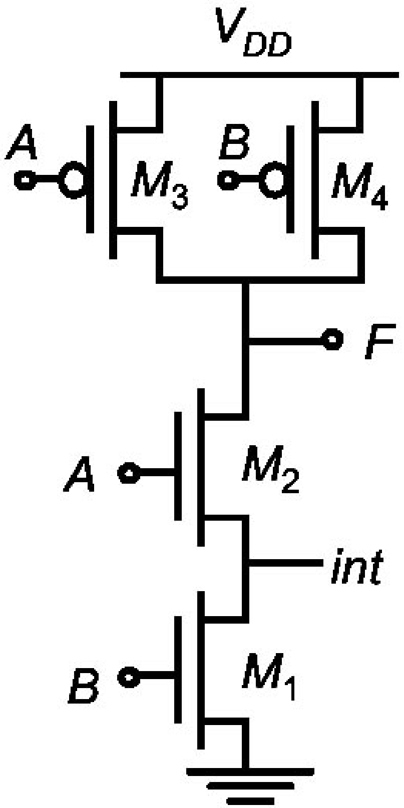


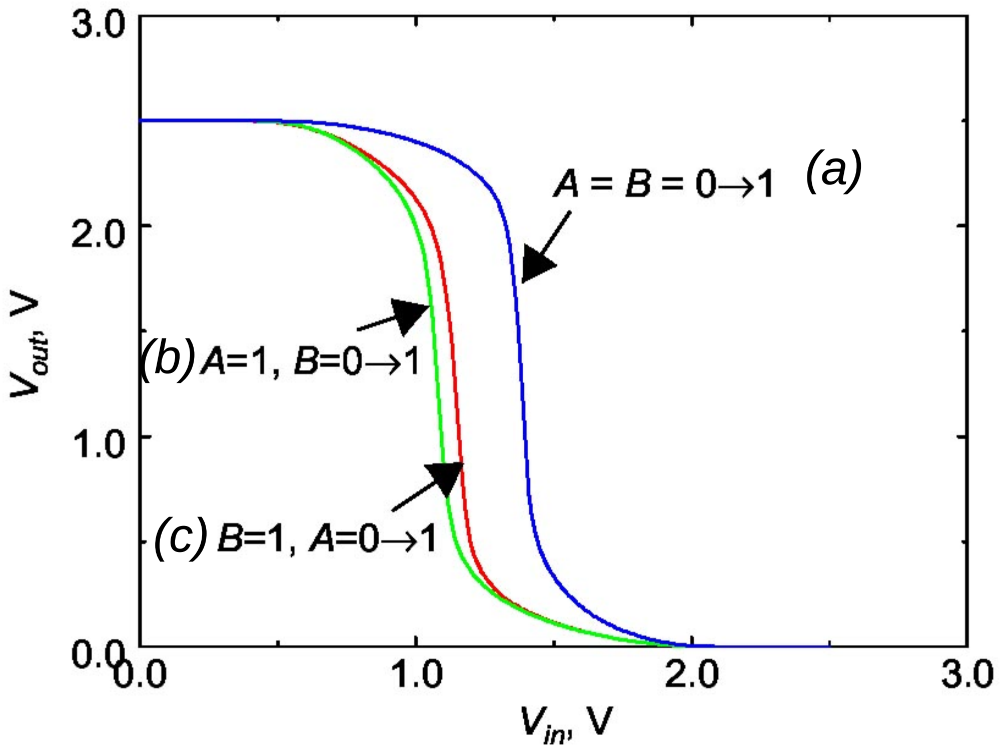
$t_{PHL} = 0.69(R_N \times 2)C_L$ **Series** connection

$t_{PLH} = 0.69 R_p C_L$ **One input** goes low

$t_{PLH} = 0.69(R_p/2)C_L$ **Two inputs** go low, **parallel** connection

Data-dependent VTC: 2nd order effects

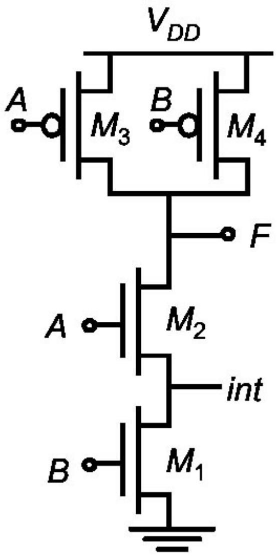


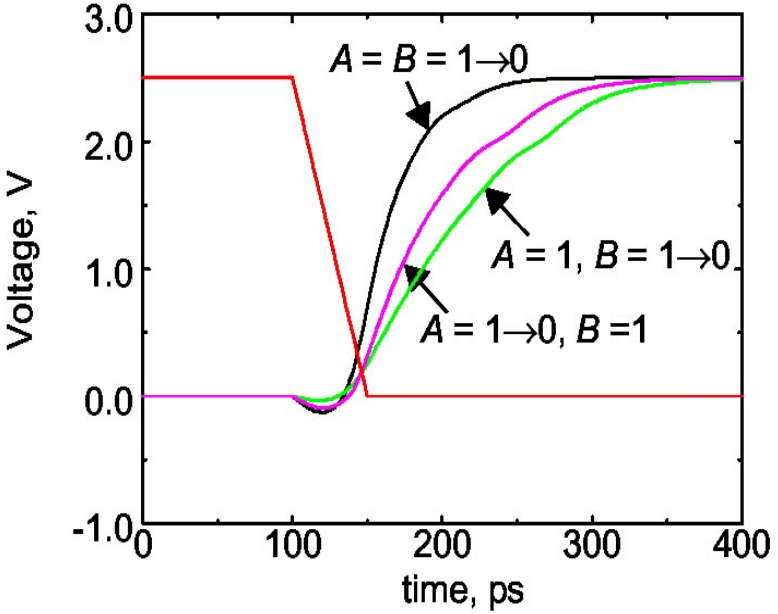


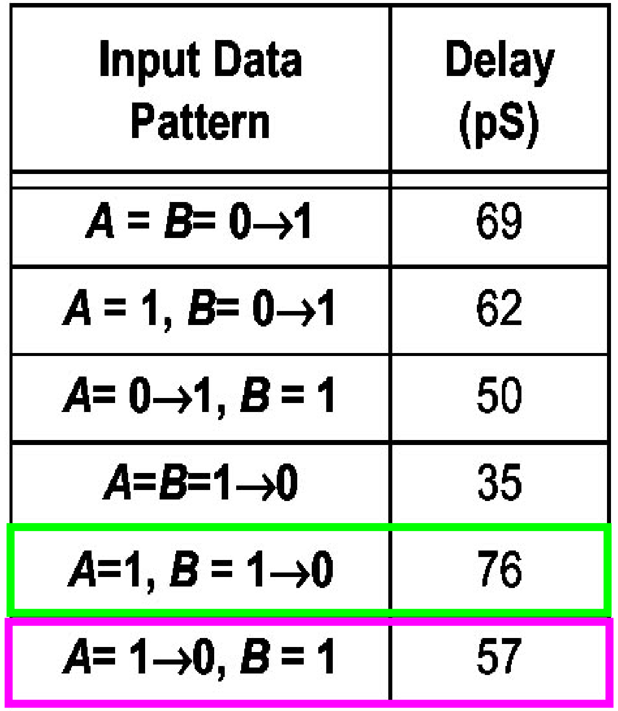
- Charge at ‘int’
- Body effect in M_2
- Short-circuit currents

- **Don’t need to be able to work with these effects**
- **But remember: there is more going on than shown by our simple, 1^{st} order model**

Data-dependent Timing (2).





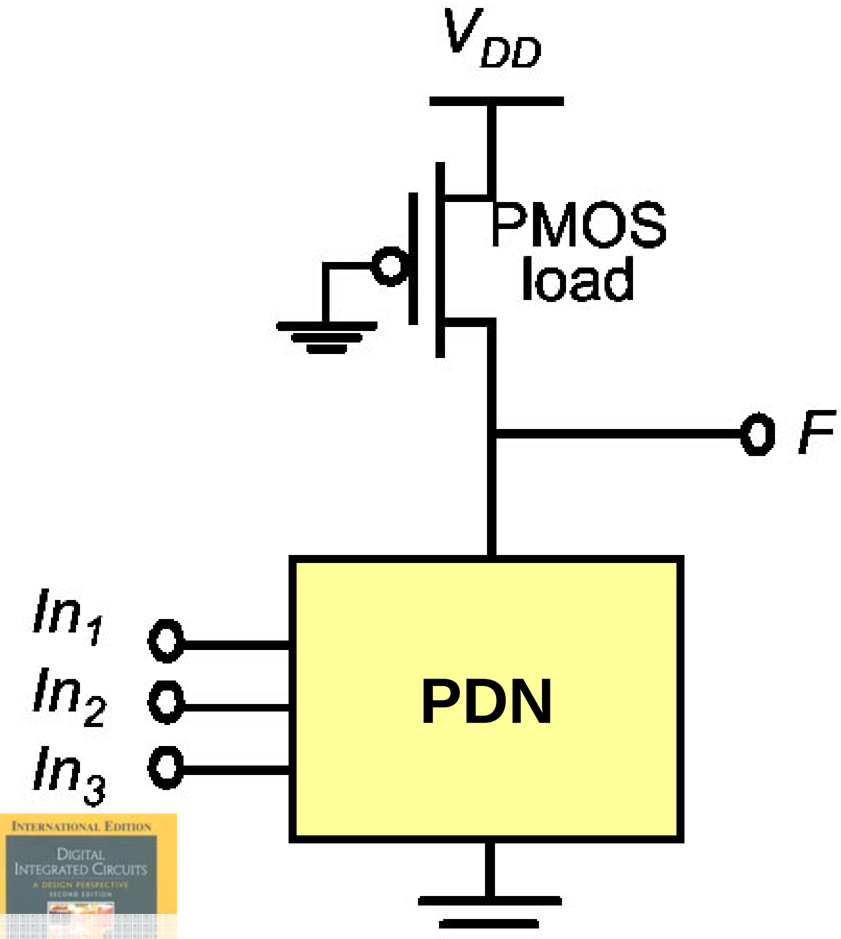


Input Data Pattern	Delay (pS)
A = *B*= 0→1	69
A = 1, *B*= 0→1	62
A= 0→1, *B* = 1	50
A=*B*=1→0	35
A=1, *B* = 1→0	76
A= 1→0, *B* = 1	57

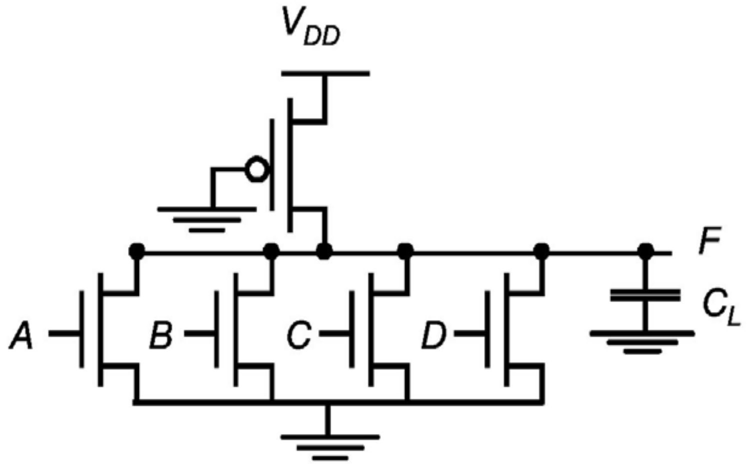
Ratioed logic

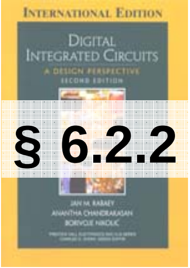
Pass transistor logic

Pseudo NMOS Ratioed Logic



- ☺ **Reduced area**
- ☺ **Reduced capacitances**
- ☹ **Increased V_{OL}**
- ☹ **Reduced noise margins**
- ☹ **Static dissipation**





§ 6.2.2

Ratioed Logic V_{OL} Computation.

Exercise: **verify these assumptions/steps**

I_{Dn} (linear) = I_{Dp} (saturation)

$$k_n\left(\left(V_{DD}-V_{Tn}\right)V_{OL}-\cancel{\frac{V_{OL}^2}{2}}\right)=k_p\left(\left(-V_{DD}-V_{Tp}\right)V_{DSAT}-\cancel{\frac{V_{DSAT}^2}{2}}\right)$$

Ignore quadratic terms (they are relatively small)

$$k_n\left(\cancel{V_{DD}-V_{Tn}}\right)V_{OL}\approx k_p\left(-\cancel{V_{DD}-V_{Tp}}\right)V_{DSAT}$$

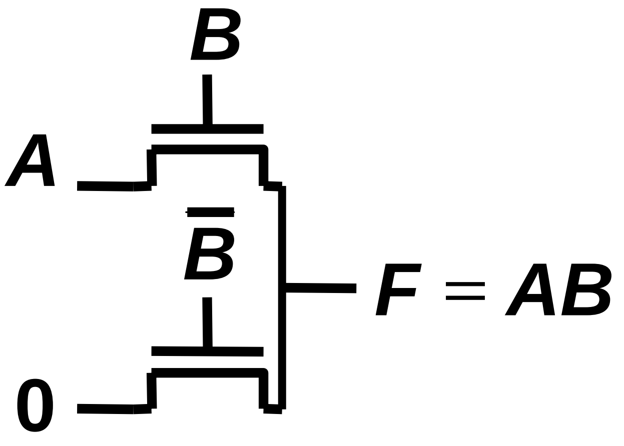
Ignore, because approximately equal

$$V_{OL}\approx\frac{k_p}{k_n}\left|V_{DSAT}\right|\approx\frac{\mu_p W_p}{\mu_n W_n}\left|V_{DSAT}\right|$$

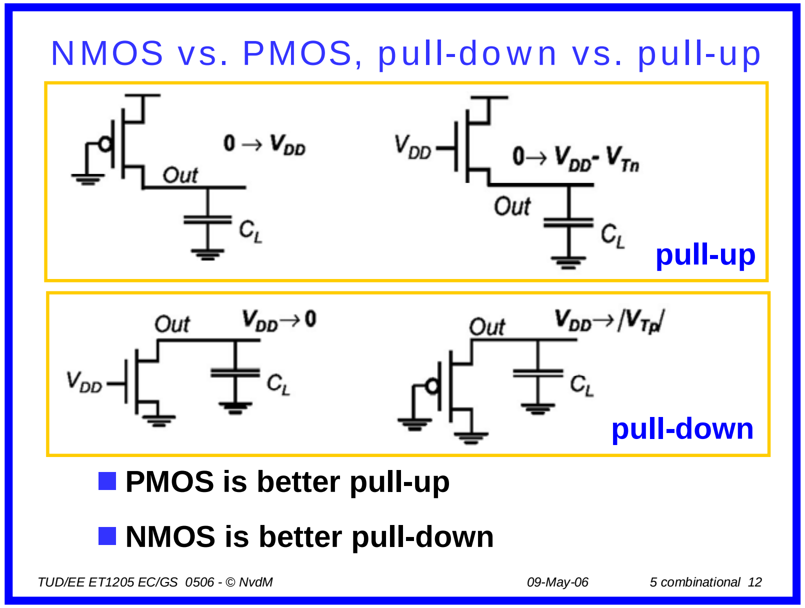
Pass-transistor and Pass-gate circuits

Pass Transistor Logic

- Save area, capacitances
- Need complementary inputs (extra inverters)



But remember:



NMOS vs. PMOS, pull-down vs. pull-up

pull-up

pull-down

- PMOS is better pull-up
- NMOS is better pull-down

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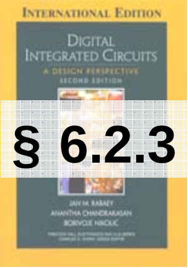
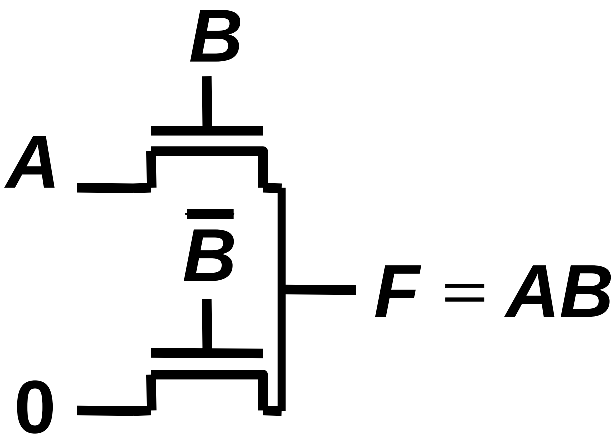


§ 6.2.3

Pass Transistor Logic

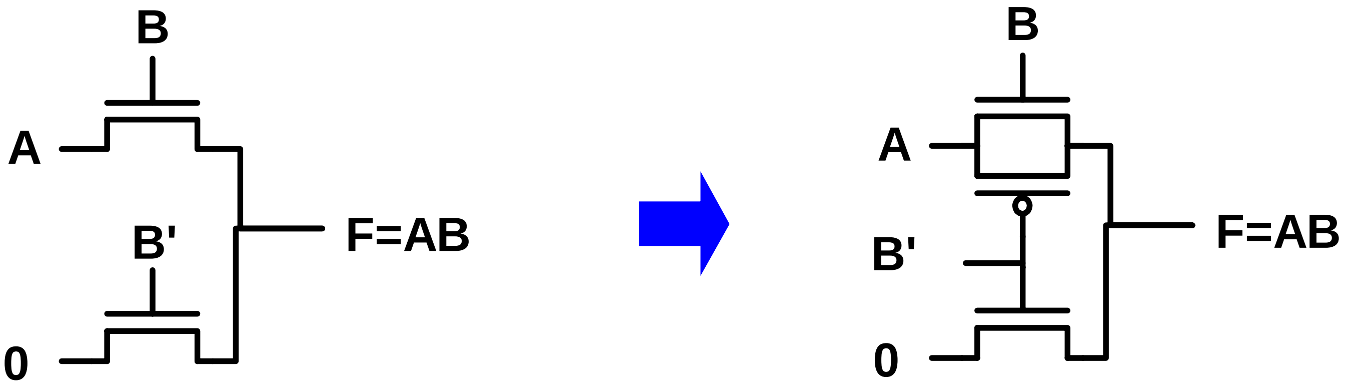


- **Save** **area, capacitances**
- **Need complementary inputs (might mean extra inverters)**
- **Reduced** **V_{OH}, noise margins**
- $V_{OH} = V_{DD} - \left(V_{Tno} + \gamma\left(\left(\sqrt{|2\phi_f| + V_{OH}}\right) - \sqrt{|2\phi_f|}\right)\right)$
- **Static dissipation** **in subsequent static inverter/buffer**
- **Disadvantages (and advantages) may be reduced by** **complementary pass gates** **(NMOS + PMOS parallel)**

Exercise: **Why is there static dissipation in next conventional gate?**

Pass Gates

- **Remedy: use an N-MOS and a P-MOS in parallel**

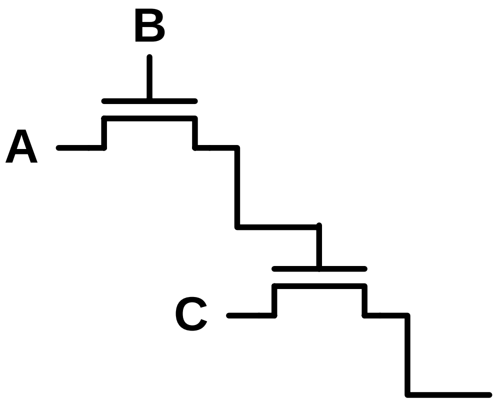


- **Pass gates eliminate some of the disadvantages of simple pass-transistors**
- **But also some of the advantages**
- **Design remains a trade-off!**

Pass-gate a.k.a. Transmission-gate

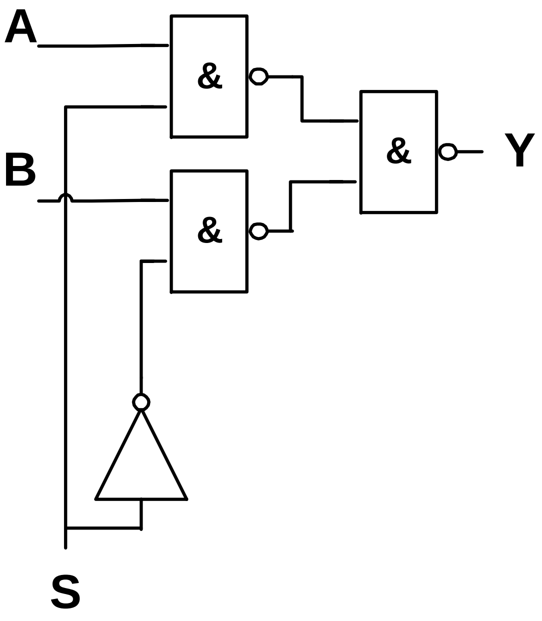
Exercise

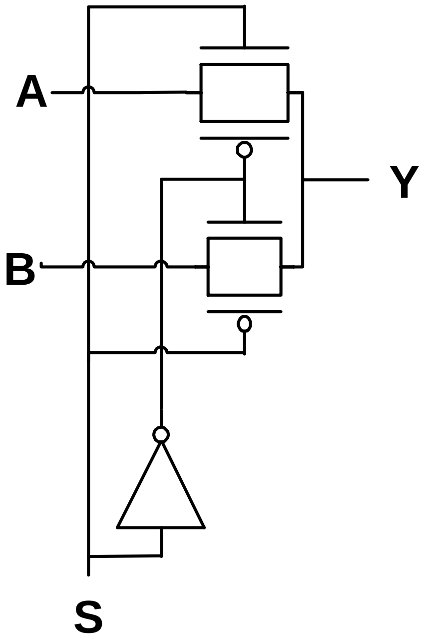
- **Discuss what happens when you connect the output of a single pass-transistor (not a pass-gate) to the input of another pass-transistor stage (i.e. the gate of another pass-transistor). Why should you never use such a circuit?**



Pass Transistor Logic.

- **Most typical use: for multiplexing, or path selecting**
- **Assume in circuit below it is required to either connect A or B to Y, under control by S**
- **Y = AS + BS' (S' is easier notation for S-bar = S-inverse = $\overline{S}$)**
- **Y = ((AS)' (BS)')' allows realization with 3 NAND-2 and 1 INV: 14 transistors**
- **Pass gate needs only 6 (or 8) transistors (see also Katz, section 4.2)**





Summary

- **Conventional Static CMOS basic principles**
- **Complementary static CMOS**
  - **Complex Logic Gates**
  - **VTC, Delay and Sizing**
- **Ratioed logic**
- **Pass transistor logic**