MODULE 8

MODULARITY

Course Material for Modularity

Chapter 11

P = primair, I = Illustratie, O = overslaan

P	11.1	Introduction	560
P	11.2	Datapaths in Digital Processor Architectures	560-561
P	11.3	The Adder	561
P	11.3.1	The Binary Adder: Definitions	561-564
P	11.3.2	The Full Adder: Circuit Design Consideration	564-578
I	11.3.2	Manchester Carry Chain Adder	568-570
O	11.3.3	The Binary Adder: Logic Design Considerations	571-586
P	11.4	The Multiplier	586
P	11.4.1	The Multiplier: Definitions	586-587
P	11.4.2	Partial-Product Generation	587-589
P	11.4.3	Partial-Product Accumulation	589-592
O		The Tree Multiplier	592-593
I	11.4.4	Final Addition	593-594
P	11.5	The Shifter	595
P	11.5.1	Barrel Shifter	595-596
P	11.5.2	Logarithmic Shifter	596
O	11.6	Other Arithmetic Operators	596-600
O	11.7	Power and speed trade-offs	600-618
P	11.8	Perspective: Design as a Trade-off	618-619
P	11.9	Summary	619-620

Outline

- Background on Modular Design
  - Hierarchy, reuse, regularity
  - Architecture, bit-slicing
- Adder Design
- Multiplier Design
- Shifter Design
- Layout Strategies (regularity)
- Design as a Trade-Off

contains a lot of reminders

Get further appreciation of some system level design issues

Adder Design

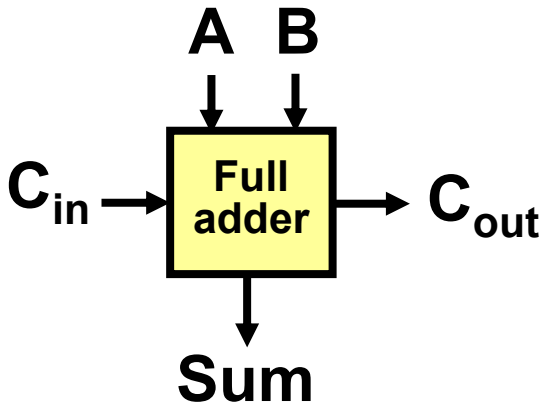
- Adders are fundamental building blocks
  - Digital filtering (DSP): MP3 en/decoder, GSM, GPS, …
  - Data processing
  - Multiplication
  - Address arithmetic
  - …
- Good performance is key
- Many architectures
  - ✓ Static adder
  - × Dynamic adder (Manchester Carry Chain)
  - × Pipelined Adder
  - × Carry-Bypass, Carry Lookahead, Carry Select
  - × …
- Design trade-offs, optimization
  - ✓ Architecture level
  - ✓ Logic level
  - ✓ Circuit level
  - ✓ Layout level

Also see Digital Systems – Katz § 5.5

Most effective

Least effective

Full-Adder



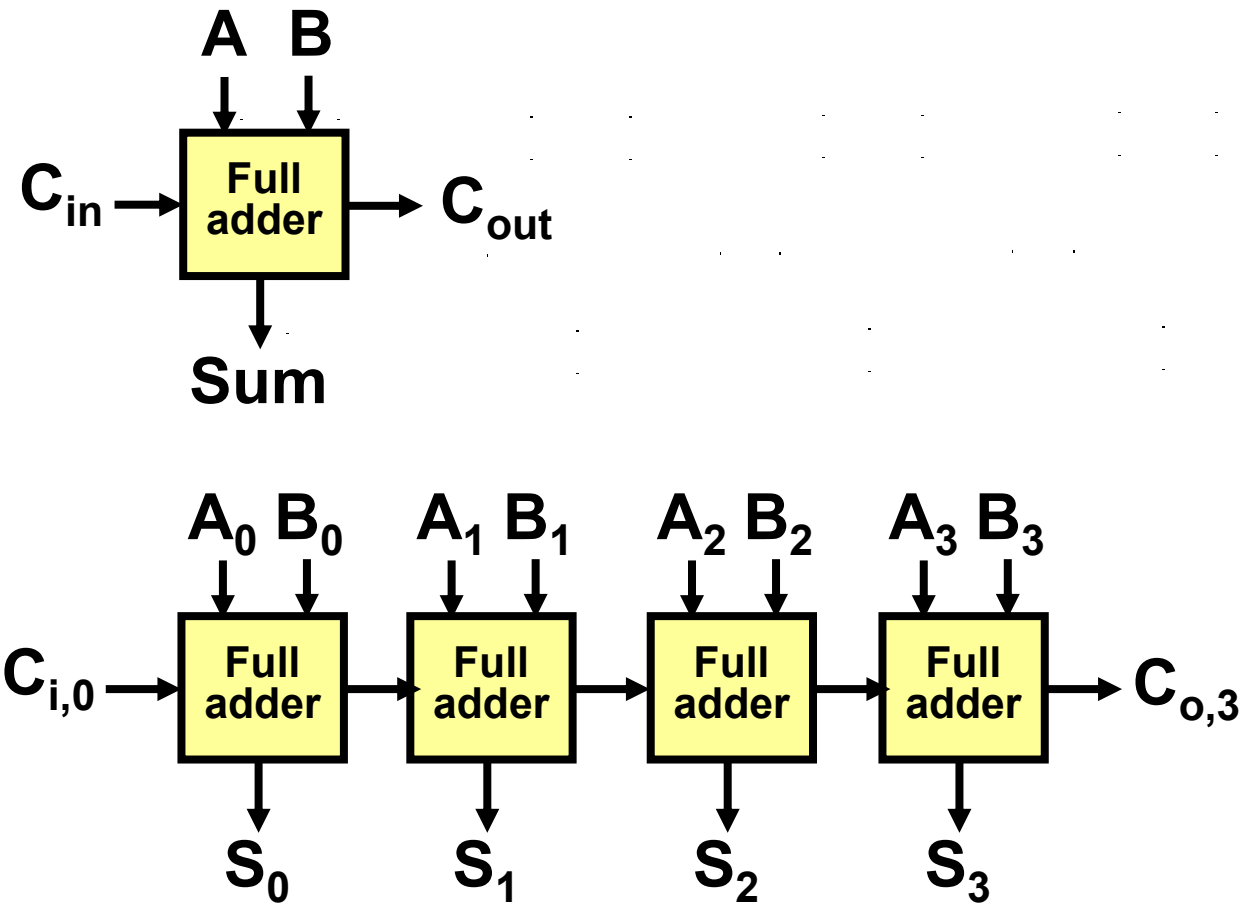
Add three one-bit numbers

Equivalently: count # 1's in A, B, C_i

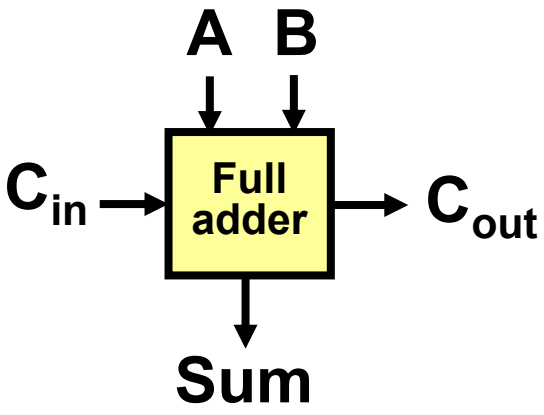
Output as 2-bit number <C_oS>

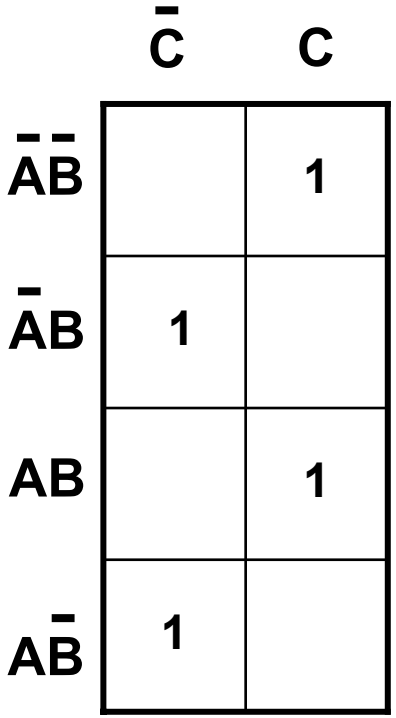
A	B	C_i	S	C_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The Ripple-Carry Adder

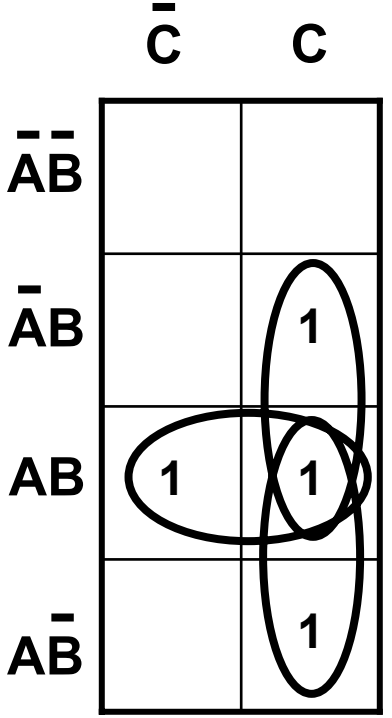


The Binary Adder





(a) SUM



(b) CARRY

AB + BC + AC

$$S = A \oplus B \oplus C_i$$
$$= \bar{A}\bar{B}C_i + \bar{A}B\bar{C}_i + A\bar{B}\bar{C}_i + ABC_i$$
$$C_0 = AB + BC_i + AC_i$$

AND-OR expressions for sum and carry

Naïve Complementary CMOS Implementation

- **Use DeMorgan to convert AND-OR expressions for *SUM* and *CARRY* to NAND-NAND**
- **$PQ + RS = \overline{\overline{PQ}\ \overline{RS}}$ (example)**

Q: What is advantage of NAND-NAND over NOR-NOR? Consider drive strength vs. area

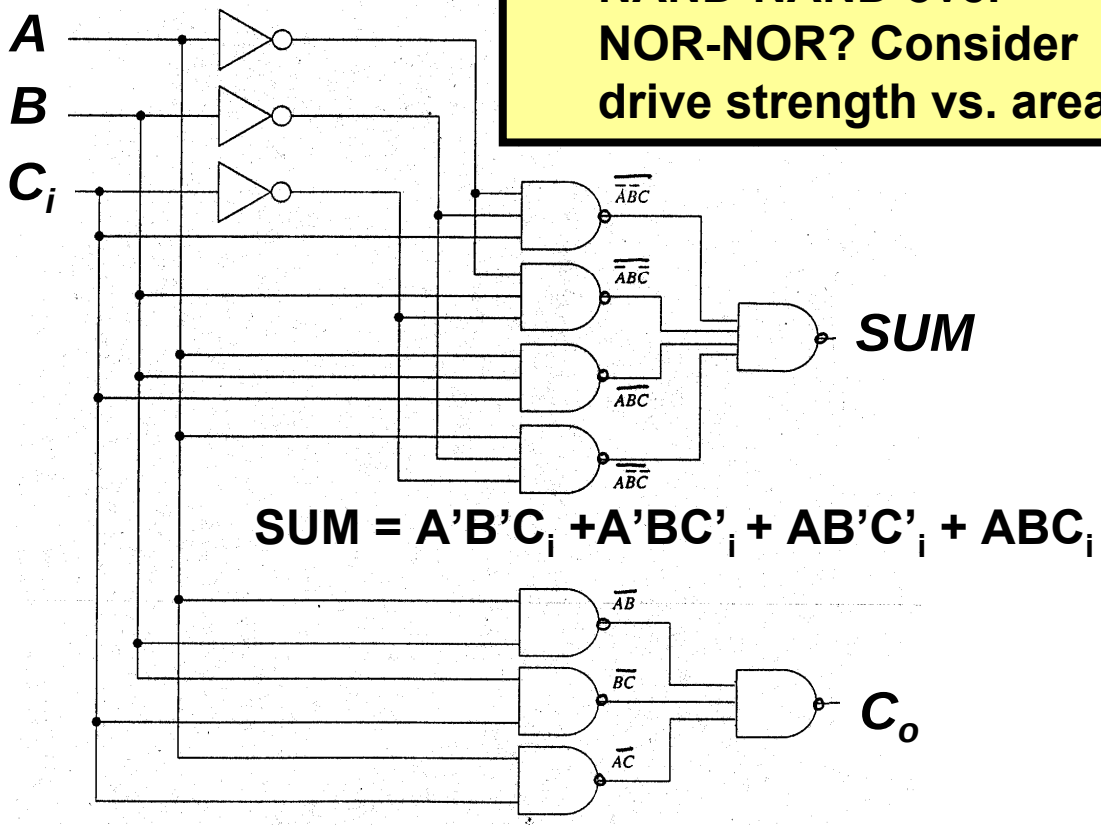
Transistor Count

3 × INVERT

3 × NAND-2

5 × NAND-3

1 × NAND-4



SUM = A'B'C_i +A'BC'$_i$ + AB'C'$_i$ + ABC$_i$

Can do better using more clever boolean factoring, but…

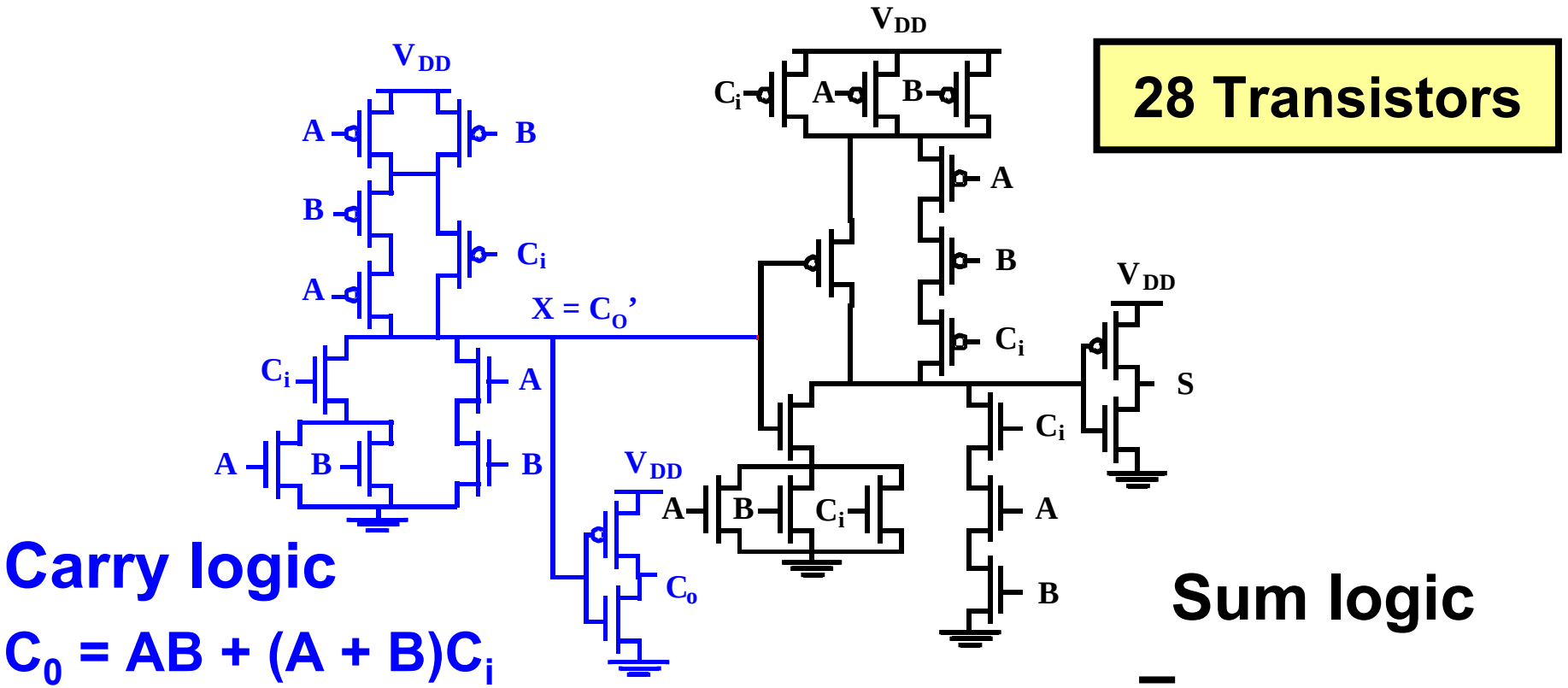
Full-Adder Boolean Factoring

$$S = A\bar{B}\bar{C}_i + \bar{A}B\bar{C}_i + \bar{A}\bar{B}C_i + ABC_i$$
$$= ABC_i + \bar{C}_0(A + B + C_i)$$

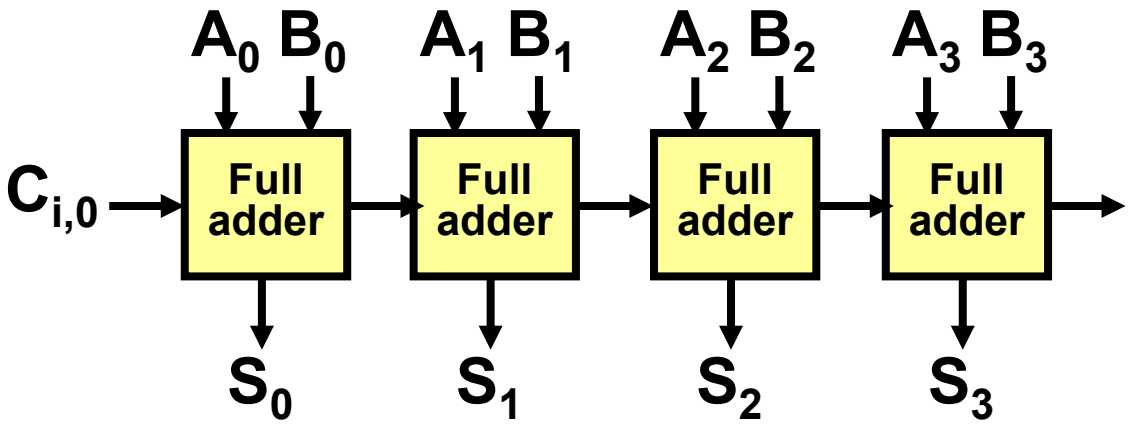
$$C_0 = AB + BC_i + AC_i$$
$$= AB + (A + B)C_i$$

A	B	C_i	S	C_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Improved Complementary Static Full Adder

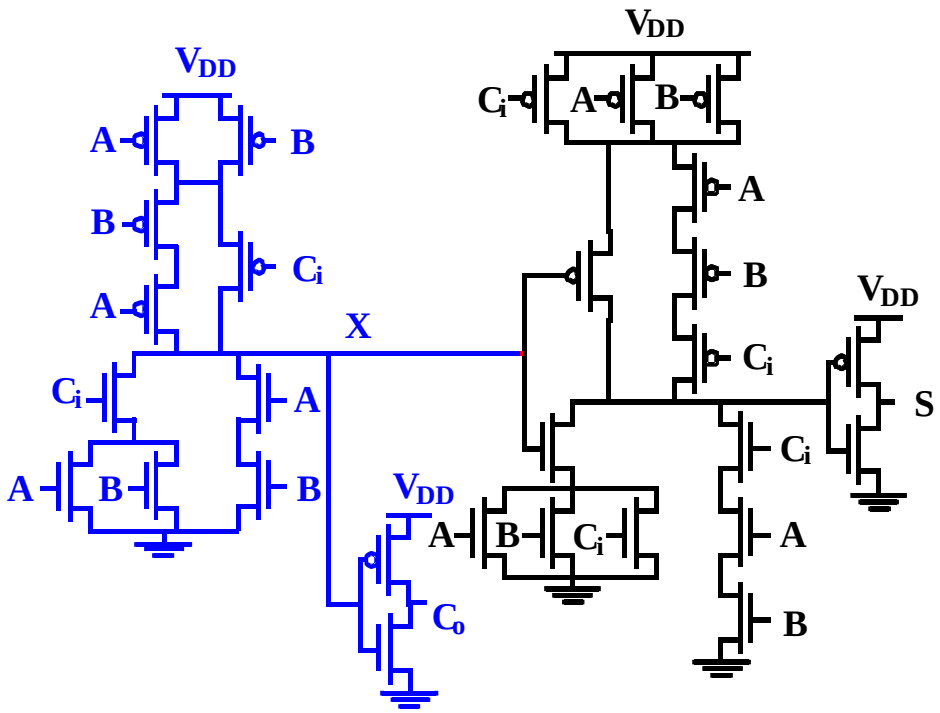


Ripple-Carry Adder Delay



- Worst case delay through full carry path (ripple carry)
- Linear with the number of bits (N)
- $T_{adder} = (N-1)\ T_{carry} + Max\ (T_{carry}, T_{sum})$
- $T_{adder} = O(N)$ "T_{adder} is of Order N" *means linear with N*
- **Goal: Make the fastest possible carry path circuit**

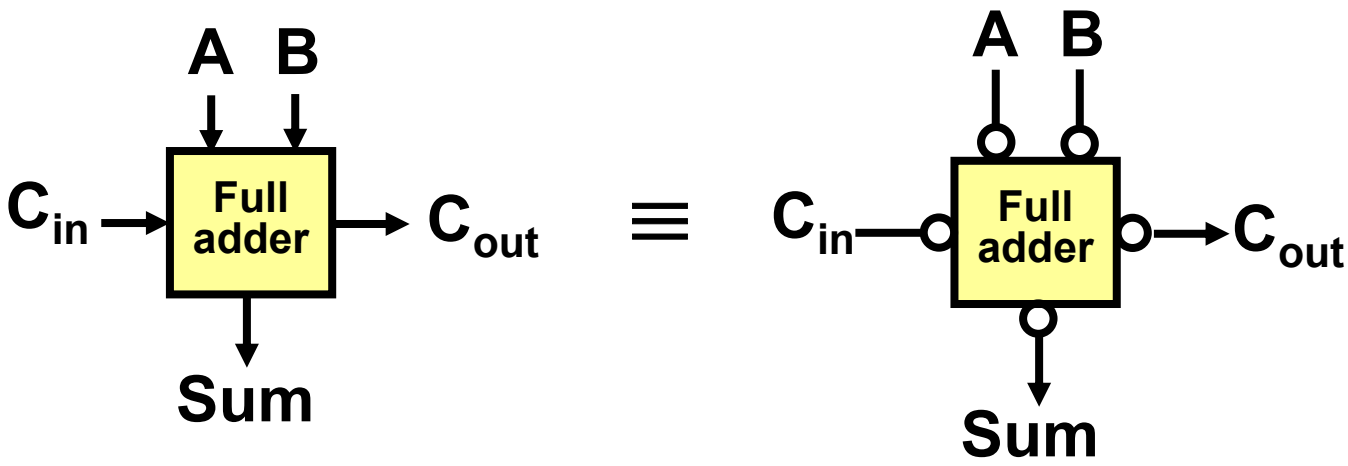
Adder Evaluation



Carry Chain:

- Long PMOS chains
- High C at X
- 2 (inverting) stages

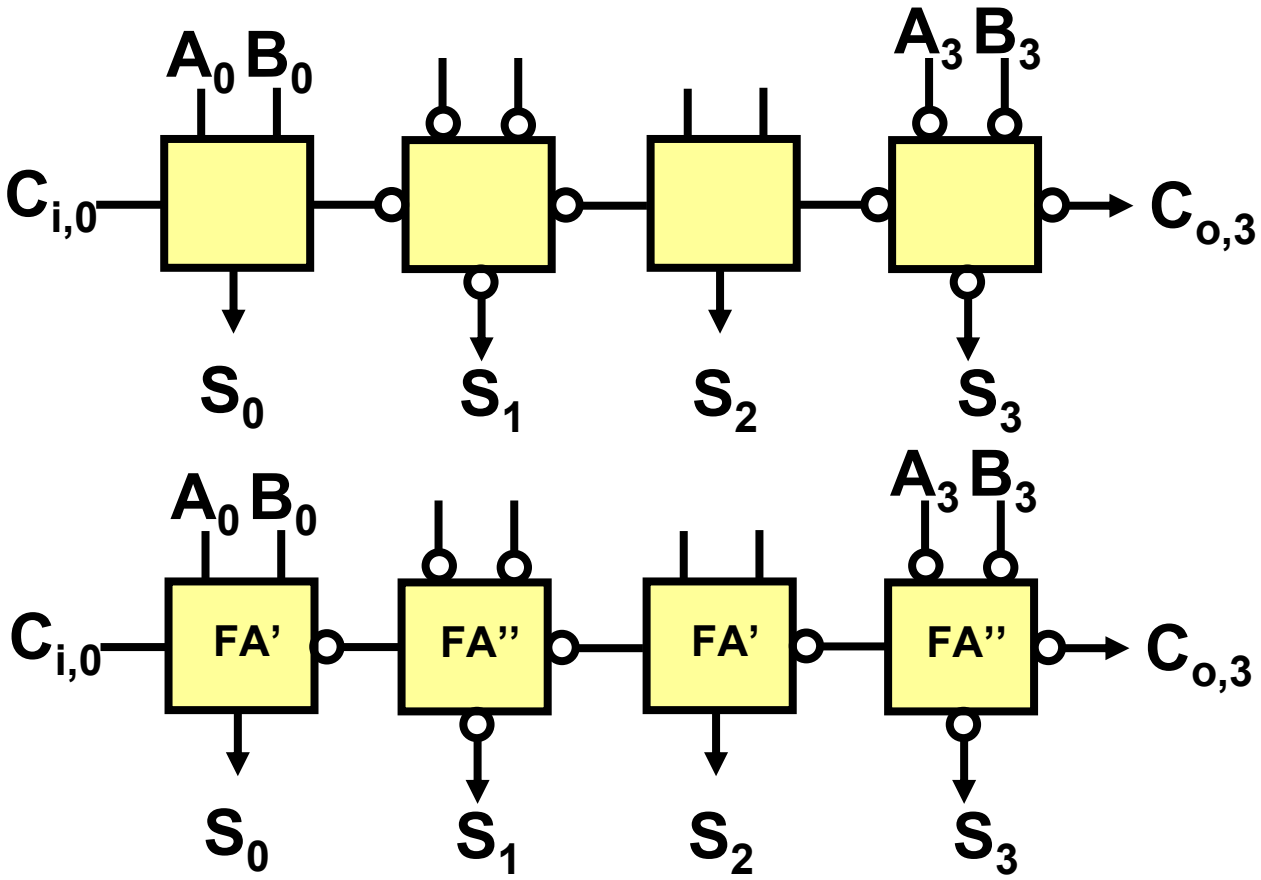
Inversion Property



$$\overline{S}(A,B,C_i) = S(\overline{A},\overline{B},\overline{C_i})$$

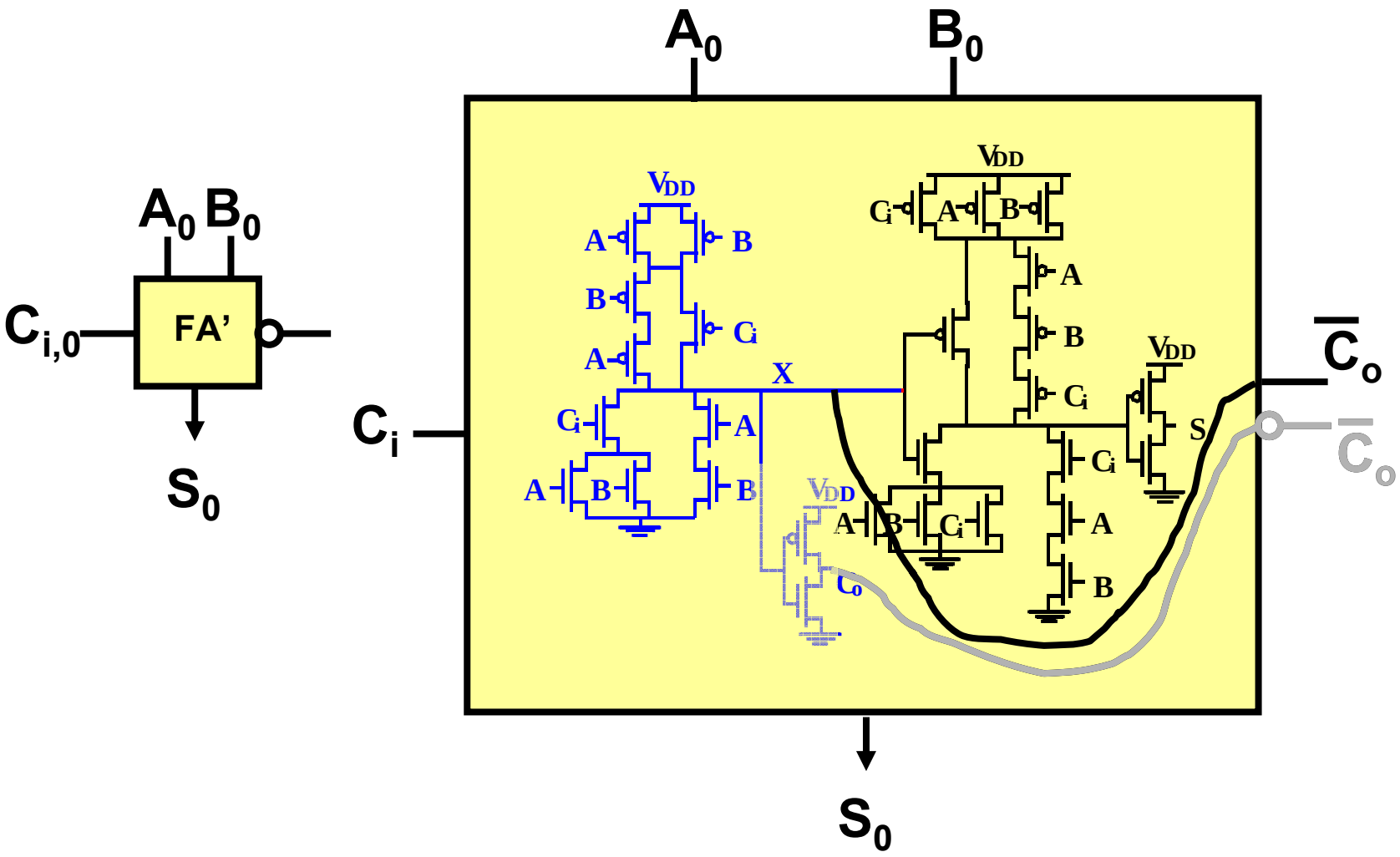
$$\overline{C_0}(A,B,C_i) = C_0(\overline{A},\overline{B},\overline{C_i})$$

Minimize Critical Path by Reducing Inverting Stages



- Can eliminate inverter in carry from each FA
- Need 2 different types of cells, but both with inverting carry – will require only one stage per bit

Eliminate Inverter In Carry



Multiplier Design

- Multipliers are fundamental building blocks too
  - Digital filtering (DSP): MP3 en/decoder, GSM, GPS, …
  - Data processing
  - Address arithmetic
  - …
- Good performance is key, often they are the performance bottleneck
- Multipliers are complex arrays of adders
- Many architectures
  - ✓ Basic Array Multiplier
  - ✗ Bit-serial
  - ✗ Booth-encoding multiplier
  - ✗ Baugh-Wooley multiplier
  - ✗ Wallace tree multiplier
  - ✗ …
- Design trade-offs, optimization
  - ✓ Architecture level, Logic level, Circuit level, Layout level

The Binary Multiplication

$$X = \sum_{i=0}^{M-1} X_i 2^i \qquad Y = \sum_{j=0}^{N-1} Y_j 2^j$$

$$Z = X \times Y = \sum_{k=0}^{M+N-1} Z_k 2^k$$

$$= \left(\sum_{i=0}^{M-1} X_i 2^i \right) \left(\sum_{j=0}^{N-1} Y_j 2^j \right)$$

$$= \sum_{i=0}^{M-1} \left(\sum_{j=0}^{N-1} X_i Y_j 2^{i+j} \right)$$

ADD

AND

SHIFT

Example: 42 x 11 = 462

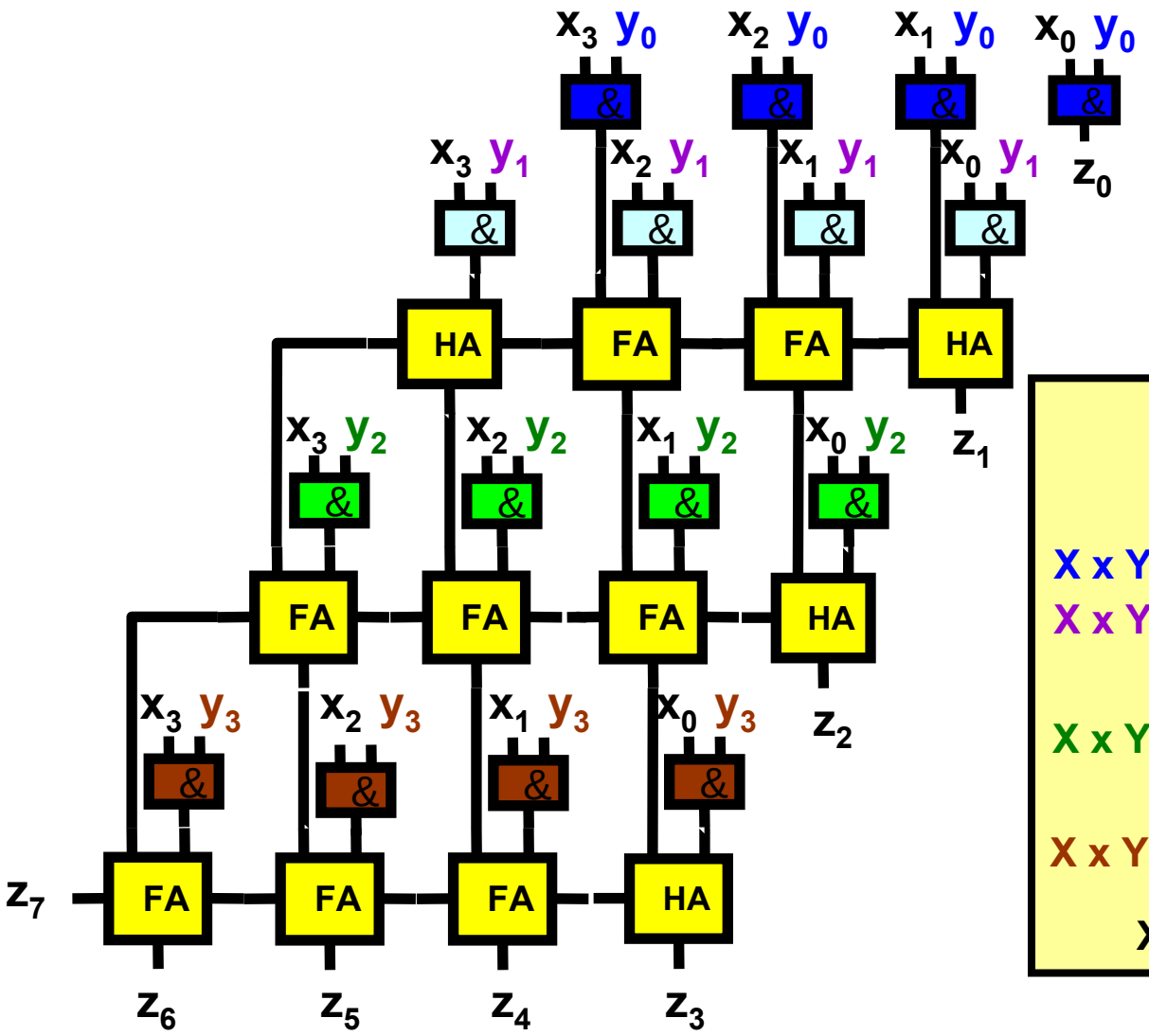
```
  1 0 1 0 1 0
      1 0 1 1  x
  -------------
  1 0 1 0 1 0
1 0 1 0 1 0
0 0 0 0 0 0
1 0 1 0 1 0
-------------------  +
1 1 1 0 0 1 1 1 0
```

Each partial product formed by bitwise AND operation

Partial products are shifted before being added

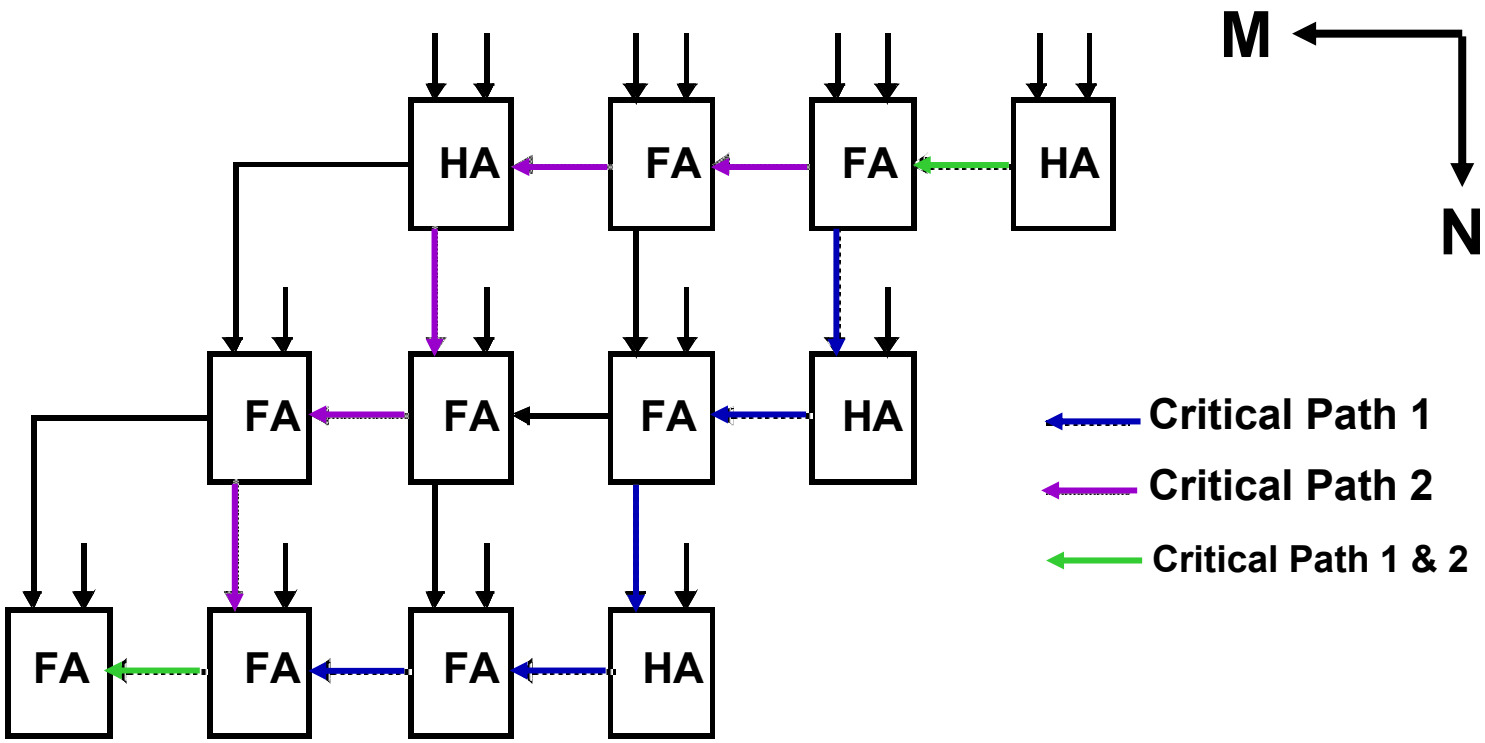
- **Conclusion:** similar to decimal multiplication
- $$X_{10} \times Y_{10} = \sum_{i=0}^{M-1} \left(\sum_{j=0}^{N-1} X_i Y_j 10^{i+j} \right)$$

The Array Multiplier



X	1 0 1 0	
Y	1 0 1 1	
		x
X x Y_0 x 2^0	1 0 1 0	
X x Y_1 x 2^1	1 0 1 0	+
	0 1 1 1 0	
X x Y_2 x 2^2	0 0 0 0	+
	1 0 1 1 1 0	
X x Y_3 x 2^3	1 0 1 0	+
X x Y	1 0 0 1 1 1 0	

The MxN Array Multiplier — Critical Path



- $t_{mult} \approx [(M - 1) + (N - 2)]t_{carry} + (N - 1)t_{sum} + t_{and}$
- **Requires comparable carry and sum delays**

Adder Cells in Array Multiplier

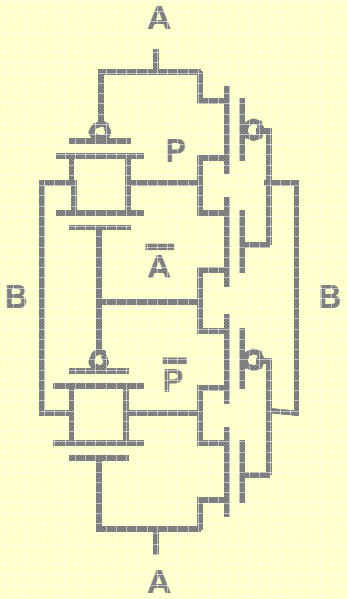
Identical Delays for Carry and Sum

A	B	C_i	S	C_o	*Carry status*
0	0	0	0	0	delete
0	0	1	1	0	delete
0	1	0	1	0	propagate
0	1	1	0	1	propagate
1	0	0	1	0	propagate
1	0	1	0	1	propagate
1	1	0	0	1	generate
1	1	1	1	1	generate

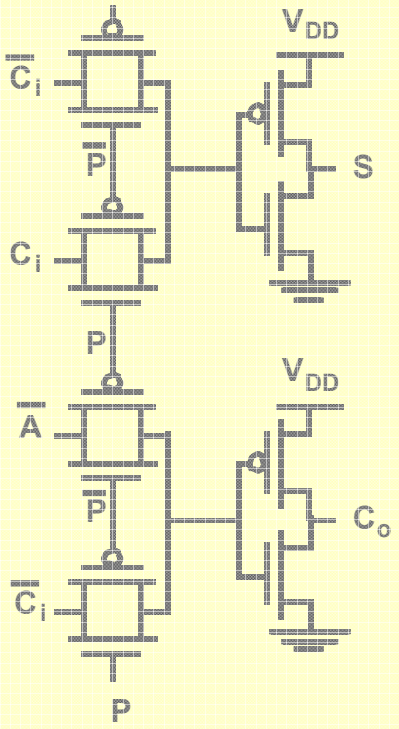
$P = A \oplus B$

If $P = 1$ then $S = \overline{C}_i$, $C_o = C_i$

If $P = 0$ then $S = C_i$, $C_o = A$

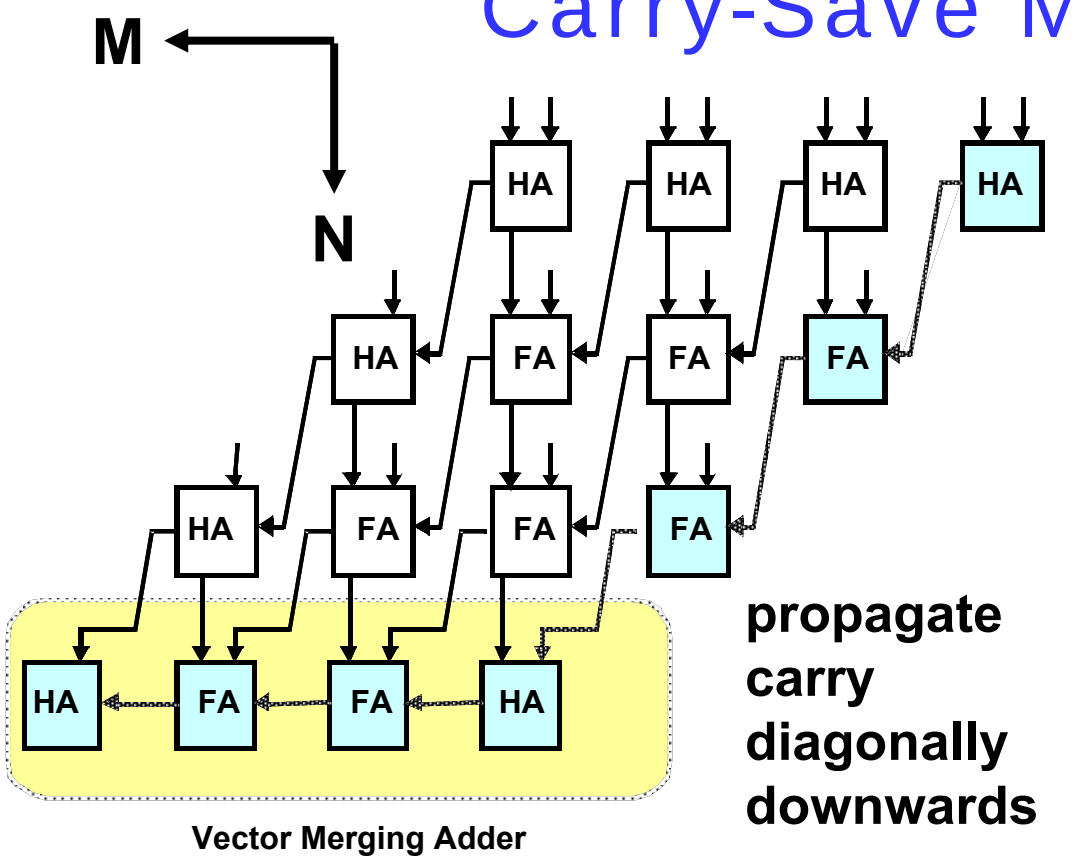


2 x transmission gate XOR for P, $\overline{P}$ (Fig 11.17)

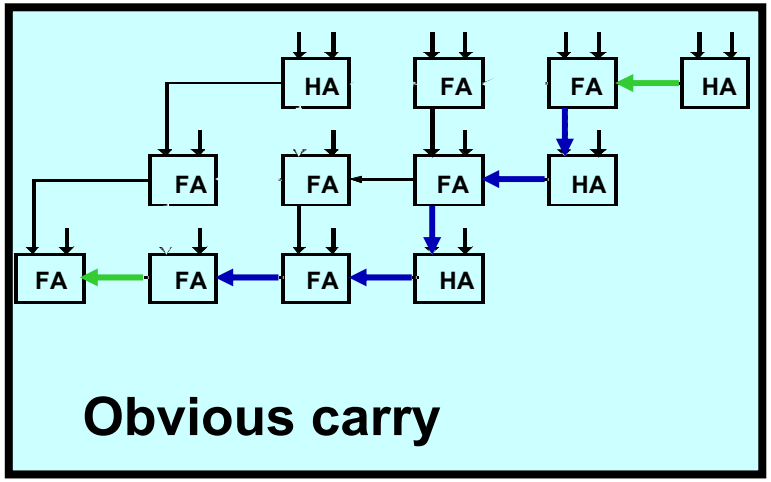


Multiplexers for S, C_o

Carry-Save Multiplier

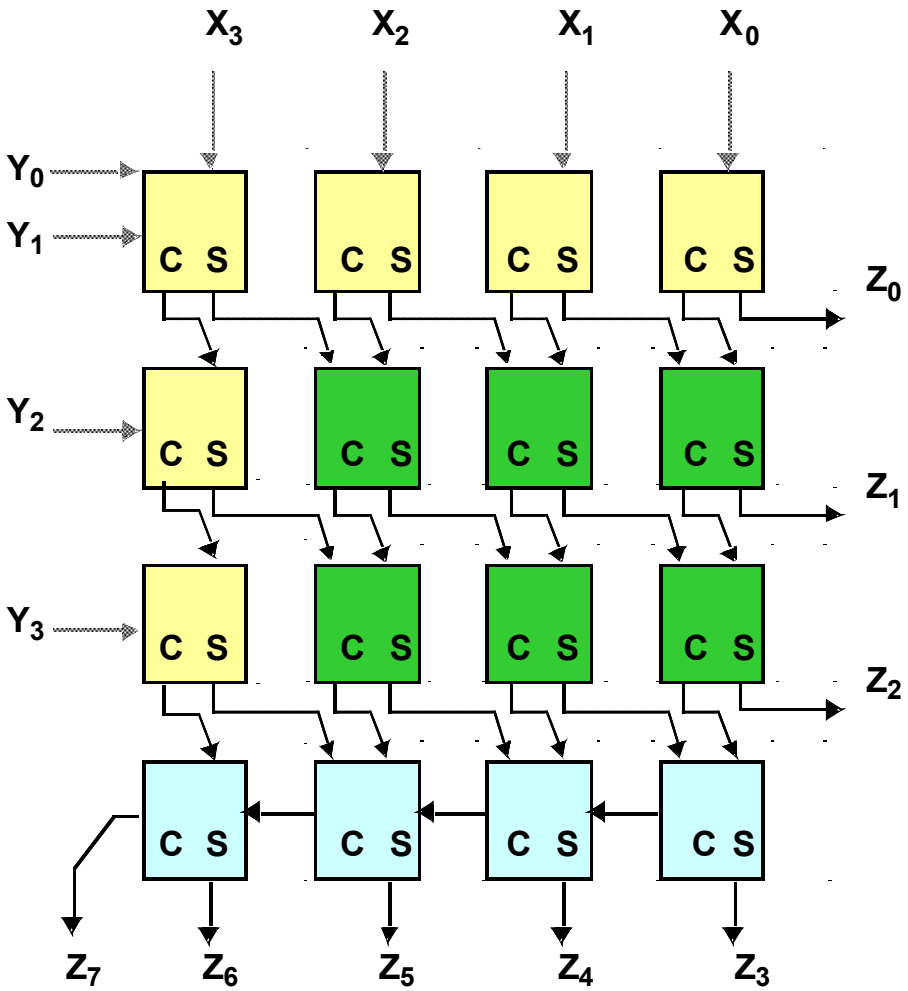


propagate carry diagonally downwards



- $t_{mult} \approx (N - 1)t_{carry} + t_{and} + t_{merge}$ (assuming $t_{add} \approx t_{carry}$)
- Use fastest possible adder for final vector merging
- Will be larger, use more power, etc, but need only one row!

Multiplier Floorplan



HA Multiplier Cell

FA Multiplier Cell

Vector Merging Cell

X signals are routed vertically across each column ("broadcast")

Y signal are broadcasted horizontally across rows

Regularity!

Multipliers — Summary.

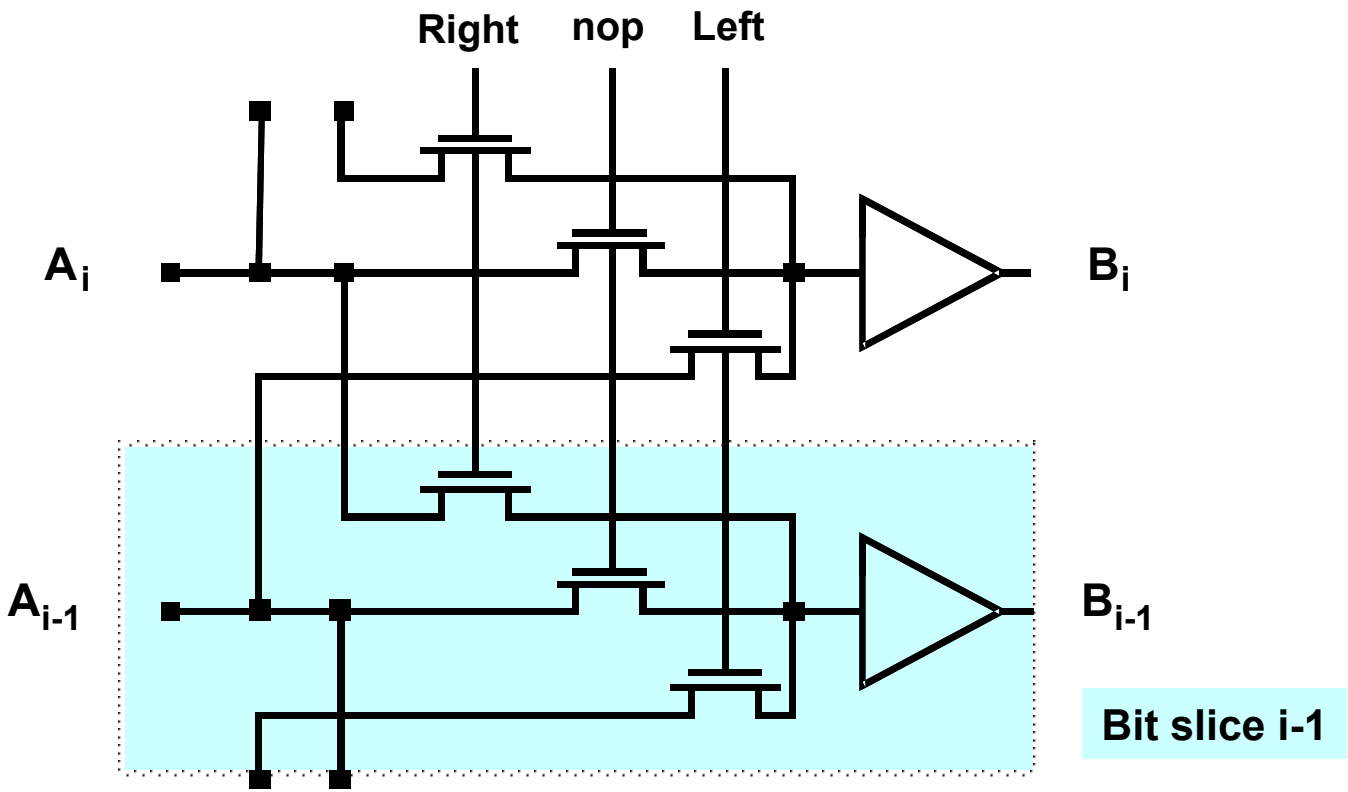
- **Optimization Goals Different Vs Binary Adder**
- **Once Again: Identify Critical Path**
- **Other possible techniques**
  - **- Logarithmic versus Linear (Wallace Tree Mult)**
  - **- Data encoding (Booth)**
  - **- Pipelining**

GLIMPSE AT SYSTEM LEVEL OPTIMIZATION

Shifter Design

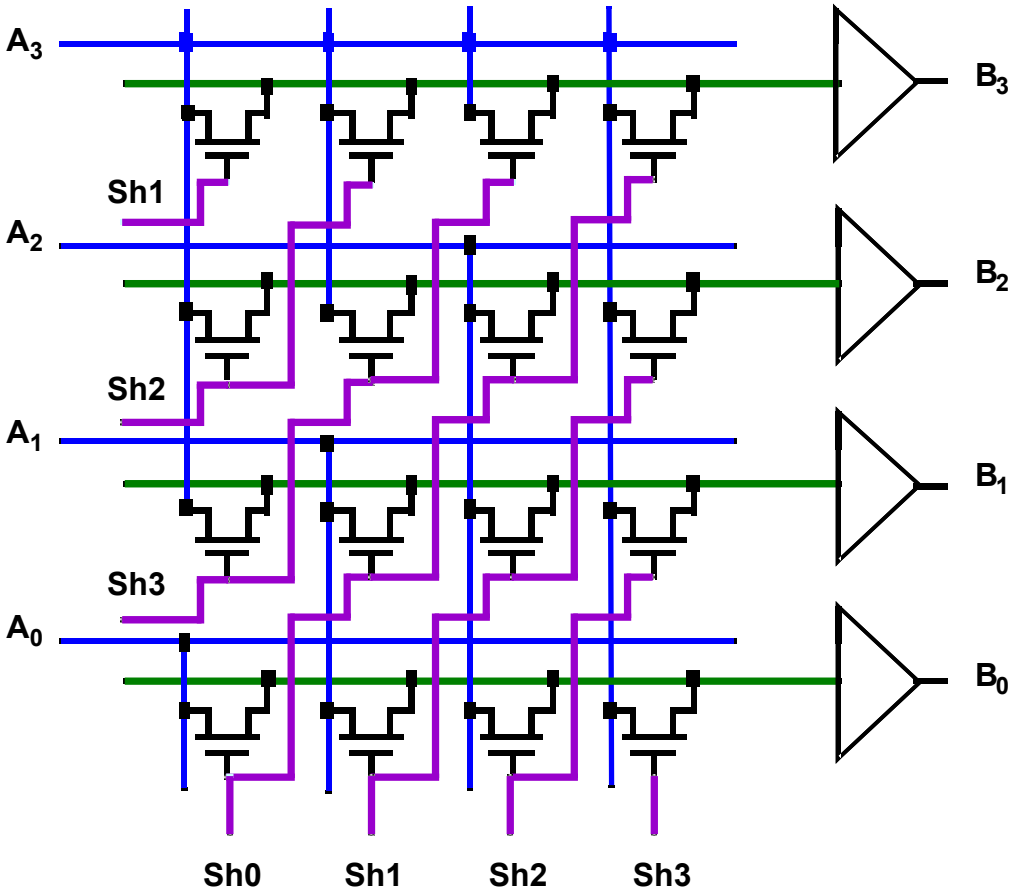
- Shifters are fundamental building blocks too
  - Floating point units
  - Scalers
  - Multiplication by constant numbers (add and shift)
  - …
- Constant shifting is only interconnect
- Programmable shifting requires active circuitry
- Usually dominated by interconnect
- Architectures
  - Barrel Shifter
  - Logarithmic Shifter
  - …
- Design trade-offs, optimization
  - Architecture level, Logic level, Circuit level, Layout level
  - Simpler compared to Adder, Multiplier, hence less rewarding
- Good example of pay-off of structural design

The Binary Shifter



- Multibit shifters by cascading
- M stages for M-bit shift
- Complex and slow for larger M
- More structured approach needed

The Barrel Shifter

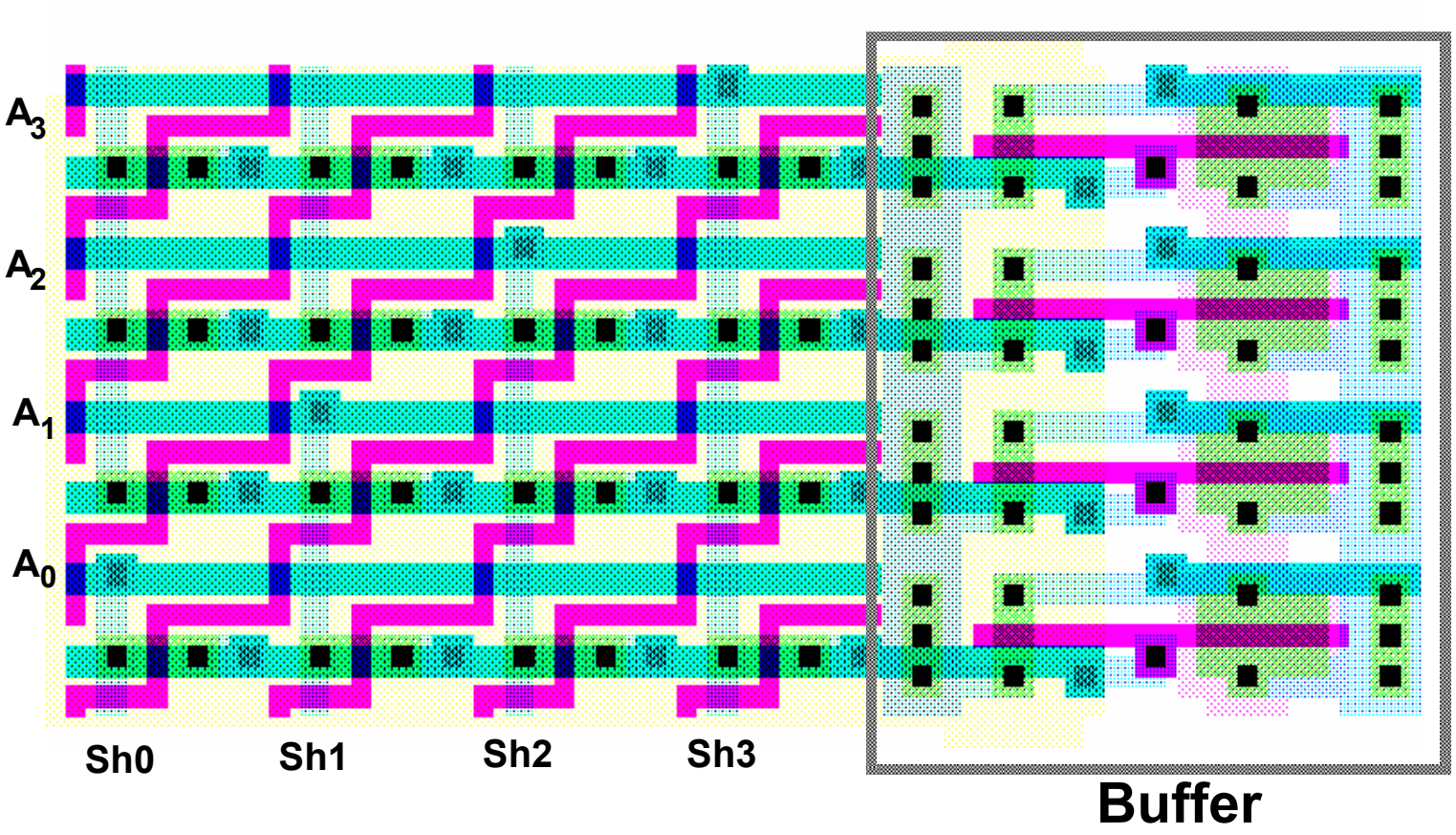


- Input data wire
- Output data wire
- Control wire

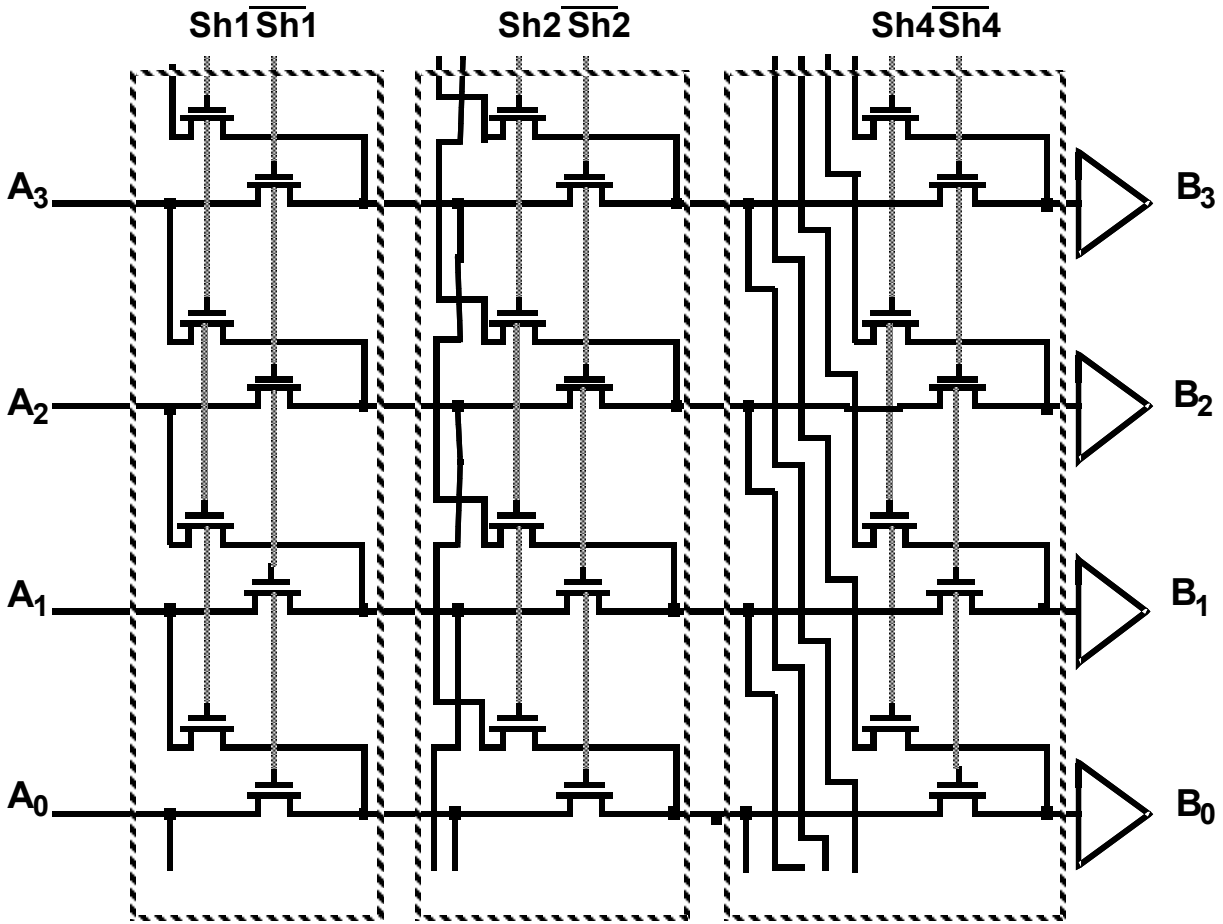
- Shift 1-N positions
- One control bit high

- Need M stages for M-bit shift
- Signal passes only one pass-transistor => delay?
- Area Dominated by Wiring, not (always) by # transistors

4x4 Barrel Shifter

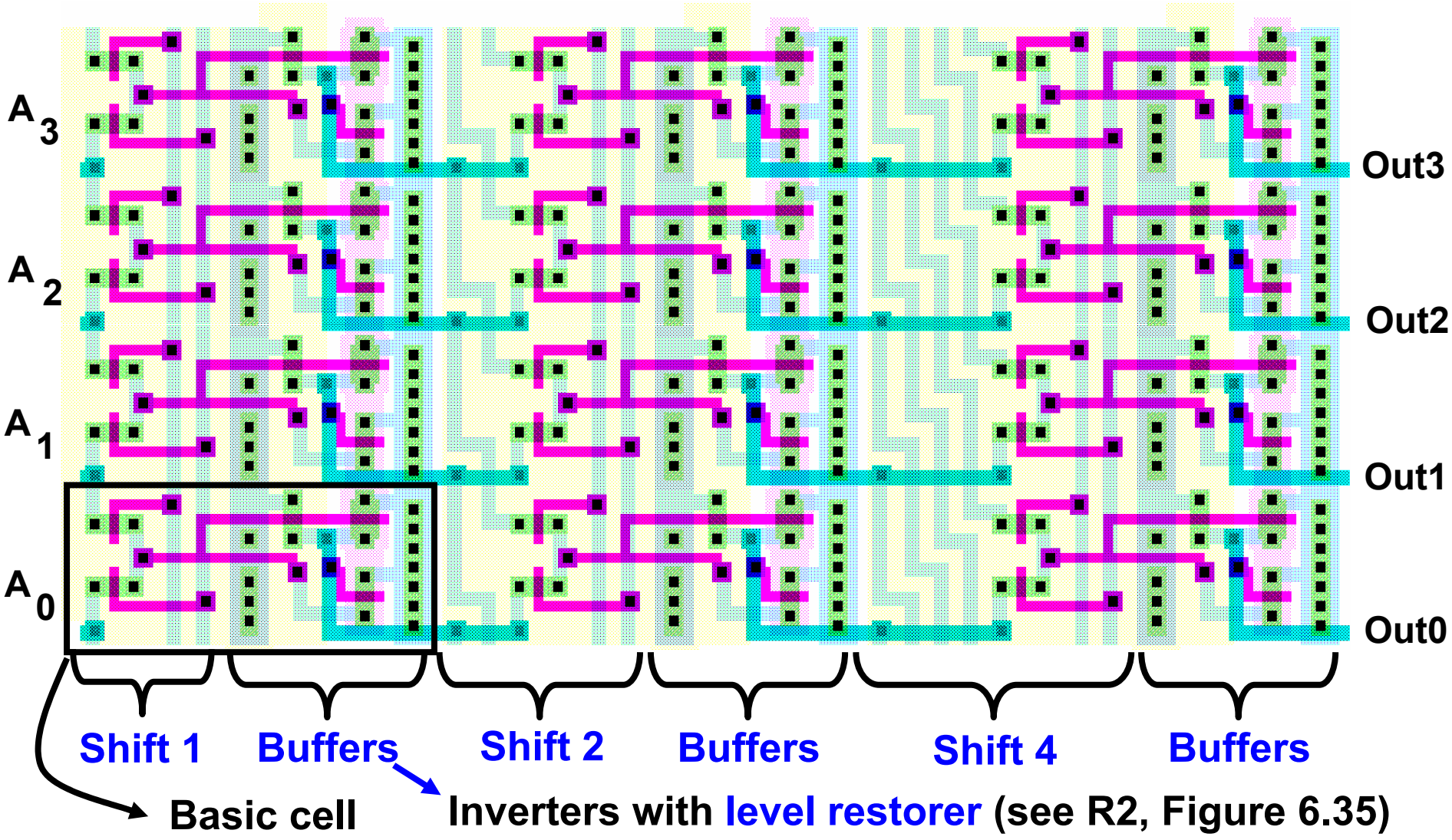


Logarithmic Shifter



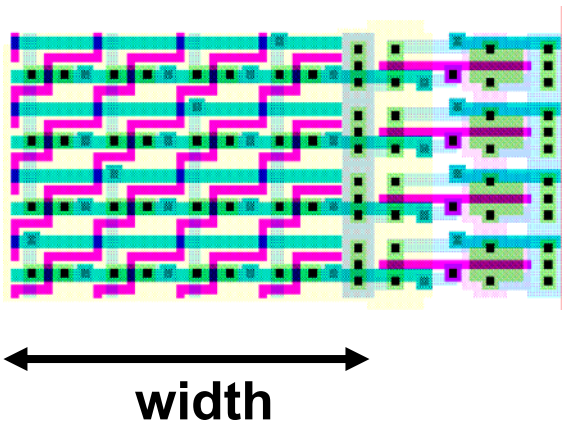
- **Section i shifts $2^{(i-1)}$ bits**
- **Need only *$log_2 M$* stages for M-bit shift**

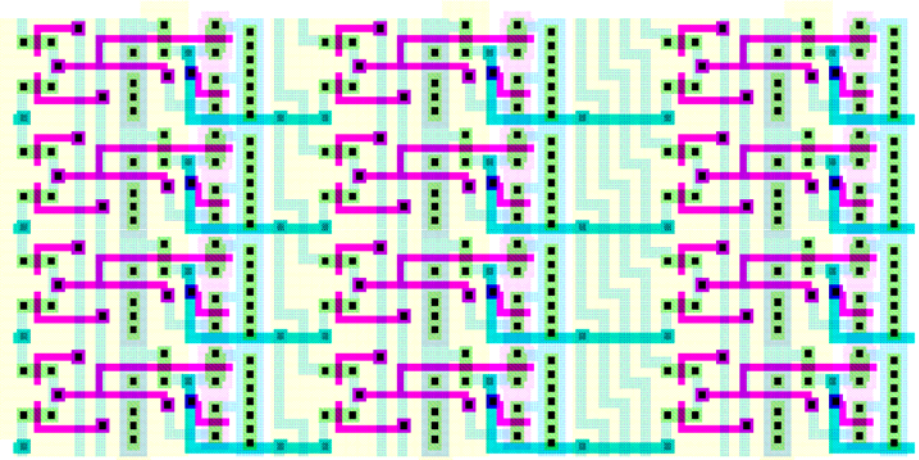
0-7 bit Logarithmic Shifter



Exercise: decipher layout of basic cell and draw transistor circuit

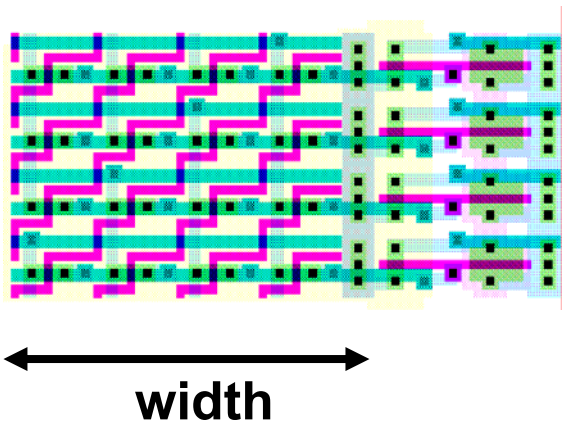
Size Comparison

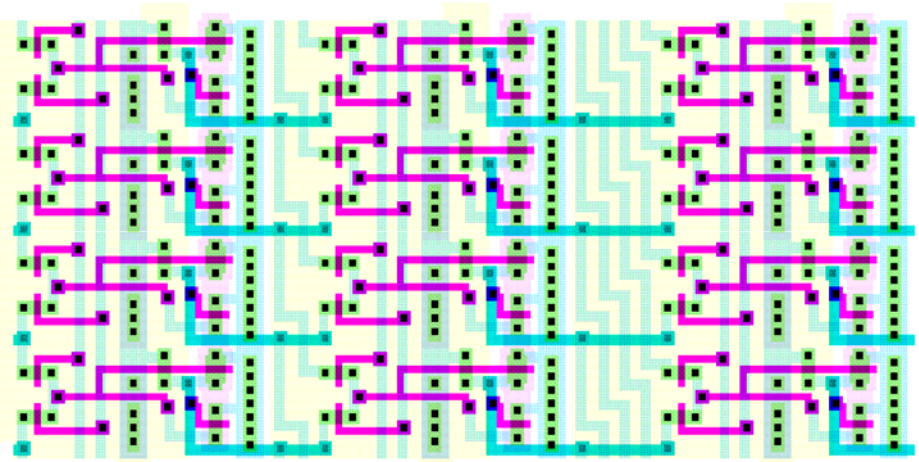




- *M* is maximum bit displacement, $K = log_2 M$
- For large *M*, width is dominated by vertical metal wires
- Disregard buffer size, only count vertical wires
- Barrel shifter needs 1 control and 1 data wire per stage
- # Wires: *2M*
- Log shifter needs 2 control + 2^{i-1} data wires for stage *i*
- # Wires: $2K+(1+2+4+\ldots+2^{K-1}) = 2K+2^K-1 = 2\ log_2 M + M - 1$
- Log shifter will have smaller area for larger *M*!

Speed Comparison



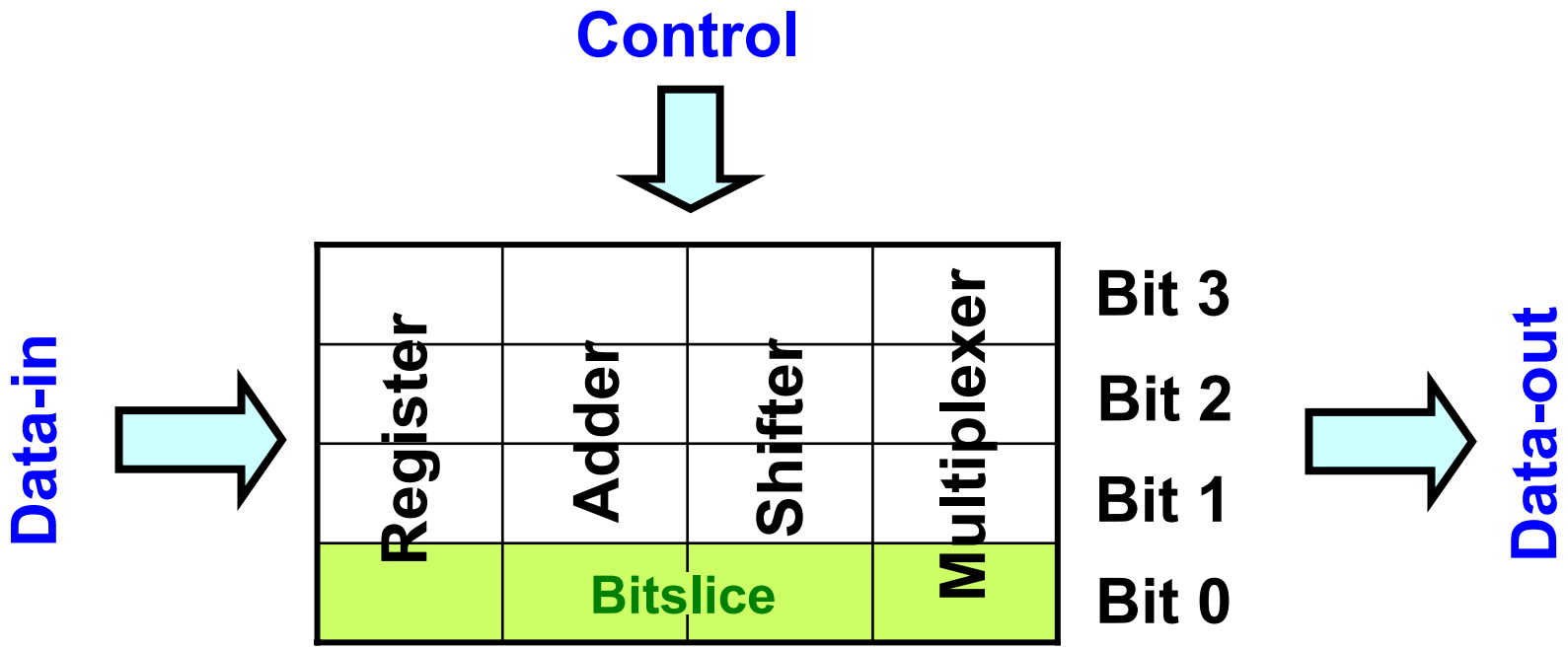


Exercise

- **Discuss the relative speeds of both shifters, as a function of *M* (see discussion in book). Consider:**
  - **Number of sections**
  - **Input capacitance at the buffers (including diffusion areas of the driving pass-transistors)**
  - **Number of buffers (necessity of buffers)**
  - **The number of pass-transistors the signal has to pass**
  - **…**

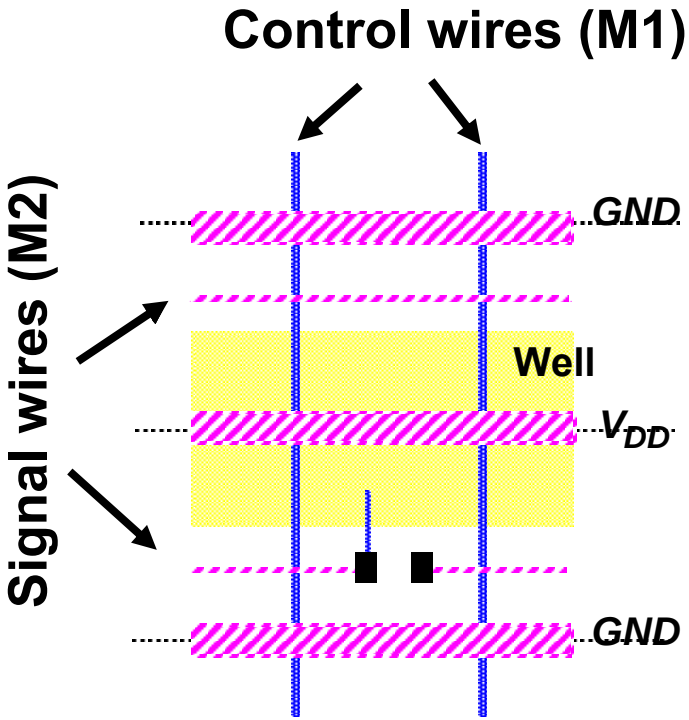
Layout Strategies (regularity)

Bit-Sliced Design



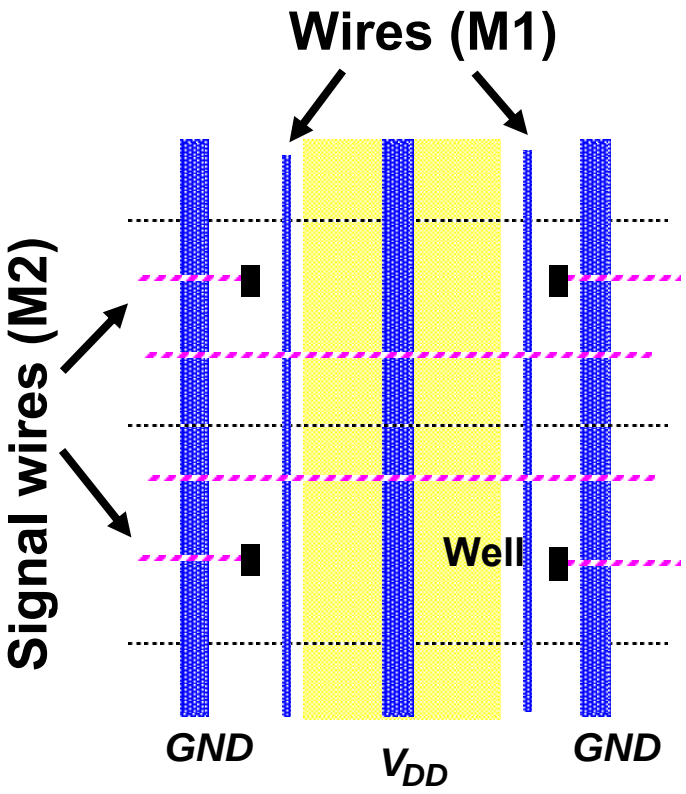
- **Tile** identical processing elements
- Rows for each **bit**
- Columns for each **function**
- **Control** from top (often with ***control-slice***)
- (Example orientation)

Layout Strategies for Bit-Sliced Datapaths



Approach I —

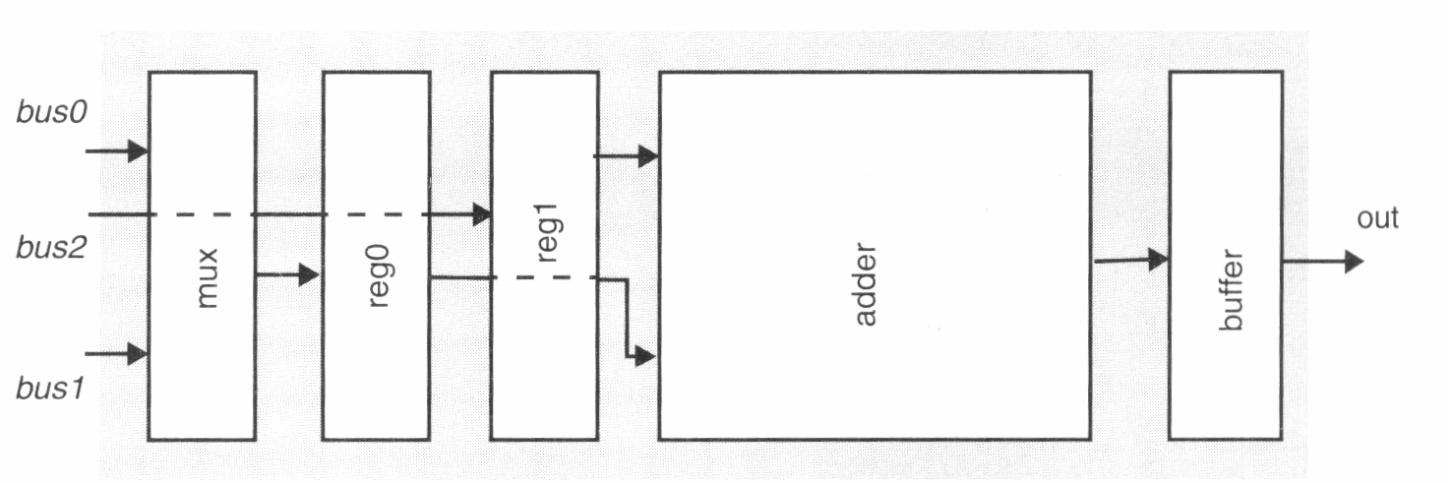
Signal and power lines parallel



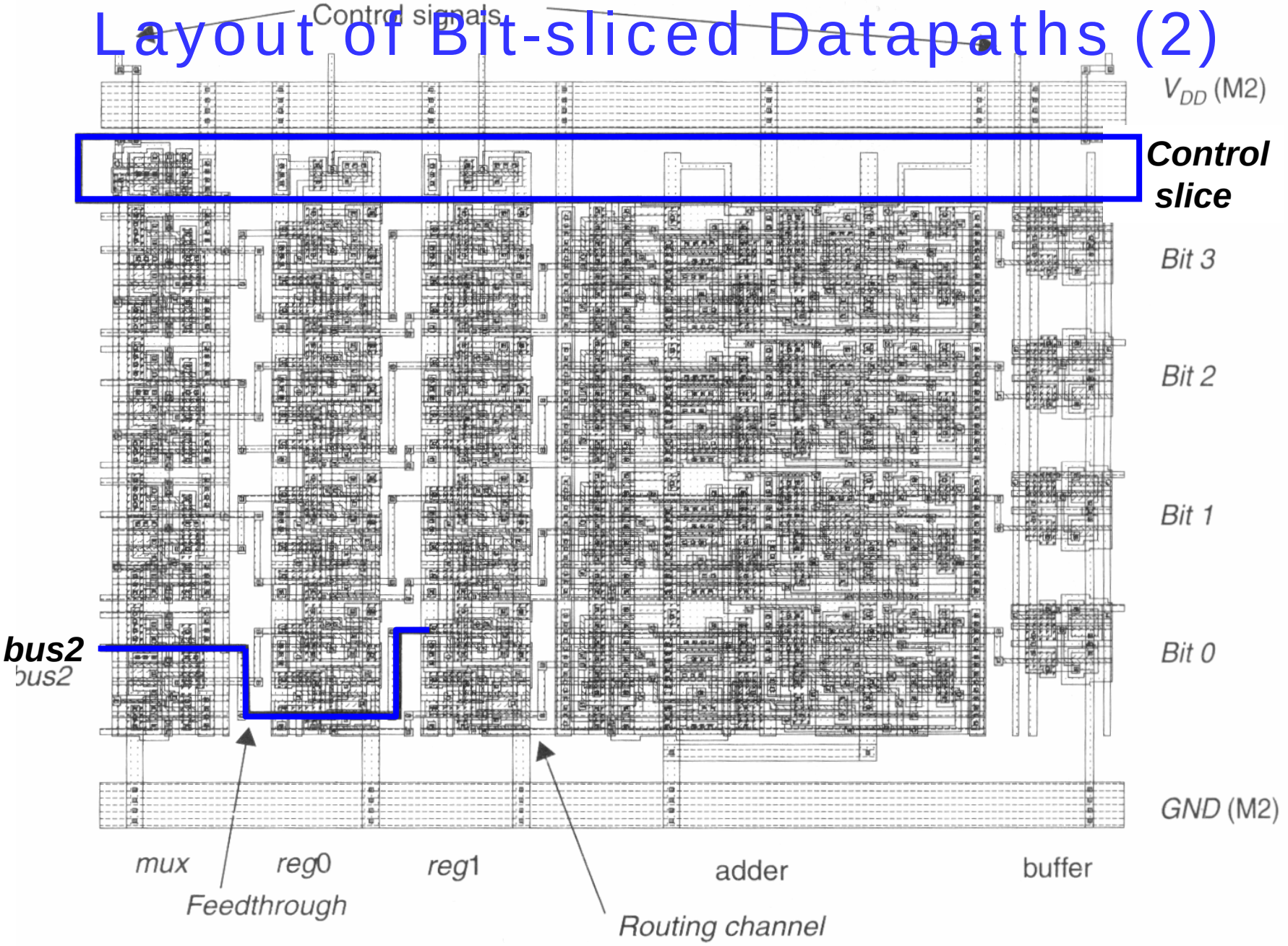
Approach II —

Signal and power lines perpendicular

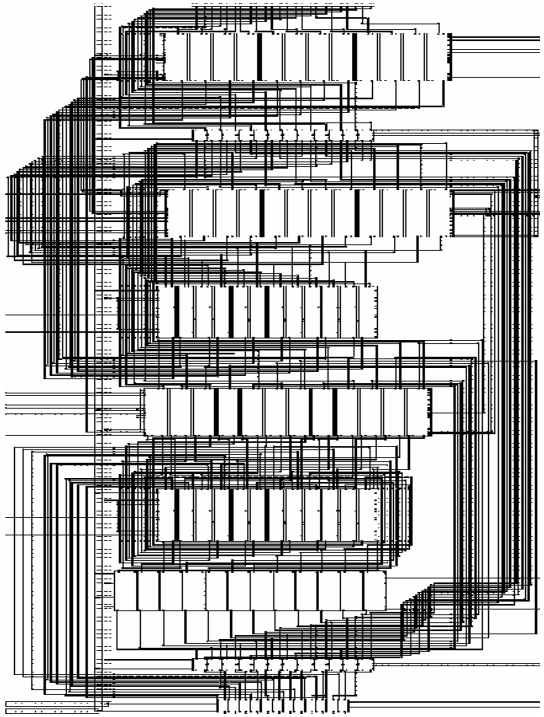
Layout of Bit-sliced Datapaths



Layout of Bit-sliced Datapaths (2)

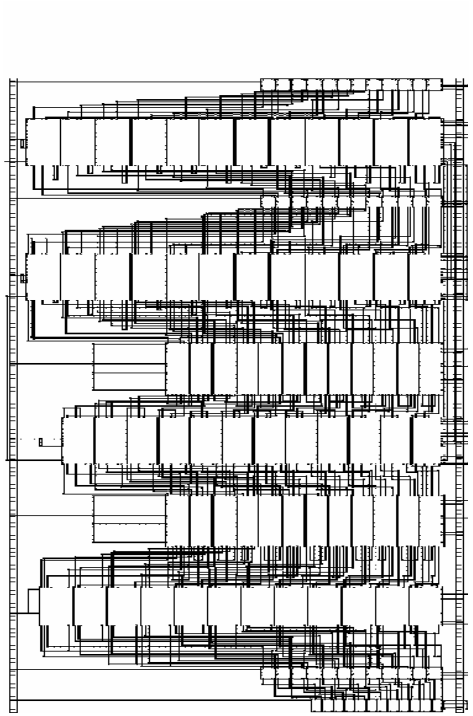


Layout of Bit-sliced Datapaths (3)



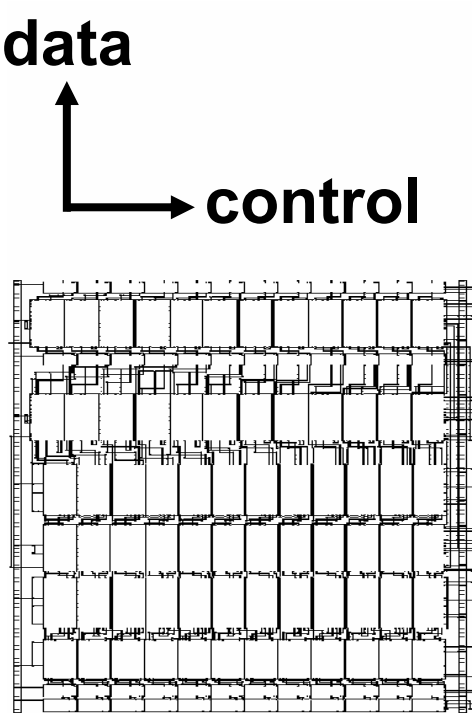
Unoptimized

Area: 4.2mm^2



With feedthroughs

Area: 3.2mm^2

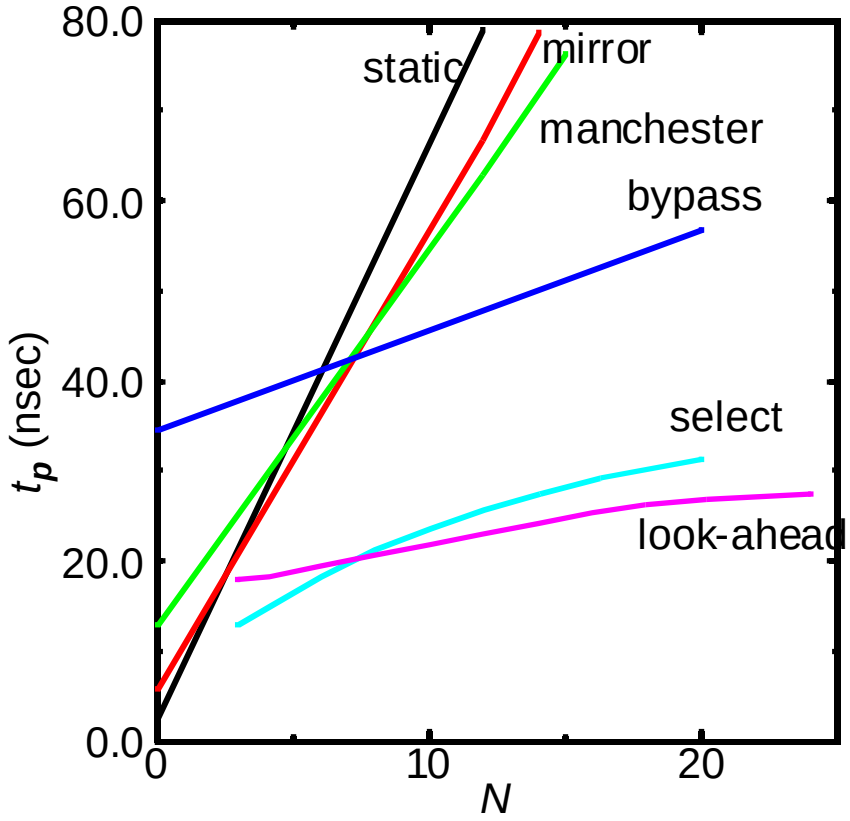


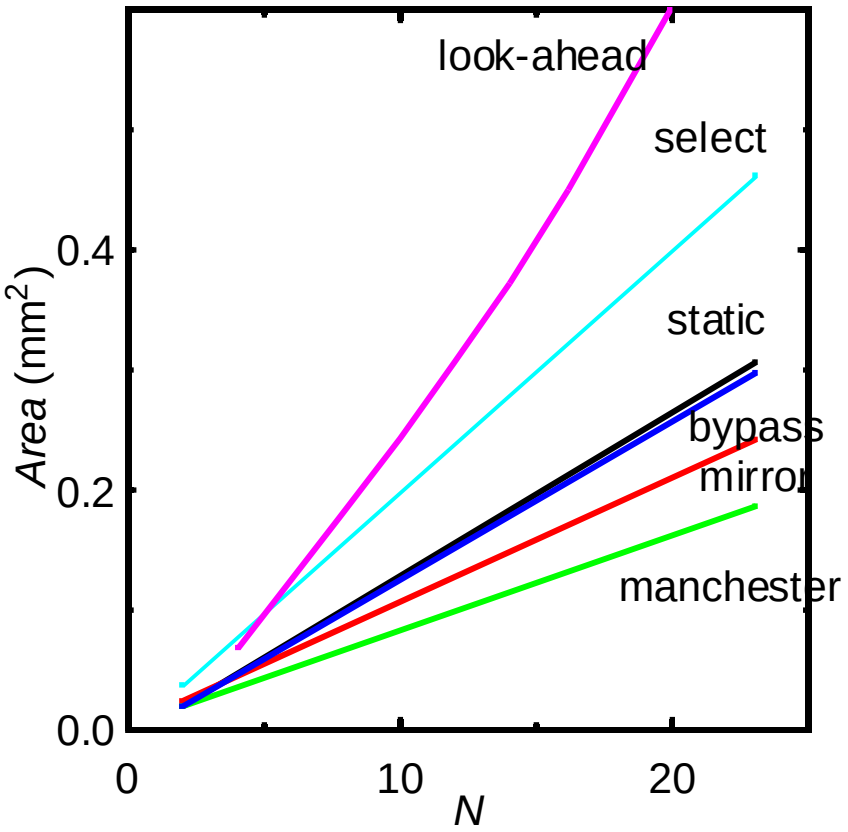
+ Equalized cell height

Area: 2.2mm^2

- **Good layout really counts!**
- **Feedthroughs less (but still) useful with multiple metal layers**

Design as a Trade-Off





VLSI Design

- Select right structure
- Determine and optimize critical timing path for speed
- Optimize rest for area (cost) and/or power and/or design time
- Consider layout aspects

Regularity and modularity are a VLSI designer's best friends

Summary

- Background on Modular Design
  - Hierarchy, reuse, regularity
  - Architecture, bit-slicing
- Adder Design
- Multiplier Design
- Shifter Design
- Layout Strategies (regularity)
- Design as a Trade-Off

Got further appreciation of some system level design issues?

The End


"Thats all Folks!"
Cartoon Songs From
Merry Chips & Looney Circuits