CMOS INVERTER

Course Material for Inverter

Chapter 5, 2nd ed.

P = primair, I = Illustratie, O = overslaan

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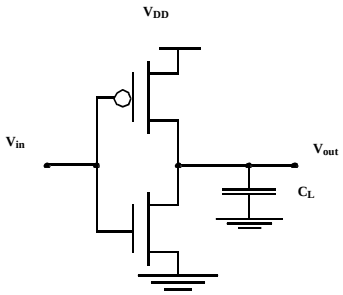
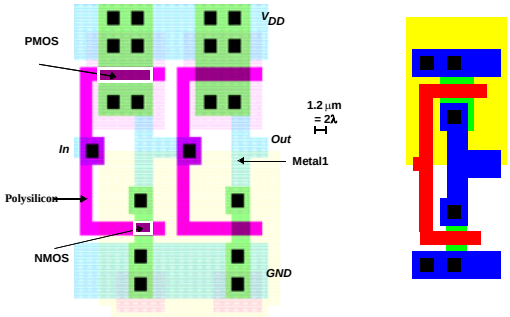
The CMOS Inverter - Outline

- First Glance
- Digital Gate Characterization
- Static Behavior (Robustness)
  - VTC
  - Switching Threshold
  - Noise Margins
- Dynamic Behavior (Performance)
  - Capacitances
  - Delay
- Power
  - Dynamic Power, Static Power, Metrics

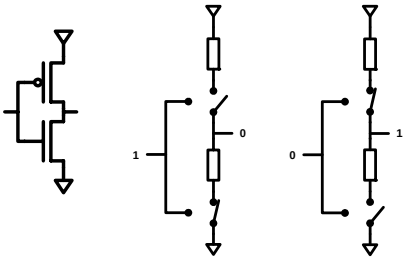
The CMOS Inverter: A First Glance



CMOS Inverters (1)



CMOS Inverter Operation Principle

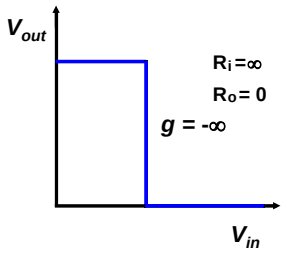


$V_{OH} = V_{DD}$ $V_{OL} = 0$

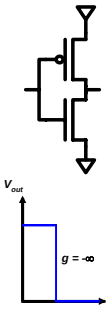
Digital Gate Fundamental Parameters

- Functionality
- Reliability, Robustness
- Area
- Performance
  - Speed (delay)
  - Power Consumption
  - Energy

The Ideal Inverter



Static CMOS Properties



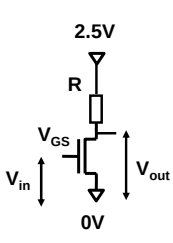
Basic inverter belongs to class of static circuits: output always connected to either V_{DD} or V_{SS}. Not ideal but:

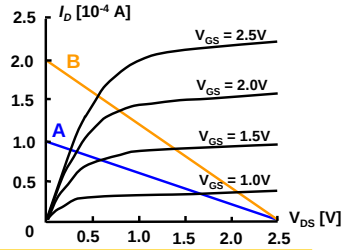
- Rail to rail voltage swing
- Ratio less design
- Low output impedance
- Extremely high input impedance
- No static power dissipation
- Good noise properties/margins

Exercise: prioritize the list above

Voltage Transfer Characteristic (VTC)

Load Line (Ckt Theory)



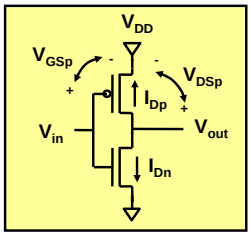


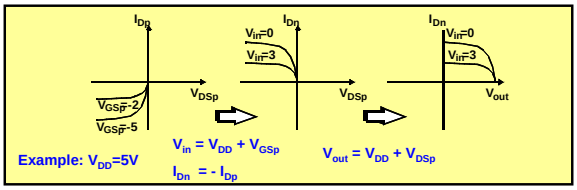
Exercise:
The blue load line A corresponds to R =
The orange load line B corresponds to R =
With load line A and V_{GS} = 1V, V_{out} =
Draw a graph $V_{out}(V_{in})$ for load line A and B

PMOS Load Lines

Goal: Combine I_{Dn} and I_{Dp} in one graph

Kirchoff:
$V_{in} = V_{DD} + V_{GSp}$
$I_{Dn} = -I_{Dp}$
$V_{out} = V_{DD} + V_{DSp}$



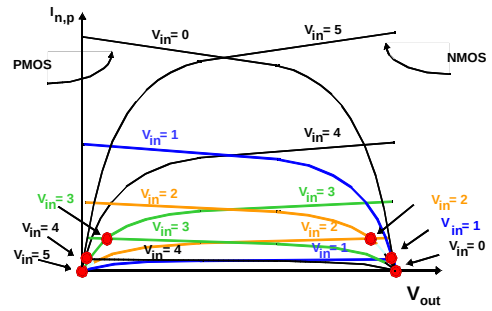


Example: V_{DD}=5V

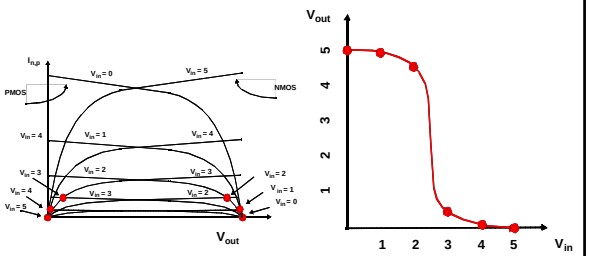
$V_{in} = V_{DD} + V_{GSp}$
$I_{Dn} = -I_{Dp}$

$V_{out} = V_{DD} + V_{DSp}$

CMOS Inverter Load Characteristics

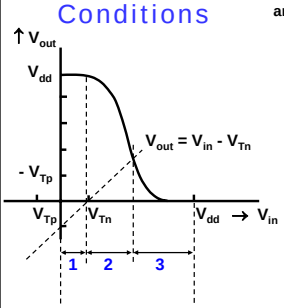


CMOS Inverter VTC



NMOS Operating Conditions

Need to know for proper dimensioning, analysis of noise margin, etc.



NMOS

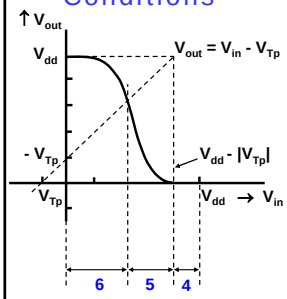
1 $V_{in} = V_{GS} < V_{Tn} \Rightarrow$ off

2 $V_{out} > V_{in} - V_{Tn}$
$V_{DS} > V_{GS} - V_{Tn}$
$V_{GD} < V_{Tn}$ $\Rightarrow$ saturation

3 $V_{out} < V_{in} - V_{Tn} \Rightarrow$ resistive

PMOS Operating Conditions

Need to know for proper dimensioning, analysis of noise margin, etc.



PMOS

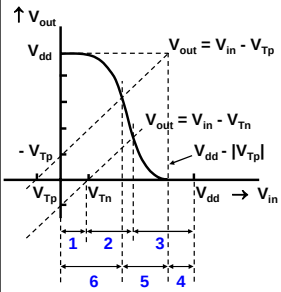
4 $V_{in} > V_{DD} + V_{Tp} \Rightarrow$ off

5 $V_{out} < V_{in} - V_{Tp} \Rightarrow$ saturation

6 $V_{out} > V_{in} - V_{Tp} \Rightarrow$ resistive

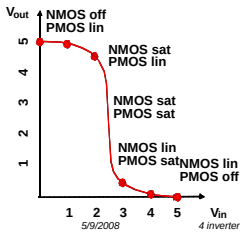
Exercise: check results for PMOS

Operating Conditions



NMOS 1 off
2 saturation
3 resistive

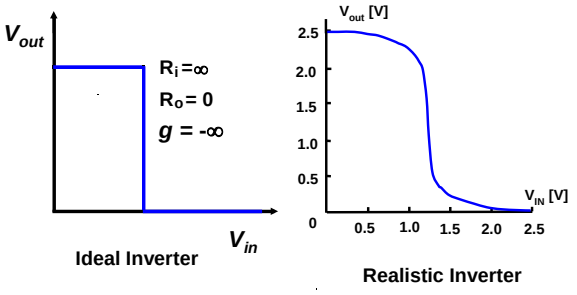
PMOS 4 off
5 saturation
6 resistive



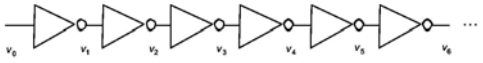
Inverter Static Behavior

- Regeneration
- Noise margins
- Delay metrics

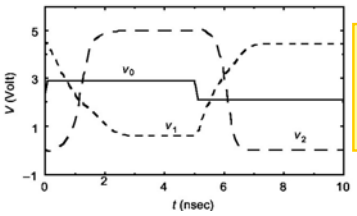
The Realistic Inverter



The Regenerative Property



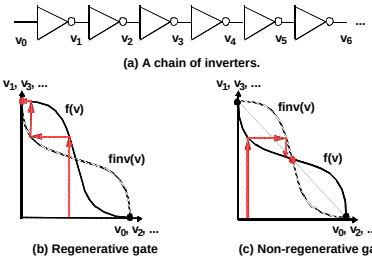
A chain of inverters



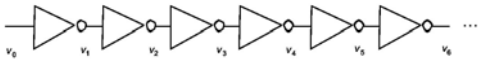
The regenerative property

- **Regenerative Property:** ability to regenerate (repair) a weak signal in a chain of gates

The Regenerative Property (2)



The regenerative Property (3)

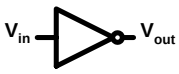


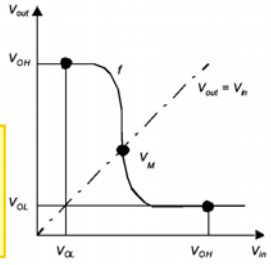
Exercise: what is the output voltage of a chain of 4 inverters with a piece-wise linear VTC passing through (0, 10), (4,8), (6,2) and (10,0) [Volt], as the result of an input voltage of 5.5 [Volt].

Exercise: discuss the behavior for an input of 5 [Volt]

Inverter Switching Treshold

- Not the device threshold $V_m = f(R_{onn}, R_{onp})$
- Point of $V_{in} = V_{out}$

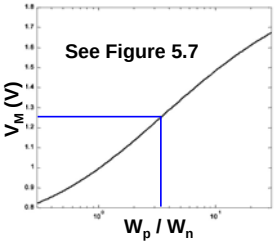




- Try to set W_n, L_n, W_p, L_p so that VTC is symmetric as this will improve noise margins

optimize NMOS-PMOS ratio

Simulated Gate Switching Threshold



Electrical Design Rule
$W_p \approx 2.5\ W_n$

- Assumes $L_p = L_n$
- Should be applied consistently

- Symmetrical VTC $\Rightarrow V_m \approx \frac{1}{2} V_{DD} \Rightarrow W_p/W_n \approx$ ☐
- In practice: choose somewhat smaller value of W_p/W_n
- Why? ☐

Inverter Switching Threshold
Analytical Derivation

- V_M is V_{in} such that $V_{in} = V_{out}$
- $V_{DS} = V_{GS} \Leftrightarrow V_{GD} = 0 \Rightarrow$ saturation
  - Assume $V_{DSAT} < V_M - V_T$ (velocity saturation)
  - Ignore channel length modulation
- V_M follows from
  - $I_{DSATn}(V_M) = - I_{DSATp}(V_M)$

§ 5.3.1

Inverter Switching Threshold
Analytical Derivation (ctd)

$$I_{DSATn}(V_M) = - I_{DSATp}(V_M) \qquad I_D = kV_{DSAT}(V_{GS} - V_T - V_{DSAT}/2)$$

$$\Leftrightarrow k_n V_{DSAT_n}(V_M - V_{Tn} - V_{DSATn}/2) = -k_p V_{DSAT_P}(V_M - V_{DD} - V_{Tp} - V_{DSATp}/2)$$

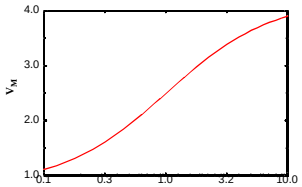
$$\Leftrightarrow \frac{k_p}{k_n} = \frac{-V_{DSAT_n}(V_M - V_{Tn} - V_{DSATn}/2)}{V_{DSAT_P}(V_M - V_{DD} - V_{Tp} - V_{DSATp}/2)} \qquad k = \frac{W}{L}k'$$

$$\Rightarrow \frac{(W/L)_p}{(W/L)_n} = \left| \frac{k'_n V_{DSAT_n}(V_M - V_{Tn} - V_{DSATn}/2)}{k'_p V_{DSAT_P}(V_M - V_{DD} - V_{Tp} - V_{DSATp}/2)} \right|$$

- See Example 5.1:
- $(W/L)_p = 3.5\,(W/L)_n$ for typical conditions and $V_M = ½\ V_{DD}$
- Usually: $L_n = L_p$

Gate Switching Threshold w/o Velocity Saturation

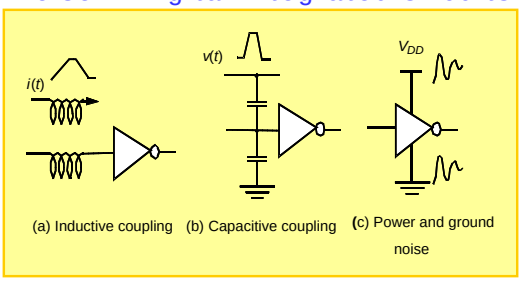
- Long channel approximation
- Applicable with low V_{DD}



Exercise (Problem 5.1): derive V_M for long-channel approximation as shown below

$$V_M = \frac{r(V_{DD} - V_{Tp} + V_{Tn})}{1+r} \quad with \quad r = \sqrt{\frac{-k_p}{k_n}}$$

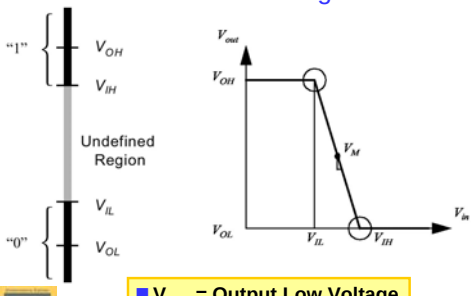
Noise in Digital Integrated Circuits



(a) Inductive coupling (b) Capacitive coupling (c) Power and ground noise

- Study behavior of static CMOS Gates with noisy signals

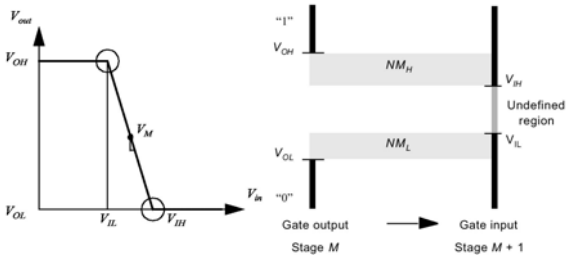
Noise Margins



§ 1.3.2

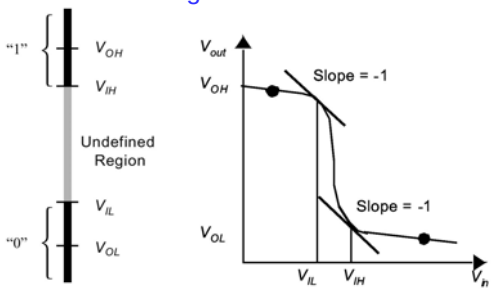
- V_{OL} = Output Low Voltage
- V_{IL} = Input Low Voltage
- V_{OH}, V_{IH} = …

Noise Margins



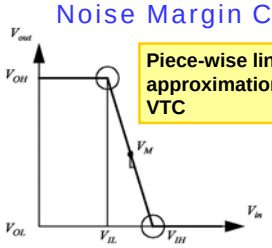
- $NM_H = V_{OH} - V_{IH}$ = High Noise Margin
- $NM_L = V_{IL} - V_{OL}$ = Low Noise Margin

Noise Margin for Realistic Gates



Exercise: explain significance of slope = -1 for noise margin

Noise Margin Calculation



Piece-wise linear approximation of VTC

g = gain factor (slope of VTC)

We know how to compute V_M
Next: how to compute g

$$V_{IH} - V_{IL} = -\frac{(V_{OH} - V_{OL})}{g} = \frac{-V_{DD}}{g}$$

§ 5.3.2

$$V_{IH} = V_M - \frac{V_M}{g} \qquad V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

$$NM_H = V_{DD} - V_{IH} \qquad NM_L = V_{IL}$$

Noise Margin Calculation (2)

- Approximate *g* as the slope in V_{out} vs. V_{in} at $V_{in} = V_M$

$$k_n V_{DSAT_n}(V_{in} - V_{Tn} - V_{DSATn}/2)(1 + \lambda_n V_{out}) +$$

$$k_p V_{DSAT_p}(V_{in} - V_{DD} - V_{Tp} - V_{DSATp}/2)(1 + \lambda_p V_{out} - \lambda_p V_{DD}) = 0$$

$$g = \left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=V_M} \approx \frac{1+r}{(V_M - V_T - V_{DSATp}/2)(\lambda_n - \lambda_p)} \qquad r = \frac{k_p V_{DSATp}}{k_n V_{DSATn}}$$

- Mostly determined by technology
- See example 5.2

- **Exercise:** verify calculation
- **Exercise:** explain why we add channel length modulation to the I_D expressions (we did not do this to determine V_M)

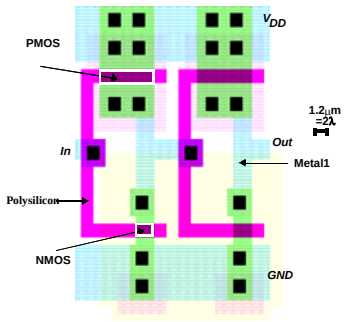
Dynamic Behavior (Performance)

- Capacitances
- Delay

§ 5.4

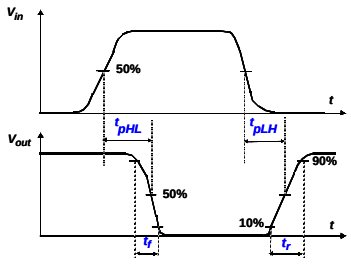
- But: we take much simpler model for capacitances compared to book.
- See the syllabus.
- § 5.4.1 of book is only illustration.

CMOS Inverters



- What causes the delay?
- What is R_{on} ?
- Where are the capacitances?
- Which capacitances determine T_p?

Delay Definitions

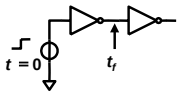


$$t_p = \frac{1}{2}(t_{pHL} + t_{pLH})$$

§ 1.3.3

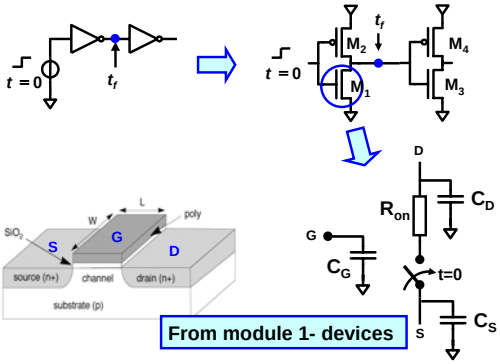
t_p : property of gate
t_r, t_f : property of signal

CMOS Inverter Rise/Fall Delay



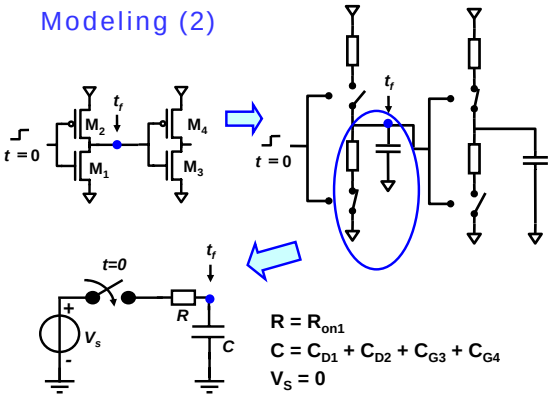
- Goal: determine t_f, t_r and t_p
- First: compute relevant capacitances
- Second: determine equivalent R_{on}
- Third: compute RC delay (τ) and scale result
- Assume: ideal source, step input

Modeling



From module 1- devices

Modeling (2)

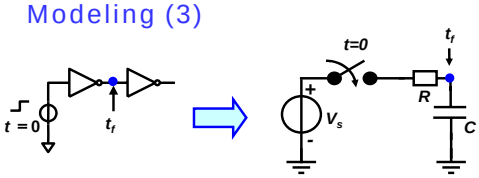


$R = R_{on1}$

$C = C_{D1} + C_{D2} + C_{G3} + C_{G4}$

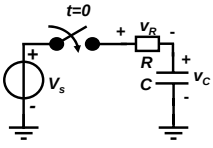
$V_S = 0$

Modeling (3)

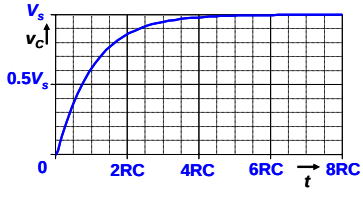


- Delay can be modeled using RC circuit
  - First order simplified model
- R is (linearized) on-resistance of active transistor
- C is sum of (linearized) C's that are switched
  - Typically, C_G of driven gate, C_D of driving gate
  - Plus relevant interconnect C
  - Might need to include interconnect R in model
- V_s is final voltage of delay node (initial voltage determined by previous state)

RC Delay Review (1)



$$v_C(t) = v_C(\infty) + (v_C(t_0) - v_C(\infty))e^{-(t-t_0)/\tau}$$



$$RC\frac{dv_C}{dt} = V_S - v_C$$

$$v_C = V_S(1 - e^{-\frac{t}{RC}})$$

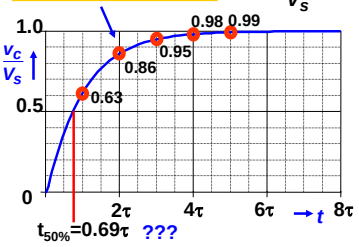
RC Delay Review (2)

- Response can be normalized with respect to $\tau = RC$ and $V_S = v_C(t = \infty)$

$$v_C = V_S(1 - e^{-\frac{t}{RC}})$$

$$\frac{v_C}{V_S} = (1 - e^{-t/\tau})$$

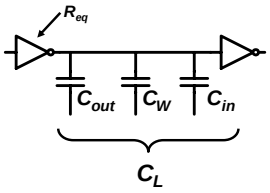
Example: $(1 - e^{-2}) = 0.86$



Each τ-step gives 63% of remaining swing

swing	time
0–50%	0.69τ
0–63%	1.0τ
10%–90%	2.2τ
0–90%	2.3τ

Inverter Propagation Delay Summary

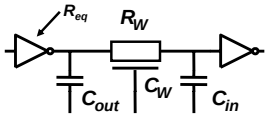


$$t_{pHL} = 0.69 R_{eqn} C_L$$

$$t_{pLH} = 0.69 R_{eqp} C_L$$

$$t_p = \frac{1}{2}\left(t_{pHL} + t_{pLH}\right)$$ **Propagation time**

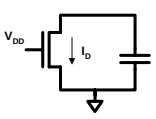
Propagation Delay w. Wire Resistance



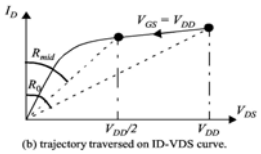
$$t_{PHL} = 0.69 R_{eqn}(C_{out} + 0.5 C_W) + 0.69(R_{eqn} + R_W)(0.5 C_W + C_{in})$$

- **See module 3, interconnect**

Equivalent R_{on} (R_{eq})



(a) Schematic



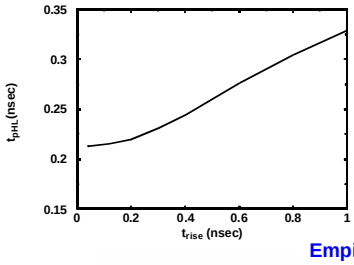
(b) trajectory traversed on ID-VDS curve.

$$R_{eq} = \frac{1}{2}\left[\frac{V_{DD}}{I_{DSAT}\left(1+\lambda V_{DD}\right)} + \frac{V_{DD}/2}{I_{DSAT}\left(1+\lambda V_{DD}/2\right)}\right]$$

$$\approx \frac{3}{4}\frac{V_{DD}}{I_{DSAT}}\left(1 - \frac{5}{6}\lambda V_{DD}\right)$$

See 3-devices, example 3.8

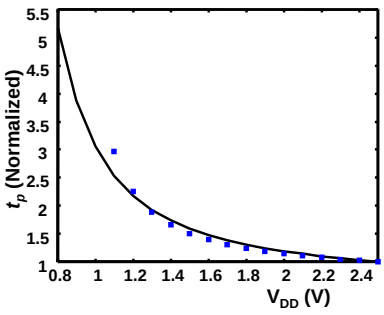
Impact of Rise Time on Delay



$$t_{pHL} = \sqrt{t_{pHL(step)}^2 + (t_r/2)^2}$$

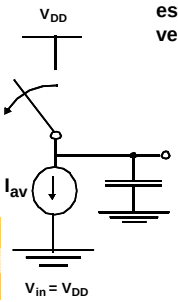
Empirical from the first edition of book

Delay as a function of V_{DD}



Fg.5.17

CMOS Inverter Propagation Delay



Alternative current-based estimate for T_p, assuming velocity saturation

$$T_{pHL} = \frac{C_L \frac{V_{swing}}{2}}{I_{av}} \approx \frac{C_L V_{DD}}{2 I_{DSAT}\left(1 + \frac{3}{4}\lambda V_{DD}\right)}$$

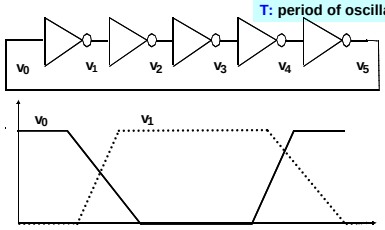
Pr. 5.4

Exercise (Problem 5.4): Derive the expression above

Ring Oscillator

Frequently used to obtain T_p by measurement (or simulation)

N: number of inverters
T_p: propagation delay
T: period of oscillation

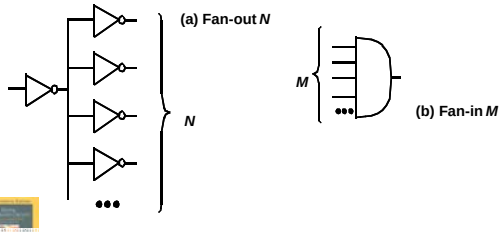


§ 1.3.3

$T = 2 \times T_p \times N \Leftrightarrow T_p = T/2N$

Exercise: explain the factor 2 in the expression above

Fan-in and Fan-out



§ 1.3.2

Power (§ 5.5)

- Dynamic Power
- Static Power
- Metrics

CMOS Power Dissipation

- Power dissipation is a very important circuit characteristic
- CMOS has relatively low static dissipation
- Power dissipation was the reason that CMOS technology won over bipolar and NMOS technology for digital IC's
- (Extremely) high clock frequencies increase dynamic dissipation
- Low V_T increase leakage
- Advanced IC design is a continuous struggle to contain the power requirements!

§ 1.3.4 § 5.5

Power Density

Estimate

- Furnace: 2000 Watt, r=10cm ➔ P ≈ 6Watt/cm^2
- Processor chip: 100 Watt, 3cm^2 ➔ P ≈ 33Watt/cm^2

Power-aware design, design for low power, is blossoming subfield of VLSI Design

Where Does Power Go in CMOS

- Dynamic Power Consumption
  - Charging and discharging capacitors
- Short Circuit Currents
  - Short circuit path between supply rails during switching
- Leakage
  - Leaking diodes and transistors
  - May be important for battery-operated equipment

Dynamic Power

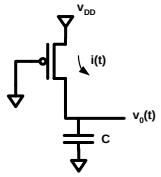
Dynamic Power

- E_i = energy of switching event i
  - (to first order) independent of switching speed
  - depends on process, layout
- Power = Energy/Time

$$P = \frac{1}{T}\sum_i E_i$$

- E_i = Power-Delay-Product P-D
  - important quality measure
- Energy-Delay-Product E-D
  - combines power*speed performance

Low-to-High Transition Energy



Equivalent circuit for low-to-high transition

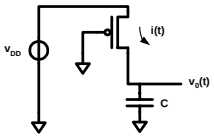
E_C - Energy stored on C

$$E_C = \int_0^\infty i v_0 dt \qquad v_0 = v_0(t) \qquad i = i(t) = C\frac{dv_0}{dt}$$

$$= \int_0^\infty C v_0 \frac{dv_0}{dt} dt$$

$$= \int_0^{V_{DD}} C v_0 dv_0 = \frac{1}{2} C v_0^2 \Big|_0^{V_{DD}} = \frac{1}{2} C V_{DD}^2$$

Low-to-High Transition Energy



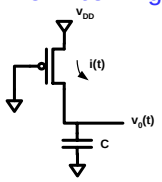
$E_{V_{DD}}$ Energy delivered by supply

$$E_{V_{DD}} = \int_0^\infty i(t) V_{DD} dt = \int_0^{V_{DD}} C V_{DD} \frac{dv_0}{dt} dt = C V_{DD}^2$$

$$E_{V_{DD}} = C V_{DD}^2 \qquad E_C = \frac{1}{2} C V_{DD}^2$$

Where is the rest?

Low-to-High Transition Energy



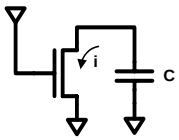
E_{diss} Energy dissipated in transistor

$$E_{diss} = \int_0^\infty i(V_{DD} - v_0) dt$$

$$= \int_0^\infty i V_{DD} dt - \int_0^\infty i v_0 dt$$

$$= E_{V_{DD}} - E_C$$

High-to-Low Transition Energy



Equivalent circuit

Exercise: Show that the energy that is dissipated in the transistor upon discharging C from V_{DD} to 0 equals $E_{diss} = ½CV_{DD}^2$

CMOS Dynamic Power Dissipation

$$Energy = E_{charge} + E_{discharge}$$

$$Power = \frac{Energy}{Time} = \frac{Energy}{transition} \times \frac{\#\,transitions}{time}$$

$$= C V_{DD}^2 \times f$$

- Independent of transistor on-resistances
- Can only reduce C, V_{DD} or f to reduce power
- In this formula, f accounts for switching activity (not necessarily a simple regular waveform)

Summary

- First Glance
- Digital Gate Characterization (§ 1.3)
- Static Behavior (Robustness) (§ 5.3)
  - VTC
  - Switching Threshold
  - Noise Margins
- Dynamic Behavior (Performance) (§ 5.4)
  - Capacitances
  - Delay
- Power (§ 5.5)
  - Dynamic Power, Static Power, Metrics