DOA Estimation And Beamforming Using Under-Sampled AVS Arrays

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Introduction

Acoustic vector sensors (AVSs) are capable of measuring acoustic particle velocity and pressure. Its applications include:
- Acoustic situational awareness
- Noise localization and tracking
- Indoor Acoustics
- NVH analysis in automotive

Considering under-sampled AVS array, the questions addressed in this work include:

- Study the effect of grating lobes for unambiguous DOA estimation
- Performance analysis of classical and MVDR beamformer based DOA estimation
- The behavior of the beamformers for the interference cancellation tasks.

Preliminaries

The data with M AVSs and D far-field narrow-band sources can be modeled as:

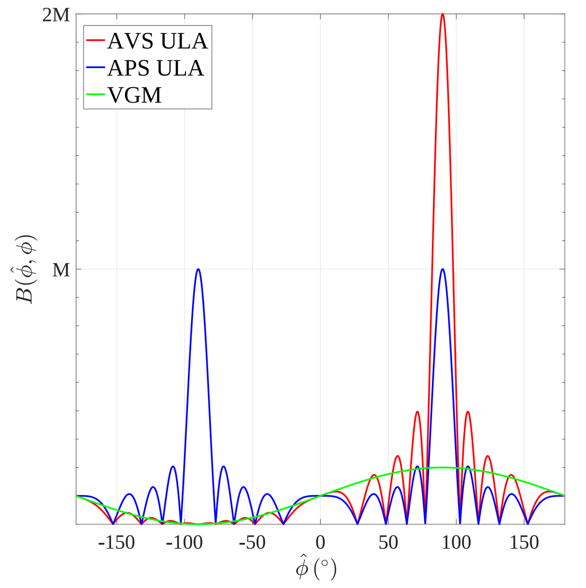
$$\boldsymbol{y}(t) = \sum_{i=1}^{D} \underbrace{(\boldsymbol{a}_p(\phi_i) \otimes \boldsymbol{h}(\phi_i))}_{\boldsymbol{a}(\phi_i)} s_i(t) + \boldsymbol{n}(t) \in \mathbb{C}^{3M\times 1}, \quad (1)$$

$$\boldsymbol{a}_p(\phi_i) = \left[e^{j2\pi(\boldsymbol{r}_1^T\boldsymbol{u}(\phi_i))} \ \ldots \ e^{j2\pi(\boldsymbol{r}_M^T\boldsymbol{u}(\phi_i))}\right]^T \in \mathbb{C}^{M\times 1}, \quad \boldsymbol{h}(\phi_i) = \left[1 \ \cos(\phi_i) \ \sin(\phi_i)\right]^T.$$

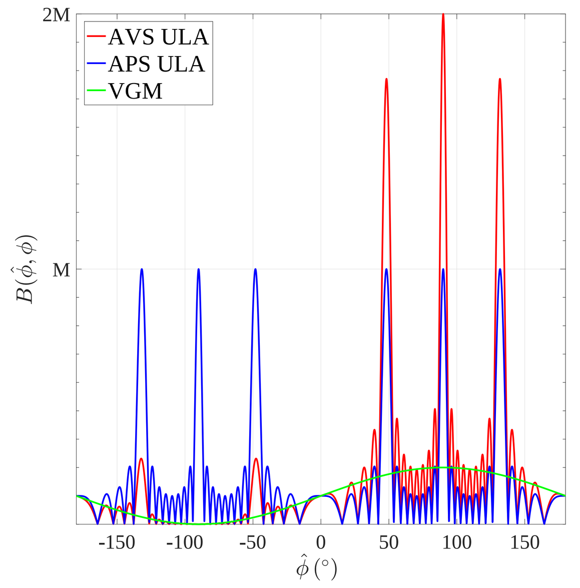
The beam pattern of an AVS array for a single source (at DOA ϕ):

$$B(\hat{\phi},\phi) = \left|\boldsymbol{a}^H(\hat{\phi})\boldsymbol{a}(\phi)\right| = \underbrace{\left(1+\cos(\phi-\hat{\phi})\right)}_{\mathsf{VGM}(\hat{\phi},\phi)} \cdot \underbrace{\left|\sum_{m=1}^{M} e^{j2\pi\left(\boldsymbol{r}_m^T\left(\boldsymbol{u}(\phi)-\boldsymbol{u}(\hat{\phi})\right)\right)}\right|}_{B_p(\hat{\phi},\phi)}. \quad (2)$$

The beam pattern of an AVS array in terms of an equivalent APS array and the VGM term,



(a) $r = 0.5$, $M = 9$, $\phi = 90°$



(b) $r = 1.5$, $M = 9$, $\phi = 90°$

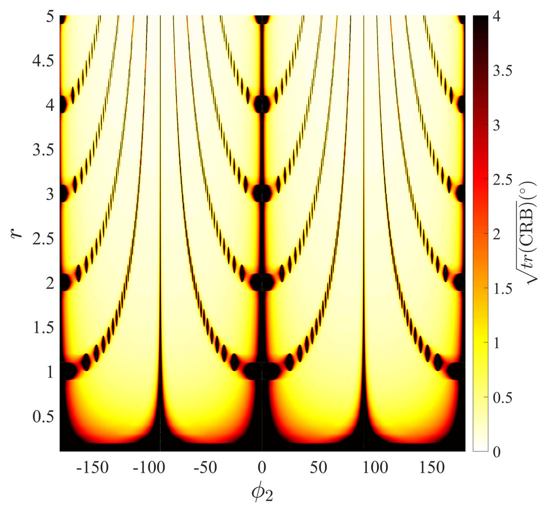
Cramér-Rao lower bound for DOA estimation

For a scenario with two uncorrelated sources (ϕ_1, ϕ_2), the CRLB can be written as:

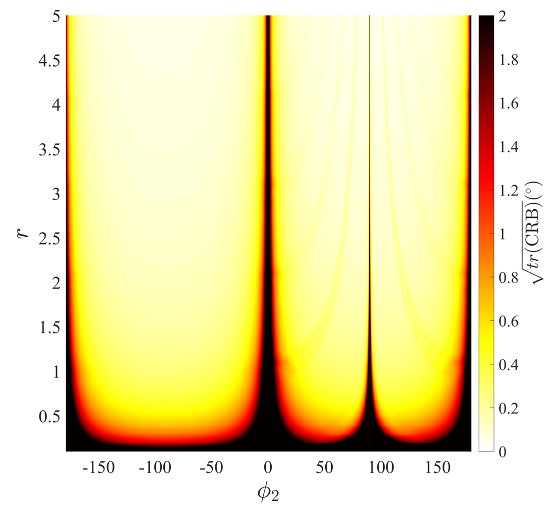
$$\mathsf{CRLB}(\boldsymbol{\phi}) \approx \frac{\sigma_n^2}{2N}\left(\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\right)^{-1}, \quad (3)$$

$$\frac{K_q}{\sigma_q^2} = 8\pi^2 \sum_{i=1}^{M}\left(\boldsymbol{r}_i^T \frac{\partial \boldsymbol{u}(\phi_q)}{\partial \phi_q}\right)^2 + M - \frac{2M\,(B_p(\phi_1,\phi_2))^2 \sin^2(\phi_1-\phi_2)}{4M^2 - (B_p(\phi_1,\phi_2))^2 (\mathsf{VGM}(\phi_1,\phi_2))^2}; \ \forall q = 1,2.$$

For two sources scenario with $M = 9$, $N = 10$, and $\phi_1 = 90°$,

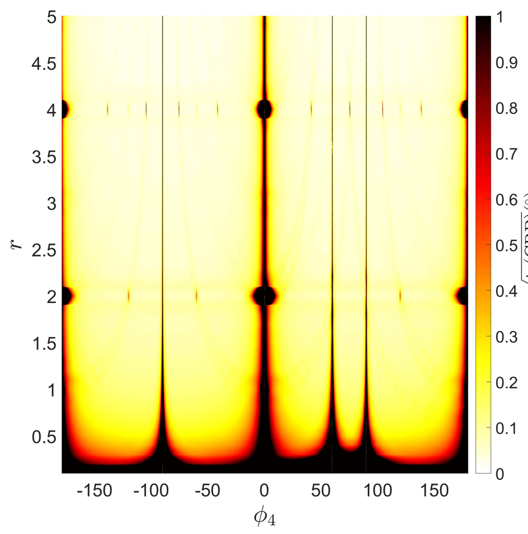


(a) APS ULA, SNR = 0 dB

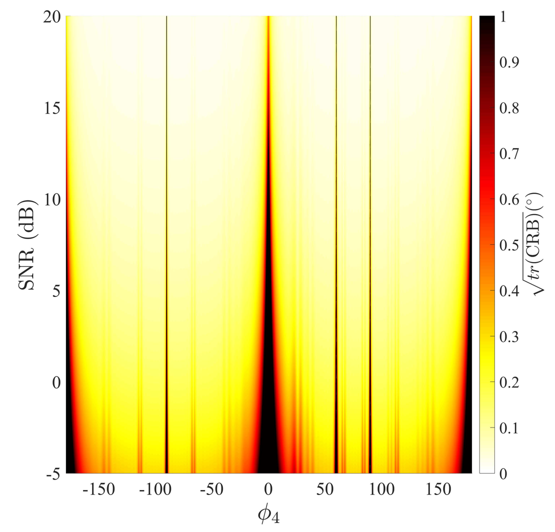


(b) AVS ULA, SNR = 0 dB

For multi-source scenario with $M = 9$, $N = 10$, $\phi_1 = 90°$, $\phi_2 = 60°$ and $\phi_3 = -90°$,



(a) AVS ULA, SNR = 10 dB



(b) AVS ULA, $r = 2.5$

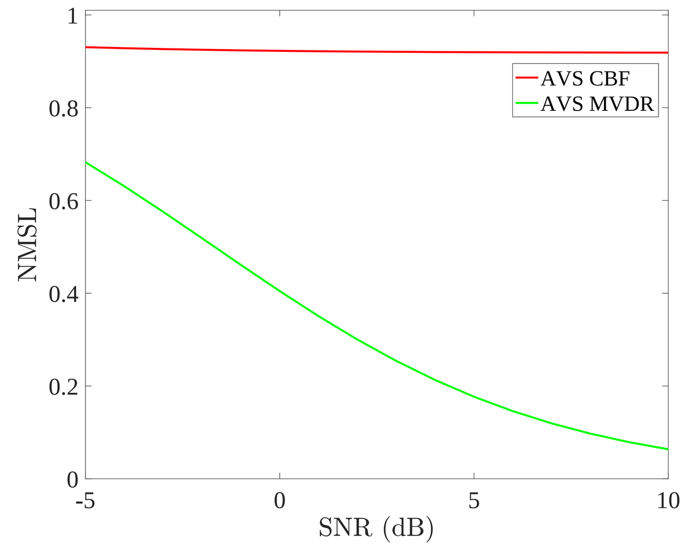
Classical and MVDR beamformer

Under single source scenario, the spectrum estimate based on Classical (CBF) and MVDR beamformer at the grating/main lobe locations:

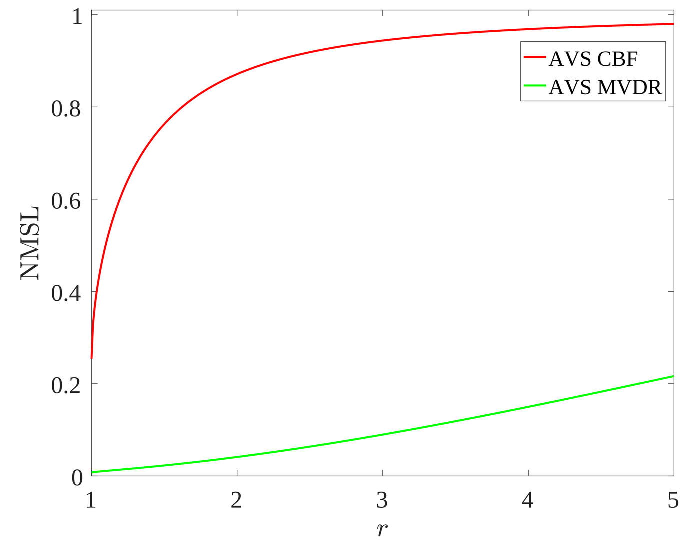
$$\mathsf{CBF}_v(\hat{\phi},\phi) = \frac{M^2(\mathsf{SNR})\left(\mathsf{VGM}(\hat{\phi},\phi)\right)^2 + 2M}{\mathsf{SNR}}, \quad (4)$$

$$\mathsf{MVDR}_v(\hat{\phi},\phi) = \frac{2M(\mathsf{SNR})+1}{2M(\mathsf{SNR}) + M^2(\mathsf{SNR})^2\left(4-\left(\mathsf{VGM}(\hat{\phi},\phi)\right)^2\right)}. \quad (5)$$

For $M = 9$, $\phi = 90°$, the variation of maximum side lobe level (MSL) for CBF and MVDR,



(a) $r = 2.5$



(b) SNR = 10 dB

Interference cancellation

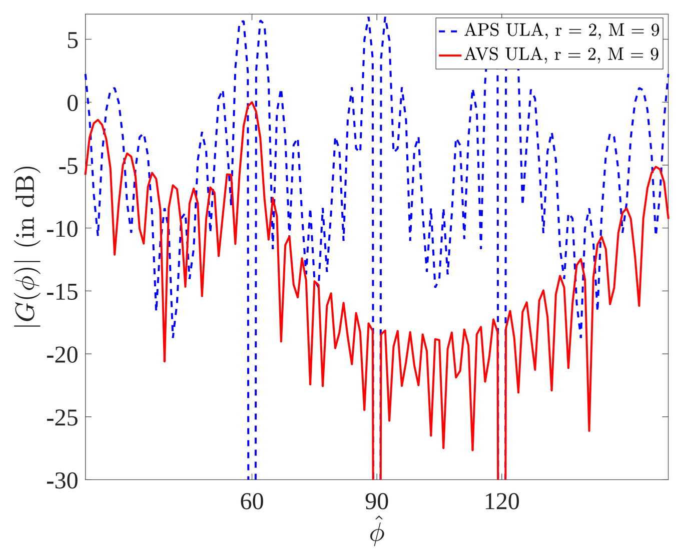
The beam pattern synthesis problem for an AVS array can be written as:

$$G(\phi) = \left|\sum_{m=1}^{M} w_{mp} e^{j2\pi(\boldsymbol{r}_m^T\boldsymbol{u}(\phi))} + \cos(\phi)\sum_{m=1}^{M} w_{mx} e^{j2\pi(\boldsymbol{r}_m^T\boldsymbol{u}(\phi))} + \sin(\phi)\sum_{m=1}^{M} w_{my} e^{j2\pi(\boldsymbol{r}_m^T\boldsymbol{u}(\phi))}\right| = \left|\boldsymbol{w}^H\boldsymbol{a}(\phi)\right|. \quad (6)$$

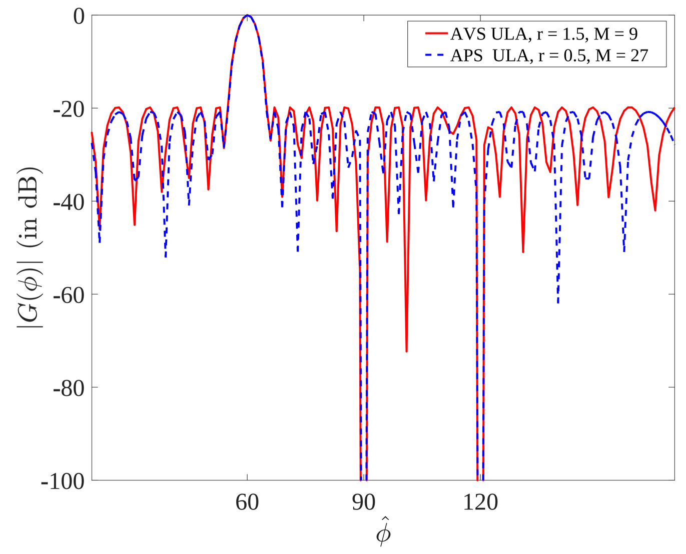
Formulation of a beamformer minimizing maximum level side lobes with distortion-less response:

$$\begin{aligned} \min_{\mathbf{w}} \ & \max_{\phi\in\mathcal{S}} |\mathbf{w}^H\mathbf{a}(\phi)|, \\ \text{subject to} \ & |\mathbf{w}^H\mathbf{a}(\phi)| \le \alpha; \ \forall\phi\in\mathcal{M} \\ & \mathbf{w}^H\mathbf{a}(\phi_0) = 1, \ \ \|\mathbf{w}\|_2 \le \beta \\ & \mathbf{w}^H\mathbf{a}(\phi) = 0; \ \forall\phi\in\mathcal{N}. \end{aligned} \quad (7)$$

Beam pattern synthesis for $\phi_0 = 60°$ and interference locations, $\phi_n = 90°, 120°$.,



(a) Zero Forcing beamformer



(b) Beamformer from (7), $\{\alpha,\beta\} = 1$

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