Sparse Sensing for Composite Matched Subspace Detection

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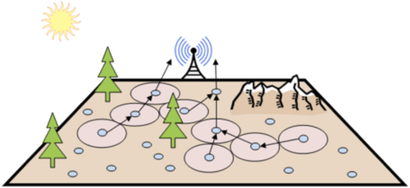
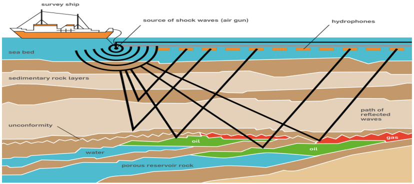
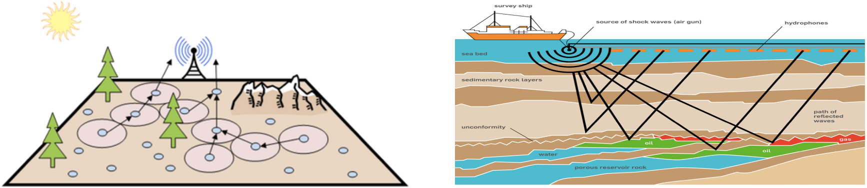


(a)



(b)

Figure : (a) Distributed sensor network. (b) Seismic imaging.



(a) (b)

Figure : (a) Distributed sensor network. (b) Seismic imaging.

Design sparse samplers for event detection.

- Why?

[1] S. Joshi, and S. Boyd. "Sensor selection via convex optimization." *TSP* 2009

[2] S.P. Chepuri, and G. Leus, "Sparse Sensing for Statistical Inference," *Foundations and Trends in Sig. Proc.* 2016

[3] A. Krause, and C. Guestrin, "Near-optimal observation selection using submodular functions," *AAAI* 2007

[4] J. Ranieri, et al., "Near-optimal sensor placement for linear inverse problems," *TSP* 2014

Sparse Sampler Design

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  - possibly many non-informative measurements

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Sparse Sampler Design

- Why?
  - possibly many non-informative measurements
  - reduces processing overhead
  - economical or physical constraints
- How?
  - **convex optimization:** through selection vector $\mathbf{w} \in \{0,1\}^M$

[Joshi-Boyd-09][1], [Chepuri-Leus-16][2]

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- How?
  - **convex optimization:** through selection vector $\mathbf{w} \in \{0, 1\}^M$
    [Joshi-Boyd-09][1], [Chepuri-Leus-16][2]
  - **submodular optimization:** greedy methods and heuristics
    [Krause-Guestrin-07][3],[Ranieri-Chebira-Vetterli-14][4]

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Prior Art

- **Sampler Design for Detection**
  - *Simple Hypothesis Test*
    - Design criteria based on distances between distributions.
      [Bajović-11][5], [Chepuri-Leus-16][6],[Coutino-Chepuri-Leus-17][7]
    - Compressed Detectors (random samplers)
      [Davenport-10][8]
  - *Composite Hypothesis Test*
    - *Compressed Subspace Detector* (random samplers)
      [Wang-08][9]

Our Focus: Deterministic Samplers

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5 D. Bajovic, et al., "Sensor selection for event detection in wireless sensor networks", *TSP* 2011

6 S.P. Chepuri and G. Leus., "Sparse Sensing for Distributed Detection". *TSP* 2016

7 M. Coutino, et al., "Near-Optimal Sparse Sensing for Gaussian Detection with Correlated Observations," *Submitted to TSP* 2017

8 M. Davenport, et al., "Signal processing with compressive measurements". *STPS* 2010

9 Z.Wang et al.,"Subspace compressive detection for sparse signals".*ICASSP* 2008.

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Matched Subspace Detector (MSD)

- **Signal Data Model**

Consider the received signal

$$\mathbf{y} = \mathbf{x} + \mathbf{v} + \mathbf{n} \in \mathbb{R}^N,$$

where

$$\textit{signal of interest} \ : \ \mathbf{x} = \mathbf{H}\boldsymbol{\theta}, \text{ with } \mathbf{H} \in \mathbb{R}^{N \times P}, \quad (1)$$

$$\textit{interference} \ : \ \mathbf{v} = \mathbf{S}\boldsymbol{\phi}, \text{ with } \mathbf{S} \in \mathbb{R}^{N \times Q}, \quad (2)$$

and

$$\boldsymbol{\theta} \in \mathbb{R}^P; \ \ \boldsymbol{\phi} \in \mathbb{R}^Q; \ \ \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$$

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and

$$\boldsymbol{\theta} \in \mathbb{R}^{P}; \ \ \boldsymbol{\phi} \in \mathbb{R}^{Q}; \ \ \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$$

The matrix $[\mathbf{H}\ \mathbf{S}]$ is considered to be full column rank, with $P + Q \leq N$.

Matched Subspace Detector (MSD)

- **Hypothesis Test**

Check: Is $\mathbf{x} \in \mathrm{span}(\mathbf{H})$ in $\mathbf{y}$?

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{y} \sim \mathcal{N}(\mathbf{S}\phi, \sigma^2\mathbf{I}) \\ \mathcal{H}_1 &: \mathbf{y} \sim \mathcal{N}(\mathbf{S}\phi + \mathbf{H}\boldsymbol{\theta}, \sigma^2\mathbf{I}) \end{aligned} . \quad (3)$$

$$\|\boldsymbol{\theta}\|_2^2 > 0 \text{ for } \mathcal{H}_i, i = 1, 2; \quad \text{unknown}; \sigma^2$$

Difficulty: Both $\boldsymbol{\theta}$ and ϕ are unknown.

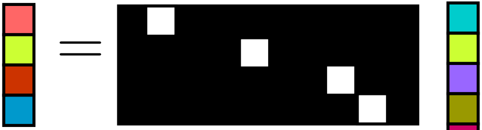
Available knowledge: Both $\mathbf{H}$ and $\mathbf{S}$ are considered known.

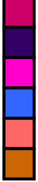
Sparse Sensing for MSD

- **Sparse Acquisition**[10]

Only subset $\mathcal{A} \subseteq \mathcal{V} = \{1, 2, \ldots, N\}$ of data is observed

$$\mathbf{y}_{\mathcal{A}} = \mathbf{\Phi}_{\mathcal{A}} \mathbf{y} \tag{4}$$





where $|\mathcal{A}| = K$, and $\mathbf{\Phi}_{\mathcal{A}} \in \{0, 1\}^{K \times N}$.

[10] This is different from other works which employ random matrices, e.g., Z.Wang et al., *"Subspace compressive detection for sparse signals"*. ICASSP 2008.

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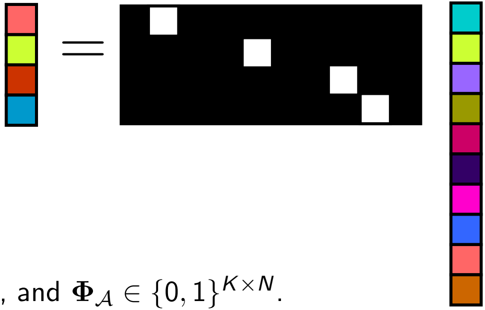

where $|\mathcal{A}| = K$, and $\mathbf{\Phi}_{\mathcal{A}} \in \{0, 1\}^{K \times N}$.

How to design $\mathcal{A}$ (of given cardinality) for best MSD performance?

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Sparse Sampler Design for MSD

- **Generalized log-likelihood ratio test (GLRT)**

GRLT of MSD:[11]

$$L(\mathbf{y}) \sim \frac{\mathbf{y}^T \mathbf{P}_{\mathbf{S}}^{\perp} \mathbf{E}_{\mathbf{HS}} \mathbf{P}_{\mathbf{S}}^{\perp} \mathbf{y}}{\mathbf{y}^T \mathbf{P}_{\mathbf{S}}^{\perp} (\mathbf{I} - \mathbf{E}_{\mathbf{HS}}) \mathbf{P}_{\mathbf{S}}^{\perp} \mathbf{y}}. \quad (5)$$

where

$$\mathbf{P}_{\mathbf{S}}^{\perp} = \mathbf{I} - \mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T, \quad (6)$$

$$\mathbf{E}_{\mathbf{HS}} = \mathbf{H}(\mathbf{H}^T\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{P}_{\mathbf{S}}^{\perp}. \quad (7)$$

[11] L. Scharf, and B. Friedlander. *"Matched subspace detectors."* IEEE Trans. on Signal Proc. 1994

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- **F-Distribution of the GRLT for MSD**

$$\frac{\bar{Q}-\bar{P}}{\bar{P}}L(\mathbf{y}) : \begin{cases} F_{\bar{P},\bar{Q}-\bar{P}}(0) & \text{under} \quad \mathcal{H}_0 \\ F_{\bar{P},\bar{Q}-\bar{P}}(\lambda^2(\boldsymbol{\theta})) & \text{under} \quad \mathcal{H}_1 \end{cases}, \tag{8}$$

$$\text{noncentrality parameter} \; : \; \lambda^2(\boldsymbol{\theta}) = \frac{1}{\sigma^2}\boldsymbol{\theta}^T\mathbf{H}^T\mathbf{P}_{\mathbf{S}}^{\perp}\mathbf{H}\boldsymbol{\theta}, \tag{9}$$

Uniform most power (UMP) invariant test is achieved by fixing a η for the GLRT.

- **Error Probabilities of the GRLT for MSD**

$$\textbf{false alarm} : P_{\mathrm{fa}} = 1 - P[F_{\bar{P},\bar{Q}-\bar{P}}(0) \le \eta]; \quad (10)$$

$$\textbf{detection probabilities} : P_{\mathrm{d}} = 1 - P[F_{\bar{P},\bar{Q}-\bar{P}}(\lambda^2(\boldsymbol{\theta})) \le \eta]. \quad (11)$$

Design Metric: P_{d} is a monotone function of $\lambda^2(\boldsymbol{\theta})$.

maximizing $\lambda^2(\boldsymbol{\theta}) \rightarrow$ maximizes the *power* of the test.

noncentrality parameter depends on the unknown parameter $\boldsymbol{\theta}$

Sparse Sampler Design for MSD

Data under $\mathcal{H}_1$ and $\mathcal{A} \subseteq \mathcal{V}$

$$\begin{aligned} \mathbf{y}_{\mathcal{A}} &= \mathbf{\Phi}_{\mathcal{A}}\big[\mathbf{H}\boldsymbol{\theta} + \mathbf{S}\boldsymbol{\phi} + \mathbf{n}\big] \\ &= \mathbf{H}_{\mathcal{A}}\boldsymbol{\theta} + \mathbf{S}_{\mathcal{A}}\boldsymbol{\phi} + \mathbf{n}_{\mathcal{A}} \in \mathbb{R}^K. \end{aligned} \quad (12)$$

- **Worst-case (Max-Min) Design** (Concave cost[12])

$$\underset{\mathcal{A}\subseteq\mathcal{V}, |\mathcal{A}|=K}{\text{maximize}} \ \lambda_{\min}(\mathbf{G}_{\mathcal{A}}), \quad (13)$$

where, by using the definition (6), the matrix $\mathbf{G}_{\mathcal{A}}$ is given by

$$\mathbf{G}_{\mathcal{A}} := \mathbf{H}_{\mathcal{A}}^T\big[\mathbf{I}_K - \mathbf{S}_{\mathcal{A}}(\mathbf{S}_{\mathcal{A}}^T\mathbf{S}_{\mathcal{A}})^{-1}\mathbf{S}_{\mathcal{A}}^T\big]\mathbf{H}_{\mathcal{A}}. \quad (14)$$

(Efficient solution through interior-point methods).

[12] M. Coutino, et al. *"Sparse sensing for composite matched subspace detection"*. CAMSAP 2017

Sparse Sampler Design for MSD

- **Log-det Design** (Concave cost[13])

$$\underset{\mathcal{A}\subseteq\mathcal{V},|\mathcal{A}|=K}{\text{maximize}} \ \ln\det(\mathbf{G}_{\mathcal{A}}). \tag{15}$$

(Efficient solution through interior-point methods).

- **Log-det Design** (Submodular cost[13])

$$\underset{\mathcal{A}\subset\mathcal{V},|\mathcal{A}|=K}{\text{maximize}} \ \ln\det(\mathbf{M}_{\mathcal{A}}). \tag{16}$$

where

$$\mathbf{M}_{\mathcal{A}} := \begin{bmatrix} \mathbf{S}^T\mathbf{I}_{\mathcal{A}}\mathbf{S} & \mathbf{S}^T\mathbf{I}_{\mathcal{A}}\mathbf{H} \\ \mathbf{H}^T\mathbf{I}_{\mathcal{A}}\mathbf{S} & \mathbf{H}^T\mathbf{I}_{\mathcal{A}}\mathbf{H} \end{bmatrix} \succeq \mathbf{0}, \tag{17}$$

and $\det(\mathbf{M}_{\mathcal{A}}) = \det(\mathbf{S}^T\mathbf{I}_{\mathcal{A}}\mathbf{S})\det(\mathbf{G}_{\mathcal{A}})$.

(Near-optimal solution through greedy algorithm).

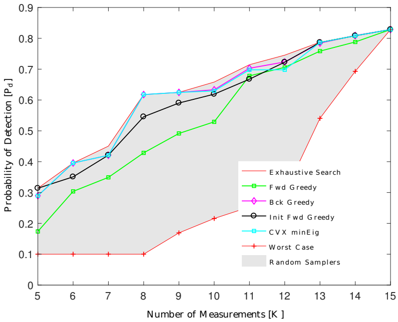
[13] M. Coutino, et al. *"Sparse sensing for composite matched subspace detection"*. CAMSAP 2017

Numerical Results

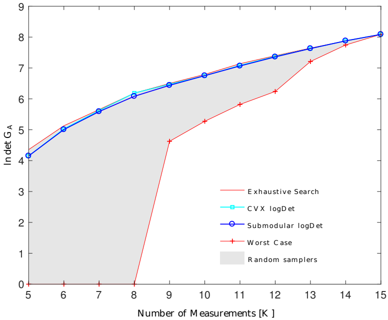
Array signal processing example (1)

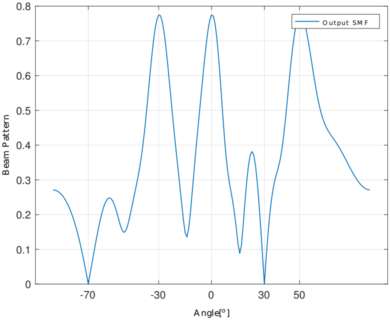
Settings:

- half wavelength linear array with $M = 15$ elements.
- Angles of interest $\Phi = \{-30^o, 0^o, 50^o\}$
- $\mathbf{H} = [\mathbf{a}(\phi_1), \mathbf{a}(\phi_2), \mathbf{a}(\phi_3)]$, $\phi_i \in \Phi$, for $i = \{1, 2, 3\}$
- Interferer matrix $\mathbf{S} = [\mathbf{a}(-70^o)\ \mathbf{a}(30^o)]$
- noise and signal power, $\sigma^2 = 1$, $\|\boldsymbol{\theta}\|^2 = 1$, respectively.
- ULA steering vector $\mathbf{a}(\phi)$.



Array signal processing example (2)





Conclusions

- We introduced *sparse sampler design* for matched subspace detectors.
- The noncentrality parameter of the GRLT is used for obtaining a *design metric*.
- As the noncentrality parameter depends on unknown parameters, we introduced two alternative designs:
  - max-min criterion $->\lambda_{\min}\{\mathbf{G}_{\mathcal{A}}\}$.
  - log-det criterion $->\ln\log\{\mathbf{G}_{\mathcal{A}}\}$.
- Samplers, for the proposed criteria, are shown to be found efficiently by the *convex* and *submodular* machinery.

- Outlook
  - Can we extend sparse sampler design for other composite hypothesis test problems?
  - Is it possible to devise convex/submodular metrics for multiple hypothesis test?

Thank you!

Questions?