Near-Optimal Greedy Sensor Selection for MVDR Beamforming with Modular Budget Constraint

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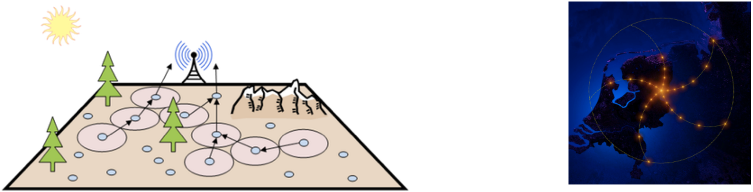

(a) (b)

Figure: (a) Distributed sensor network. (b) Array of radio telescopes.

Sparse sampler design for spatial filtering in large-scale setup required

Budgeted Sparse Sensing

- Why?
  - possibly many non-informative measurements
  - reduces processing overhead
  - economical or physical constraints
  - sensors might incur different operation costs, e.g., energy requirements
- How?
  - **convex optimization:** through selection vector $\boldsymbol{w} \in \{0,1\}^M$
    [Joshi-Boyd-09][1], [Chepuri-Leus-16][2]
  - **submodular optimization:** greedy methods and heuristics
    [Krause-Guestrin-07][3],[Ranieri-Chebira-Vetterli-14][4]

Cross-pollination possible?

[1] S. Joshi, and S. Boyd. "Sensor selection via convex optimization." *TSP* 2009

[2] S.P. Chepuri, and G. Leus, "Sparse Sensing for Statistical Inference," *Foundations and Trends in Sig. Proc.* 2016

[3] A. Krause, and C. Guestrin, "Near-optimal observation selection using submodular functions," *AAAI* 2007

[4] J. Ranieri, et al., "Near-optimal sensor placement for linear inverse problems," *TSP* 2014

Prior Art

- **Convex Methods**
  - Emura, S., "ℓ_1-constrained MVDR-based selection of nonidentical directivities in microphone array.", ICASSP 2015

    ℓ_1 regularization for filter coefficients
  - Zhang, J., et al., "Microphone Subset Selection for MVDR Beamformer Based Noise Reduction.", arXiv preprint arXiv:1705.08255 (2017),

    model-driven and data-driven by SDP formulations
- **Greedy Methods**
  - A. Bertrand and M. Moonen, "Efficient sensor subset selection and link failure response for linear mmse signal estimation in wireless sensor networks.", EUSIPCO 2010

    based on MSE cost for speech signal estimation
  - J. Szurley, et al., "Energy aware greedy subset selection for speech enhancement in wireless acoustic sensor networks.", EUSIPCO 2012

    based on SNR gain for speech enhancement

Submodularity

Notation

- finite ground set $\mathcal{V} \in \{1, \ldots, M\}$ -available sensors
- set function $f : 2^{\mathcal{V}} \rightarrow \mathbb{R}$ -performance metric
- selection variable $\boldsymbol{w} \in \{0, 1\}^M$ -selected vectors
- arbitrary subset - chosen sensor set

$$\mathcal{S} = \{m | \boldsymbol{w}_m = 1, m \in \mathcal{V}\}$$

Submodular set function (*diminishing return property*)

A set function is submodular if $\forall\, \mathcal{S} \subseteq \mathcal{T} \subset \mathcal{V}, s \in \mathcal{V} \setminus \mathcal{T}$ it holds that

$$f(\mathcal{S} \cup \{s\}) - f(\mathcal{S}) \geq f(\mathcal{T} \cup \{s\}) - f(\mathcal{T})$$

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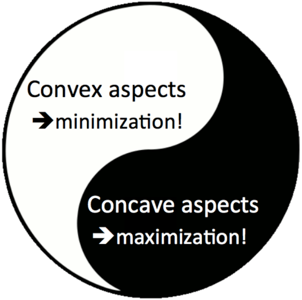
Main Results



- Dual aspects [Lovász-83][5]
- Near-optimal maximization [Nemhauser-Wolsey-Fisher-78][6]
- Exact unconstrained minimization [Fujishige-Isotani-11][7]

[5] L. Lovász, "Submodular functions and convexity," 1983.
[6] Nemhauser, G. L., et al., "An analysis of approximations for maximizing submodular set functions", 1978
[7] Fujishige, S., et al., "A submodular function minimization algorithm based on the minimum-norm base.", 2011

The MVDR Beamformer Problem

- Array Data Model

$$\boldsymbol{x} = \boldsymbol{a}(\theta)s + \boldsymbol{n} \in \mathbb{C}^M$$

  - $\boldsymbol{a}(\theta) \in \mathbb{C}^M$ – array steering vector
  - $s \sim \mathcal{CN}(0, \sigma_s^2)$ – signal of interest
  - $\boldsymbol{n} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_n)$ – noise + interference
  - θ – direction of arrival

- MVDR Beamformer

Optimization problem

$$\begin{aligned} \underset{\boldsymbol{z}\in\mathbb{C}^M}{\text{minimize}} \quad & \boldsymbol{z}^H \boldsymbol{R}_x \boldsymbol{z} \\ \text{subject to} \quad & \boldsymbol{z}^H \boldsymbol{a}(\theta) = 1 \end{aligned}$$

Optimal Solution

$$\boldsymbol{z}^* = \frac{\boldsymbol{R}_n^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\boldsymbol{R}_n^{-1}\boldsymbol{a}(\theta)}$$

where $\boldsymbol{R}_x = \sigma_s^2 \boldsymbol{a}(\theta)a(\theta)^H + \boldsymbol{R}_n \in \mathbb{C}^{M\times M}$.

alternative: sparse beamformers [Nguyen-et.al-09][8] [Emura-15][9]

[8] Nguyen, N., et al., "Sparse beamforming for active underwater electrolocation." ICASSP 2009

[9] Emura, S., "ℓ_1-constrained MVDR-based selection of nonidentical directivities in microphone array.", ICASSP 2015

Sensor Selection for MVDR

- Modular Constrained MVDR Beamformer

$$\begin{aligned} \underset{\mathcal{S}}{\text{maximize}} \quad & f(\mathcal{S}) \\ \text{subject to} \quad & B(\mathcal{S}) \leq \beta, |\mathcal{S}| = K \end{aligned}$$

$$f(\mathcal{S}) := \boldsymbol{a}_{\mathcal{S}}(\theta) \boldsymbol{R}_{n,\mathcal{S}}^{-1} \boldsymbol{a}_{\mathcal{S}}(\theta) \text{ (\textit{output SNR})}$$

$$B(\mathcal{S}) = \sum_{i \in \mathcal{S}} b_i, \quad \text{(\textit{modular set function})}$$

b_i is the cost related to the ith sensor in $\mathcal{S}$.

Sensor Selection for MVDR

Design Problem

Cost function: Output SNR

$$f(\boldsymbol{w}) = \mathbf{a}^H(\theta)[\mathbf{S}^{-1} - \mathbf{S}^{-1}(\mathbf{S}^{-1} + a^{-1}\text{diag}(\mathbf{w}))^{-1}\mathbf{S}^{-1}]\mathbf{a}(\theta),$$

where $\boldsymbol{R}_n = \boldsymbol{S} + a\boldsymbol{I}$.

Convex Problem [Chepuri-Leus-16][10]

$$\begin{aligned} \underset{\mathbf{w},t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & \\ & \mathbf{w}^T\mathbf{b} \leq \beta, \ \|\mathbf{w}\|_1 = K, \\ & \mathbf{w} \in [0,1]^{M\times 1} \\ & \begin{bmatrix} \mathbf{S}^{-1} + a^{-1}\text{diag}(\mathbf{w}) & \mathbf{S}^{-1}\mathbf{a}(\theta) \\ \mathbf{a}^H(\theta)\mathbf{S}^{-1} & t \end{bmatrix} \succeq 0, \end{aligned}$$

[10] S.P. Chepuri, and G. Leus, "Sparse Sensing for Statistical Inference," *Foundations and Trends in Sig. Proc.* 2016

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Sensor Selection for MVDR

Design Problem

Submodular Problem [this work]

$$\begin{aligned} \underset{\mathcal{S}}{\text{maximize}} \quad & \ln\det(\boldsymbol{M}_{\mathcal{S}}) \\ \text{subject to} \quad & B(\mathcal{S}) \leq \beta, \ |\mathcal{S}| = K \end{aligned}$$

where

$$\mathbf{M}_{\mathcal{S}} = \begin{bmatrix} \mathbf{S}^{-1} + a^{-1}\mathbf{I}_{\mathcal{S}} & \mathbf{S}^{-1}\mathbf{a}(\theta) \\ \mathbf{a}^{H}(\theta)\mathbf{S}^{-1} & \mathbf{a}^{H}(\theta)\mathbf{S}^{-1}\mathbf{a}(\theta) \end{bmatrix}$$

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Sensor Selection for MVDR

Proposed Submodular Design

$$f(\mathcal{S}) = \begin{cases} 0 & \mathcal{S} = \emptyset \\ \ln\det \begin{bmatrix} \mathbf{S}^{-1} + a^{-1}\mathbf{I}_{\mathcal{S}} & \mathbf{S}^{-1}\mathbf{a}(\theta) \\ \mathbf{a}^H(\theta)\mathbf{S}^{-1} & \mathbf{a}^H(\theta)\mathbf{S}^{-1}\mathbf{a}(\theta) \end{bmatrix} & \text{otherwise} \end{cases}$$

- The proposed cost set function is [this work]
  - monotone and normalized
  - submodular in $\mathcal{S}$,

  therefore $\rightarrow$ near-optimal optimization through greedy heuristics.
- Moreover,
  - it has a recursive description that allows linear-time optimization and does not require inversion of $\boldsymbol{S}$ [Coutino-Chepuri-Leus-17].
  - establish a link with state-of-the-art convex methods.

Sensor Selection for MVDR

Proposed Submodular Design

- with uniform cost, i.e, $\beta = K, b_i = b_j \; \forall \; i, j$ [Nemhauser, et al.-78]

$$f(\mathcal{S}_{\mathrm{uc}}) \geq (1 - 1/e) f(\mathcal{S}_{\mathrm{opt}})$$

- for non-uniform costs [Leskovec-07][11]

$$\max\{\mathrm{f}(\mathcal{S}_{\mathrm{uc}}), f(\mathcal{S}_{dc})\} \geq \frac{1}{2}(1 - 1/e) f(\mathcal{S}_{\mathrm{opt}})$$

Algorithm 1: Cost-Benefit Greedy.

Result: $\mathcal{A} : |\mathcal{A}| = K$

```
initialization A = ∅, k = 0;
while k < K and B(A) < β do
    a* = arg max_{a∈V} (f(A∪{a}) − f(A)) / b_a;
    if B(A ∪ {a*}) ≤ β then
        A = A ∪ {a*};
        k = k + 1;
    end
    V = V \ a*;
end
```

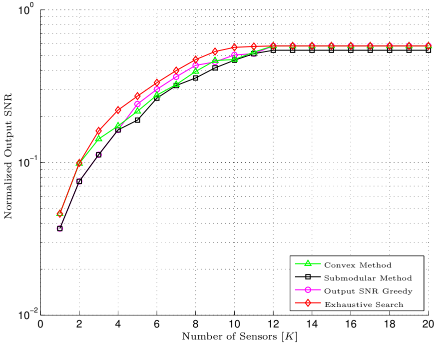
– $\mathcal{S}_{\mathrm{dc}}$ -cost benefit solution
– $\mathcal{S}_{\mathrm{uc}}$ -uniform cost solution $\rightarrow a^* = \arg\max_{a \in \mathcal{V}} f(\mathcal{A} \cup \{a\}) - f(\mathcal{A})$

[11] Leskovec, J., et al. "Cost-effective outbreak detection in networks." SIGKDD, 2007.

Array signal processing example (1)

Settings:

- half wavelength linear array with $M = 20$ elements.
- $b_i \sim \mathcal{U}[0,1]\ \forall\ i$
- $\beta = 0.8 \sum_{i \in \mathcal{V}} b_i$
- DoA of interest $\theta = -20^o$
- interferer at $\theta_i = -10^o$
- white Gaussian noise at $-10dB$



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Array signal processing example (2)

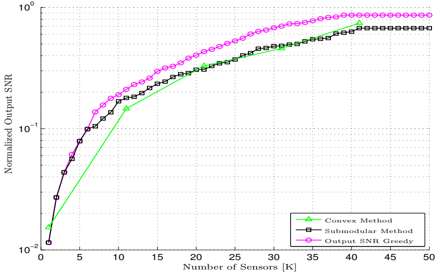

Figure: Output SNR for the different methods with $M = 50$.

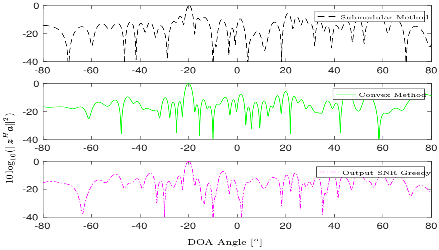

Figure: Beam pattern when $K = 21$ sensors are selected out of $M = 50$.

Conclusions

- Using similar techniques as in convex relaxations, it is possible to find submodular surrogates for optimizing complex cost set functions.
- The submodular machinery allows the application of a greedy heuristic, of linear complexity, for finding near-optimal solutions.
- The proposed greedy approach provides performance comparable to the one based on convex relaxation at a significantly reduced complexity.
- Outlook
  - Can we systematically find submodular relaxations for different set functions as in the convex cases?
  - Are there stronger theoretical guarantees for the submodular surrogate functions?

Thank you!

Questions?