BLIND SUBSPACE-BASED UPLINK RECEIVER ALGORITHM FOR WIDEBAND CDMA

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We derive a blind subspace-based receiver algorithm suitable for asynchronous wideband CDMA with periodic codes in dispersive multipath channels. The algorithm is roughly equivalent to a recently presented algorithm by Wang and Poor (IEEE Tr. Comm., Jan. 1998), but is often computationally more efficient. The algorithm is readily extended to multicode and multirate systems.

1. INTRODUCTION

The easiest way to estimate MMSE equalizers for CDMA is to use a training sequence: this information, along with an accurate estimate of the covariance matrix of the received data and some i.i.d. assumptions on the symbols is sufficient. In actual wideband CDMA proposals, a rather large portion of the bandwidth is reserved for this training overhead. Although it provides for a robust channel (equalizer) estimation, it has been shown that this information is not essential: it is possible to estimate the channels blindly.

Several techniques have recently been proposed for blind multi-user CDMA detection [1–12]. Most of these assume periodic codes; we will do the same here. In principle, blind MIMO equalization ideas are directly applicable. However, more information is present, since for CDMA the channels are not arbitrary, but consist of the convolution of the physical channel with the known user code. This gives a chance to greatly reduce the complexity and enhance the performance of general blind algorithms.

Many of these blind techniques work only on channels with limited delay spread (much less than the symbol period). Two similar algorithms that provide an elegant solution to more general situations have recently been proposed in [11, 12]. The propagation channel is blindly estimated, based on the subspace structure associated with the code of the desired user, and the signal subspace of the observed data covariance matrix. In addition, the performance of this algorithm has been shown to be better than that of the 'minimum output energy' blind equalizer, especially for low SNRs [11]. Synchronization requirements are minimal. Moreover, it is very straightforward to reap the benefits of multiple antennas.

In this paper, we derive an new subspace-based algorithm that will generate roughly the same result as in [12], but in a computationally more efficient way if the number of users is smaller than half the maximal load. The algorithm is readily extended to multicode and multirate systems.

2. DATA MODEL

In CDMA systems, source signals (symbol sequences) are expanded by known codes, different for each user. Consider the source sequence of user q, $\mathbf{s}^q = [s_0^q \;\; s_1^q \;\; \cdots]$, with associated analog signal $s^q(t) = \sum s_k^q \delta(t-kT)$, where T is the symbol period, normalized to $T=1$. The expanded source sequence is $[s_0^q\mathbf{c}^q \;\; s_1^q\mathbf{c}^q \;\; \cdots]$, where $\mathbf{c}^q = [c_1^q \cdots c_P^q]$ is the code vector for this source, of length P chips. The corresponding analog signal is $u^q(t) = c^q(t) * s^q(t)$, with $c^q(t) = \sum c_k^q \delta(t-kT_c)$, where $T_c = T/P$.

The signal that is broadcast is $p(t) * c^q(t) * s^q(t)$, where $p(t)$ is the pulse shape function, typically a raised-cosine with pulse width $T_c = T/P$. Assume we have M receiving antennas, and let $\mathbf{x}(t)$ be a stack of the M received signals. Correspondingly, let $\mathbf{g}^q(t)$ be a stack of the M impulse responses of the overall "physical" multipath channel, including the pulse shape function. The received complex baseband signal is

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \mathbf{h}^q(t) * s^q(t)\,, \qquad \mathbf{h}^q(t) = \mathbf{g}^q(t) * c^q(t)\,,$$

where $\mathbf{h}^q(t)$ is the channel seen by the symbols. Over a single symbol period this can be written as

$$\begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+\frac{1}{P}) \\ \vdots \\ \mathbf{x}(k+\frac{P-1}{P}) \end{bmatrix} = \sum_q \underbrace{\begin{bmatrix} \mathbf{h}^q(0) & \cdots \mathbf{h}^q(L-1) \\ \mathbf{h}^q(\frac{1}{P}) & \cdot \\ \vdots & \vdots \\ \mathbf{h}^q(\frac{P-1}{P}) & \cdots \mathbf{h}^q(L-\frac{1}{P}) \end{bmatrix}}_{H^q} \begin{bmatrix} s_k^q \\ s_{k-1}^q \\ \vdots \\ s_{k-L+1}^q \end{bmatrix}$$

where L is the symbol channel length. Stacking m symbol

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periods in a single observation vector gives

$$\underline{\mathbf{x}}(k) := \begin{bmatrix} \mathbf{x}(k) \\ \vdots \\ \mathbf{x}(k+\frac{P-1}{P}) \\ \hline \mathbf{x}(k+1) \\ \vdots \\ \hline \vdots \\ \mathbf{x}(k+m-\frac{1}{P}) \end{bmatrix} = \sum_q \underbrace{\begin{bmatrix} \mathbf{0} & & \boxed{H^q} \\ & \cdot^{\cdot^{\cdot}} & \\ & \boxed{H^q} & \\ \boxed{H^q} & & \mathbf{0} \end{bmatrix}}_{\mathcal{H}^q} \begin{bmatrix} s^q_{k+m-1} \\ \vdots \\ s^q_{k-L+2} \\ s^q_{k-L+1} \end{bmatrix}.$$

Note how the columns of the matrix $\mathcal{H}^q$ contain the channel vector $\mathbf{h}^q$, at several shifts, and augmented with zeros. The structure of $\mathbf{h}^q$ itself is a convolution of the user code with the physical channel:

$$\begin{aligned} \mathbf{h}^q(t) &= c^q(t) * \mathbf{g}^q(t) \\ \Leftrightarrow \begin{bmatrix} \mathbf{h}^q(0) \\ \mathbf{h}^q(\frac{1}{P}) \\ \vdots \\ \vdots \\ \mathbf{h}^q(L-\frac{1}{P}) \end{bmatrix} &= \underbrace{\begin{bmatrix} c_1^q I & & \\ \vdots & \ddots & \\ c_P^q I & & c_1^q I \\ & \ddots & \vdots \\ & & c_P^q I \end{bmatrix}}_{\mathcal{C}^q} \begin{bmatrix} \mathbf{g}^q(0) \\ \mathbf{g}^q(\frac{1}{P}) \\ \vdots \\ \mathbf{g}^q(L-1) \end{bmatrix}. \end{aligned}$$

I is the identity matrix of size $M \times M$. The code matrix $\mathcal{C}^q$ is known, and tall, of size $MPL \times M[(L-1)P+1]$.

3. BLIND CHANNEL ESTIMATION ALGORITHMS

Define the data matrix

$$\mathcal{X} := [\underline{\mathbf{x}}(0) \;\; \underline{\mathbf{x}}(1) \;\; \cdots \;\; \underline{\mathbf{x}}(N-1)].$$

$\mathcal{X}$ can be written as $\mathcal{X} = \mathcal{H}\mathcal{S}$, where $\mathcal{H} := [\mathcal{H}^1 \; \cdots \; \mathcal{H}^q]$. Many blind channel identification algorithms are based on the fact that the column span of $\mathcal{H}$ is equal to the column span of $\mathcal{X}$. This requires that $\mathcal{H}$ is a tall matrix. Since $\mathcal{H}$ has size $mMP \times Q(m+L-1)$, this is the case if

$$Q < MP, \qquad m > \frac{Q(L-1)}{MP-Q}.$$

It is also necessary that $\mathcal{S}$ is wide and full rank: this puts conditions on the number of samples and persistency of excitation that are usually not very restrictive.

Although written differently, the algorithm in [11] is basically as follows:

1. Find a basis $\hat{U}$ for the column span of $\mathcal{X}$, e.g., from an SVD of $\mathcal{X}$. Under the above conditions, it is the same as the column span of $\mathcal{H}$, hence it contains the vector $\mathbf{h}^q$ (possibly augmented with zeros).
2. Find which vector in span$\hat{U}$ can be written as $\mathcal{C}^q\mathbf{g}^q$. This identifies $\mathbf{h}^q$, and it should be unique.
3. Find a suitable equalizer (MMSE or ZF).

Without noise, this algorithm finds all user channels *exactly*, even with a low number of samples ($N > mMP$).

The algorithm in [12] can be viewed as an extension of this algorithm, combining it with the ideas of blind channel identification in [13,14]. The point is that the vector $\mathbf{h}^q$ appears multiple times in $\mathcal{H}^q$, with different shifts. Thus, in step 2 of the algorithm, we can look for a vector that is not only in span$\hat{U}$, but also in its shifts. In [12], this property is expressed in terms of a basis $\hat{G}$ of the left null space of $\mathcal{X}$: $\mathbf{h}^q$ should be orthogonal to $\hat{G}$ and certain shifts of it. A problem with this approach is that the matrices involved will be very large, especially if the number of users is small.

4. SUBSPACE INTERSECTION ALGORITHM

We will now derive a new algorithm that computes roughly the same result as in [12] but works with matrices that can be much smaller. For simplicity of exposition and without loss of generality, assume $L = 2$, $m = 2$, which is a typical situation in CDMA. Let $\hat{U}$ be an orthonormal basis of span$\mathcal{X}$, as obtained e.g., from its SVD. Then

$$\mathcal{H}^q = \begin{bmatrix} 0 & \mathbf{h}_1^q & \mathbf{h}_2^q \\ \mathbf{h}_1^q & \mathbf{h}_2^q & 0 \end{bmatrix} \in \operatorname{span}\hat{U} =: \operatorname{span}\begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{bmatrix}.$$

Thus

$$\begin{aligned} \begin{bmatrix} 0 \\ \mathbf{h}_1^q \end{bmatrix} &\in \operatorname{span}\begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{bmatrix} \\ \begin{bmatrix} \mathbf{h}_1^q \\ \mathbf{h}_2^q \end{bmatrix} &\in \operatorname{span}\begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{bmatrix} \\ \begin{bmatrix} \mathbf{h}_2^q \\ 0 \end{bmatrix} &\in \operatorname{span}\begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{bmatrix}. \end{aligned}$$

This can be combined in a single expression as

$$\begin{aligned} \begin{bmatrix} 0 \\ \mathbf{h}_1^q \\ \mathbf{h}_2^q \\ 0 \end{bmatrix} \in \operatorname{span}\begin{bmatrix} \hat{U}_1 & & \\ \hat{U}_2 & & \\ & I & \\ & & I \end{bmatrix} &\cap \operatorname{span}\begin{bmatrix} I & & \\ & \hat{U}_1 & \\ & \hat{U}_2 & \\ & & I \end{bmatrix} \\ &\cap \operatorname{span}\begin{bmatrix} I & & \\ & I & \\ & & \hat{U}_1 \\ & & \hat{U}_2 \end{bmatrix}. \end{aligned} \tag{1}$$

(The identity matrices span the full space hence pose no constraints.) This holds for $q = 1, \cdots, Q$. Further, to select the user of interest, we have the condition that $\mathbf{h}^q = \mathcal{C}^q\mathbf{g}^q$. Without constraints on $\mathbf{g}^q$, this can be formulated as another subspace intersection condition:

$$\mathbf{h}^q \in \operatorname{span}\mathcal{C}^q. \tag{2}$$

We have thus formulated the blind channel estimation problem as a subspace intersection problem. With noise, there are many ways to solve such problems. A major difference is that between Least Squares (LS) and Total Least

Squares (TLS) techniques. LS is used in cases where the solution has to be exactly in one of the subspaces (because it is noise-free), here this would be the case with the code subspace in (2). TLS is used when all subspaces are about equally noisy, here the channel subspaces in (1). Various algorithms are listed in the appendix.

As shown in the appendix, there are two ways to solve the intersection problem in a TLS sense (i.e., assuming all subspaces have equal accuracy). In the first approach, we compute the complement of the sum of complements. Thus denote by $\hat{G}$ a basis for the orthogonal complement of span$U$, and partition it as

$$\hat{U} \perp \hat{G} \qquad \Leftrightarrow \qquad \begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{bmatrix} \perp \begin{bmatrix} \hat{G}_1 \\ \hat{G}_2 \end{bmatrix} .$$

Similarly as before, we have

$$\begin{aligned} \begin{bmatrix} 0 \\ \mathbf{h}_1^q \end{bmatrix} &\perp \ \text{span} \begin{bmatrix} \hat{G}_1 \\ \hat{G}_2 \end{bmatrix} \\ \begin{bmatrix} \mathbf{h}_1^q \\ \mathbf{h}_2^q \end{bmatrix} &\perp \ \text{span} \begin{bmatrix} \hat{G}_1 \\ \hat{G}_2 \end{bmatrix} \\ \begin{bmatrix} \mathbf{h}_2^q \\ 0 \end{bmatrix} &\perp \ \text{span} \begin{bmatrix} \hat{G}_1 \\ \hat{G}_2 \end{bmatrix} . \end{aligned}$$

This can be combined in a single expression as

$$\begin{aligned} & \begin{bmatrix} 0 \\ \mathbf{h}_1^q \\ \mathbf{h}_2^q \\ 0 \end{bmatrix} \perp \begin{bmatrix} \hat{G}_1 & & \\ \hat{G}_2 & \hat{G}_1 & \\ & \hat{G}_2 & \hat{G}_1 \\ & & \hat{G}_2 \end{bmatrix} \\ \Leftrightarrow \quad & \left[\begin{array}{c|cc|c} \hat{G}_1^* & \hat{G}_2^* & & \\ & \hat{G}_1^* & \hat{G}_2^* & \\ & & \hat{G}_1^* & \hat{G}_2^* \end{array} \right] \begin{bmatrix} 0 \\ \mathbf{h}_1^q \\ \mathbf{h}_2^q \\ 0 \end{bmatrix} = 0 \\ \Rightarrow \quad & \mathcal{G}^* \mathbf{h}^q := \begin{bmatrix} \hat{G}_2^* & \\ \hat{G}_1^* & \hat{G}_2^* \\ & \hat{G}_1^* \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^q \\ \mathbf{h}_2^q \end{bmatrix} = 0 . \end{aligned}$$

The solution is given by the 'smallest' right singular vector of the matrix $\mathcal{G}^*$. This is the blind channel identification algorithm originally proposed by Moulines et al. [13].

In the second approach, we start from (1), stack the (orthonormal) bases of the subspaces that we want to intersect, and compute the largest left singular vector:

$$\begin{aligned} \begin{bmatrix} 0 \\ \mathbf{h}_1^q \\ \mathbf{h}_2^q \\ 0 \end{bmatrix} &= \max_{\text{left sv}} \left[\begin{array}{ccc|ccc|ccc} \hat{U}_1 & & & I & & & I & & \\ \hat{U}_2 & & & & \hat{U}_1 & & & I & \\ & I & & & \hat{U}_2 & & & & \hat{U}_1 \\ & & I & & & I & & & \hat{U}_2 \end{array} \right] \\ &= \max_{\text{left sv}} \left[\begin{array}{ccc|cccc} \hat{U}_1 & & & \sqrt{2}I & & & \\ \hline \hat{U}_2 & \hat{U}_1 & & & I & & \\ & \hat{U}_2 & \hat{U}_1 & & & I & \\ \hline & & \hat{U}_2 & & & & \sqrt{2}I \end{array} \right] . \end{aligned}$$

The constraints posed by the zero entries of the channel vector lead to the following simplification:

$$\begin{aligned} \mathbf{h}^q = \begin{bmatrix} \mathbf{h}_1^q \\ \mathbf{h}_2^q \end{bmatrix} &= \max_{\text{left sv}} \left[\begin{array}{ccc|cc} \hat{U}_2 & \hat{U}_1 & & I & \\ & \hat{U}_2 & \hat{U}_1 & & I \end{array} \right] \\ &= \max_{\text{left sv}} \begin{bmatrix} \hat{U}_2 & \hat{U}_1 & \\ & \hat{U}_2 & \hat{U}_1 \end{bmatrix} . \end{aligned}$$

Thus, we showed that the vectors $[\mathbf{h}^1, \cdots, \mathbf{h}^Q]$ are in the space spanned by the Q dominant left singular vectors (in the noise free case corresponding to singular values $\sqrt{2}$) of

$$\mathcal{U} := \begin{bmatrix} \hat{U}_2 & \hat{U}_1 & \\ & \hat{U}_2 & \hat{U}_1 \end{bmatrix} . \tag{3}$$

This is a new algorithm which gives the same solution as in Moulines' approach. It is attractive in cases where the subspaces to be intersected have low dimensions (this is usually the case), since then the complementary subspaces are large: $\mathcal{G}$ is typically a larger matrix than $\mathcal{U}$.

Finally, to select the user of interest, we have to implement the condition (2) that $\mathbf{h}^q \in \text{span}\mathcal{C}^q$, or $\mathbf{h}^q = \mathcal{C}^q \mathbf{g}^q$. Since $\mathcal{C}^q$ is exact, we prefer to solve this subspace constraint in a LS sense. Again, there are two approaches (see the appendix). Let U^q be an orthonormal basis with the same column span as $\mathcal{C}^q$. We can thus write $\mathbf{h}^q = U^q \mathbf{x}^q$ for some vector $\mathbf{x}^q$ related one-to-one to $\mathbf{g}^q$. In the first approach, we continue from the equation $\mathcal{G}^* \mathbf{h}^q = 0$ and substitute, giving

$$\mathcal{G}^* U^q \mathbf{x}^q \; = \; \begin{bmatrix} \hat{G}_2^* & \\ \hat{G}_1^* & \hat{G}_2^* \\ & \hat{G}_1^* \end{bmatrix} U^q \mathbf{x}^q \; = \; 0 . \tag{4}$$

The solution is given by the smallest right singular vector of $\mathcal{G}^* U^q$. This is roughly the approach presented in [12]: in that paper, the matrix $\mathcal{C}^q$ is used rather than the orthonormal U^q, so it does not find exactly the LS solution.

Alternatively, we continue from $\mathcal{U}$ in (3). Following the appendix, we find $\mathbf{x}^q$ as the largest left singular vector of

$$U^{q*} \mathcal{U} \; = \; U^{q*} \begin{bmatrix} \hat{U}_2 & \hat{U}_1 & \\ & \hat{U}_2 & \hat{U}_1 \end{bmatrix} , \tag{5}$$

and then compute $\mathbf{h} = U^q \mathbf{x}$. The corresponding singular vector is equal to $\sqrt{2}$ in the noise-free case.

In summary, the new algorithm for blind CDMA channel recovery becomes as follows.

1. Find a basis $\hat{U}$ for the principal column span of $\mathcal{X}$.
2. Let U^q be an orthonormal basis for the column span of $\mathcal{C}^q$. Compute the dominant left singular vector $\mathbf{x}^q$ of $U^{q*} \mathcal{U}$.
3. The channel of the user of interest is given by

   $$\mathbf{h}^q = \alpha U^q \mathbf{x}^q$$

   for some unknown scaling α. It can subsequently be used to construct a ZF or MMSE equalizer.

The algorithm is simply extended for values of m larger than 2. For largely loaded systems with many users of interest, we would make the algorithm more efficient by first computing the dominant singular vectors of $\mathcal{U}$, and then implementing the code constraints for each user of interest on these vectors.

Since they give the same solutions, preference for solving either (4) or (5) follows from a dimension consideration. The size of the matrix product in (4) can be derived as

$$\mathcal{G}^* U^q : \quad mMP(m+L-1) - Q(m+L-1)^2 \times M[(L-1)P+1]$$

The matrix product in (5) has dimensions

$$U^{q*}\mathcal{U} : \quad M[(L-1)P+1] \times (m+L-1)^2 Q .$$

The first is large for small number of users Q, and decreases linearly in Q. The second is small for small Q and increases linearly in Q. Thus, there is a threshold in Q below which the new approach using $\mathcal{U}$ is more efficient: this is the case when

$$\begin{array}{rrcl} & (m+L-1)^2 Q & < & mMP(m+L-1) - Q(m+L-1)^2 \\ \Rightarrow & 2(m+L-1)Q & < & mMP \\ \Rightarrow & Q & < & \dfrac{MP}{2}\dfrac{1}{1+\frac{L-1}{m}} . \end{array}$$

For CDMA, we usually have $L = 2$ and $m = 2$ (hence $Q < \frac{1}{2}MP$ to have $\mathcal{H}$ tall). In that case, the new algorithm is more efficient if $Q < \frac{1}{3}MP$. For large m, the threshold is $\frac{1}{2}MP$, half of the maximal load.

5. EXTENSIONS

5.1. Multicode systems

Many proposed CDMA systems have a provision to assign more than one code to a single user, to allow him to increase his data rate. Such a user is equivalent to two (or more) virtual users, but we can obtain an improved channel estimate since we know that the physical channel is the same for each of his codes. Assuming that a user has physical channel $\mathbf{g}$, code vectors $\mathbf{c}^1$, $\mathbf{c}^2$, with corresponding Sylvester matrices $\mathcal{C}^1$, $\mathcal{C}^2$, we have

$$\mathbf{h}^1 = \mathcal{C}^1\mathbf{g}, \qquad \mathbf{h}^2 = \mathcal{C}^2\mathbf{g} \qquad \Leftrightarrow \qquad \begin{bmatrix}\mathbf{h}^1\\ \mathbf{h}^2\end{bmatrix} = \begin{bmatrix}\mathcal{C}^1\\ \mathcal{C}^2\end{bmatrix}\mathbf{g}.$$

In the approach with $\mathcal{G}$, we can simply find a combined estimate for $\mathbf{g}$ via

$$\begin{bmatrix}\mathcal{G}^*\mathcal{C}^1\\ \mathcal{G}^*\mathcal{C}^2\end{bmatrix}\mathbf{g} = 0 .$$

To have a LS approach, we need to orthonormalize the basis formed by the stacked code matrices:

$$\begin{bmatrix}\mathbf{h}^1\\ \mathbf{h}^2\end{bmatrix} = \begin{bmatrix}\mathcal{C}^1\\ \mathcal{C}^2\end{bmatrix}\mathbf{g} = \begin{bmatrix}U^1\\ U^2\end{bmatrix}\mathbf{x} =: U^q\mathbf{x} .$$

The LS version acting on $\mathcal{G}$ then becomes

$$\begin{bmatrix}\mathcal{G}^*U^1\\ \mathcal{G}^*U^2\end{bmatrix}\mathbf{x} = 0$$

The complementary version acting on $\mathcal{U}$ finds the same $\mathbf{x}$ as

$$\mathbf{x} = \max_{\text{left sv}}[U^{1*} \quad U^{2*}]\left[\begin{array}{c|c}\mathcal{U} & \\ \hline & \mathcal{U}\end{array}\right] = \max_{\text{left sv}}[U^{1*}\mathcal{U} \quad U^{2*}\mathcal{U}] .$$

If desired, the physical channel $\mathbf{g}$ can be computed from $\mathbf{x}$.

5.2. Multirate systems

Another popular way to increase the data rate of a user is to provide him with a shorter code. For example, suppose code vector $\mathbf{c}^1$ has length $\frac{1}{2}P$. Then user 1 will transmit

$$[s_1\mathbf{c}^1 \quad s_2\mathbf{c}^1 \quad | \quad s_3\mathbf{c}^1 \quad s_4\mathbf{c}^1 \quad | \quad \cdots] ,$$

where the partitioning is in segments of length P. This can be fitted into the previous framework by writing

$$\begin{array}{l} [s_1\mathbf{c}^1 \quad s_2\mathbf{c}^1 \quad | \quad s_3\mathbf{c}^1 \quad s_4\mathbf{c}^1 \quad | \quad \cdots] \\ = \quad [s_1[\mathbf{c}^1 \quad \mathbf{0}] \quad | \quad s_3[\mathbf{c}^1 \quad \mathbf{0}] \quad | \quad \cdots] \\ \quad + [s_2[\mathbf{0} \quad \mathbf{c}^1] \quad | \quad s_4[\mathbf{0} \quad \mathbf{c}^1] \quad | \quad \cdots] \end{array}$$

where $\mathbf{0}$ denotes a row vector consisting of $\frac{1}{2}P$ concatenated zeros. It thus appears that there are two users, with symbol sequences $[s_1 \quad s_3 \quad \cdots]$ and $[s_2 \quad s_4 \quad \cdots]$, and code sequences $[\mathbf{c}^1 \quad \mathbf{0}]$ and $[\mathbf{0} \quad \mathbf{c}^1]$, respectively. The physical channels of these two virtual users are the same. This brings us back to the multicode case, with the same solution for the blind channel estimation.

A complication in this approach is that it assumes that all channels have length $L = 2$ symbol periods. This might not be the case if physical channels are shorter than $\frac{1}{2}P$ chips. In that case, the rank of $\mathcal{X}$ will be smaller than what would be expected from the previous, and the intersection scheme needs to be modified.

A. COMPUTING SUBSPACE INTERSECTIONS

Suppose we have two subspaces, $\mathcal{H}_1$ and $\mathcal{H}_2$, with orthonormal bases U_1, U_2. The orthogonal complements of these spaces are $\mathcal{H}_1^\perp$, $\mathcal{H}_2^\perp$, with orthonormal bases denoted by G_1, G_2.

We wish to compute vectors that are in the intersection of $\mathcal{H}_1$ and $\mathcal{H}_2$, i.e.,

$$\mathbf{h} \in \mathcal{H}_1 \cap \mathcal{H}_2 = [\mathcal{H}_1^\perp \dot{+} \mathcal{H}_2^\perp]^\perp .$$

The latter equation computes the intersection by taking the complements of the combined span of both complements. In this scheme, we would compute $\mathbf{h}$ as the solution to

$$\left\{\begin{array}{rcl} G_1^*\mathbf{h} & = & 0 \\ G_2^*\mathbf{h} & = & 0 \end{array}\right. \qquad \Leftrightarrow \qquad \begin{bmatrix}G_1^*\\ G_2^*\end{bmatrix}\mathbf{h} = 0 ,$$

or in the presence of noise and equal weights

$$\arg\min_{\|\mathbf{h}\|=1} \| \begin{bmatrix} G_1^* \\ G_2^* \end{bmatrix} \mathbf{h} \| .$$

The solution is given by the 'smallest' right singular vector of $[G_1 \;\; G_2]^*$.

Alternatively, we can derive

$$\begin{aligned}
\arg\min_{\|\mathbf{h}\|^2=1} \| \begin{bmatrix} G_1^* \\ G_2^* \end{bmatrix} \mathbf{h} \|^2 &= \arg\min_{\|\mathbf{h}\|^2=1} \mathbf{h}^*(G_1G_1^* + G_2G_2^*)\mathbf{h} \\
&= \arg\min_{\|\mathbf{h}\|^2=1} \mathbf{h}^*(2I - U_1U_1^* - U_2U_2^*)\mathbf{h} \\
&= \arg\max_{\|\mathbf{h}\|^2=1} \mathbf{h}^*(U_1U_1^* + U_2U_2^*)\mathbf{h} \\
&= \arg\max_{\|\mathbf{h}\|^2=1} \| \mathbf{h}^*[U_1 \;\; U_2] \|^2 .
\end{aligned}$$

Thus, the same solution $\mathbf{h}$ is given by the principal left singular vector of $[U_1 \;\; U_2]$. If there is an exact solution in the intersection, then it is seen from the derivation that the corresponding largest singular value is $\sqrt{2}$. This can be viewed as a Total Least Squares approach.

If a subspace is considered less reliable, e.g., because of more noise, then we can downscale its basis, i.e., compute the principal singular vector of

$$[U_1 \quad \tfrac{1}{\alpha}U_2], \qquad \alpha > 1 .$$

In the limit (large α), the solution will be entirely in the column span of U_1. In that case,

$$\begin{aligned}
[U_1 \quad \tfrac{1}{\alpha}U_2] &\approx U_1U_1^*[U_1 \quad \tfrac{1}{\alpha}U_2] \\
&= U_1[I \quad \tfrac{1}{\alpha}U_1^*U_2] .
\end{aligned}$$

Since U_1 is orthonormal, the principal singular vector of the product can be computed as the principal left singular vector $\mathbf{x}$ of $U_1^*U_2$, followed by setting $\mathbf{h} = U_1\mathbf{x}$. This is a Least Squares approach, as it involves a projection onto $\mathcal{H}_1$.

Another way to derive this intersection is as follows. Since $\mathcal{H}_1$ is exact, write $\mathbf{h} = U_1\mathbf{x}$. From $G_2^*\mathbf{h} = 0$ it follows that

$$G_2^*U_1\mathbf{x} = 0 .$$

Hence, $\mathbf{x}$ is given by the smallest singular vector of $G_2^*U_1$. Alternatively,

$$\begin{aligned}
\arg\min_{\|\mathbf{x}\|^2=1} \| G_2^*U_1\mathbf{x} \|^2 &= \arg\min_{\|\mathbf{x}\|^2=1} \mathbf{x}^*U_1^*G_2G_2^*U_1\mathbf{x} \\
&= \arg\min_{\|\mathbf{x}\|^2=1} \mathbf{x}^*U_1^*(I - U_2U_2^*)U_1\mathbf{x} \\
&= \arg\max_{\|\mathbf{x}\|^2=1} \mathbf{x}^*U_1^*U_2U_2^*U_1\mathbf{x} \\
&= \arg\max_{\|\mathbf{x}\|^2=1} \| \mathbf{x}^*U_1^*U_2 \|^2 .
\end{aligned}$$

Thus, we obtain the same solution by computing $\mathbf{x}$ as the largest left singular vector of $U_1^*U_2$. Without noise ($\mathcal{H}_2$ also exact), the corresponding singular value is equal to 1.

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