SIGNAL PROCESSING MODEL FOR A TRANSMIT-REFERENCE UWB WIRELESS COMMUNICATION SYSTEM

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This paper presents a signal processing model for the delay-hopped transmit-reference ultra wideband communications system introduced by Hoctor and Tomlinson. In that paper, a single-user receiver based on a bank of correlators and a sliding window integrator was proposed. However, the radio propagation channel also introduces correlations, which have a significant effect not taken into account by the Hoctor-Tomlinson receiver. Here we propose an accurate signal processing model for the transmit-reference system, including the effect of the propagation channel, as well as an algorithm to estimate the resulting effective channel coefficients.

1. INTRODUCTION

Ultra Wideband (UWB) or Impulse Radio (IR) is gaining increased popularity as a prospective transmission scheme for short-range high data rate multiple access wireless communications. Topics for additional research in signal processing are acquisition and synchronization, and the design of a feasible receiver, as high data rates and dense multipath propagation environments put significant demands on the analog processing part of the receiver. Many systems proposed in the scientific literature today can be considered unrealistic and unpractical for deployment in a near future for consumer devices (e.g., sampling and processing at GHz rates, RAKE receivers with up to 50 fingers, absence of a propagation channel, or assumption of a perfectly known channel).

A system which can be considered practical for an ad-hoc communications scheme was proposed by Hoctor and Tomlinson [1, 2], and called delay-hopped (DH) transmitted reference (TR) system. Pulses are transmitted in pairs (as doublets), where the first is fixed and considered a 'carrier' and the second is modulated by the data. The first pulse is used as a template to detect the second pulse. The distance between the pulses can be varied, which serves as an additional spreading code. The receiver correlates the received data with several shifts of it using a bank of correlation lags, integrates, samples and digitally combines the outputs of the bank.

In their paper, Hoctor and Tomlinson propose a simple receiver structure based on a matched filter. However, they did not take the effect of the propagation channel into account. The delay spread of measured channels can be up to about 200 ns [3], much longer than the time interval between two pulses in a doublet. This introduces additional correlations which have a detrimental effect on the detection.

Herein, we propose an accurate signal processing data model for the TR UWB system. The model takes the propagation channel into account, and maps it into a specific set of 'effective channel coefficients' (actually correlation coefficients). We also show how these coefficients can be estimated from the received data of a single symbol. With a more accurate data model, it is easy to design improved receivers, and we give an example of a matched filter receiver.

2. TRANSMIT-REFERENCE DATA MODEL

We describe a model for the single-user DH TR system as proposed in [1, 2], and focus on the received data of a single transmitted symbol.

2.1. Analog received signal model

In a transmit reference system, two narrow pulses $g(t)$ are transmitted in sequence, with a varying time interval of D_i, to form a doublet $d(t)$. The first pulse is fixed, the second has a modulated polarity, thus

$$d(t) = g(t) + c \cdot g(t - D_i),$$

where c is the chip value, $c \in \{+1, -1\}$. N_d identical doublets (same polarities and same delays), spaced T_d, form a chip of duration $T_c = N_d T_d$. Care is taken that T_d is larger than the channel impulse response. N_c chips, defined by a certain code, form a symbol of duration $T_s = N_c T_c$. See Fig. 1.

Let $h_p(t)$ be the radio propagation channel, and define the convolution between a monopulse and the channel as $h(t) = g(t) * h_p(t)$. The received signal from a single chip

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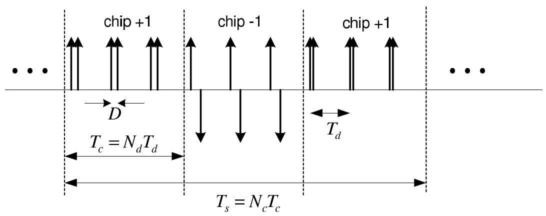


Fig. 1. Structure of a transmitted symbol.

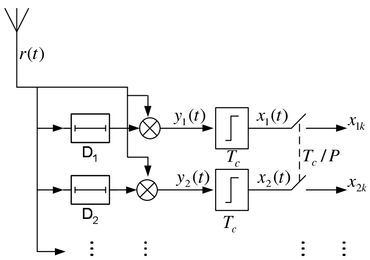


Fig. 2. Analog receiver structure—integration is over a sliding window of duration T_c, the output is sampled at P times the chiprate.

can be expressed as

$$r(t) = \sum_{k=0}^{N_d-1} h(t - kT_d) + c \cdot h(t - kT_d - D_i)\,.$$

At reception, as shown in Fig. 2, the signal is correlated in a bank of M correlators with a delayed version of itself at lags D_m, or $r(t - D_m)$, such that at the m-th correlator output we have

$$\begin{aligned} &y_m(t) = r(t)r(t - D_m) \\ &= [\textstyle\sum_{k=0}^{N_d-1} h(t - kT_d) + c \cdot h(t - kT_d - D_i)] \\ &\cdot[\textstyle\sum_{\ell=0}^{N_d-1} h(t - \ell T_d - D_m) + c \cdot h(t - \ell T_d - D_i - D_m)]\,. \end{aligned}$$

To simplify this expression, let T_h be the duration of $h(t)$, and assume $T_h < T_d - D_i - \max D_m$. While the length of the channel $h_p(t)$ is very long when compared with a monopulse duration, it is exponentially decaying, and most energy is concentrated in the first paths (effectively $T_h <$ 20ns). We can then write $y_m(t)$ as

$$\begin{aligned} y_m(t) = \sum_{k=0}^{N_d-1} & h(t - kT_d)h(t - kT_d - D_m) \\ & + h(t - kT_d - D_i)h(t - kT_d - D_i - D_m) \\ & + c \cdot h(t - kT_d - D_i)h(t - kT_d - D_m) \\ & + c \cdot h(t - kT_d)h(t - kT_d - D_i - D_m)\,. \end{aligned}$$

Subsequently, the correlated signal $y_m(t)$ is integrated over a chip period T_c by a sliding window of width $W = T_c \gg T_h$,

$$x_m(t) = \int_{t-W}^{t} y_m(\tau)d\tau\,.$$

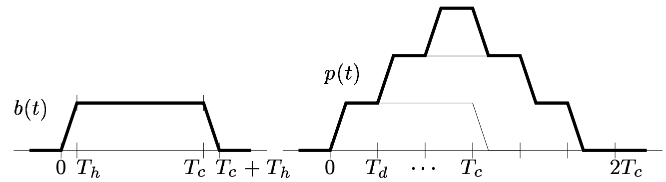


Fig. 3. $b(t)$ and $p(t)$.

Define the correlation function $\kappa(t, \Delta)$ as

$$\kappa(t, \Delta) = \int_{t-W}^{t} h(\tau)h(\tau - \Delta)d\tau\,,$$

then $x_m(t)$ can be expressed as

$$\begin{aligned} x_m(t) = \sum_{k=0}^{N_d-1} & \kappa(t - kT_d, D_m) \\ & +\kappa(t - kT_d - D_i, D_m) \\ & +c \cdot \kappa(t - kT_d - D_i, D_m - D_i) \\ & +c \cdot \kappa(t - kT_d, D_i + D_m)\,. \end{aligned} \tag{1}$$

In general, the shape of $\kappa(t, \Delta)$ depends on the correlation properties of the channel. For channels with uncorrelated taps, we may assume that

$$\begin{aligned} E_h \kappa(t, 0) &= P_h\, b(t) \\ E_h \kappa(t, \Delta) &= 0\,, \quad (\Delta \neq 0)\,, \end{aligned} \tag{2}$$

where E_h denotes the expectation operator over the distribution of the channel, $P_h = \int h(\tau)^2 d\tau$ is the energy in the impulse response of the channel, and $b(t)$ is aproximated by a 'brick' function,

$$b(t) = \begin{cases} 1, & T_h \leq t \leq W, \\ 0, & t < 0 \text{ or } t > T_h + W, \\ \text{linear slope}, & \text{elsewhere.} \end{cases}$$

where the integration length $W = T_c = N_d T_d$ is chosen to be the same as the chip duration (see Fig. 3). In this case, if $D_i = D_m$, then the dominant term in $x_m(t)$ is the third term in (1), and we obtain

$$\begin{aligned} & x_i(t) \approx \sum_{k=0}^{N_d-1} \kappa(t - kT_d - D_i, 0)c \approx \alpha_i\, p(t)\, c \\ & x_j(t) \approx 0, \quad j \neq i\,, \end{aligned}$$

where

$$p(t) = \sum_{k=0}^{N_d-1} b(t - kT_d)$$

has a staircase triangular shape with support on $0 \leq t \leq 2T_c$, and is data independent (see Fig. 3). It is the effective channel impulse response. This leads to the data model considered in [1, 2].

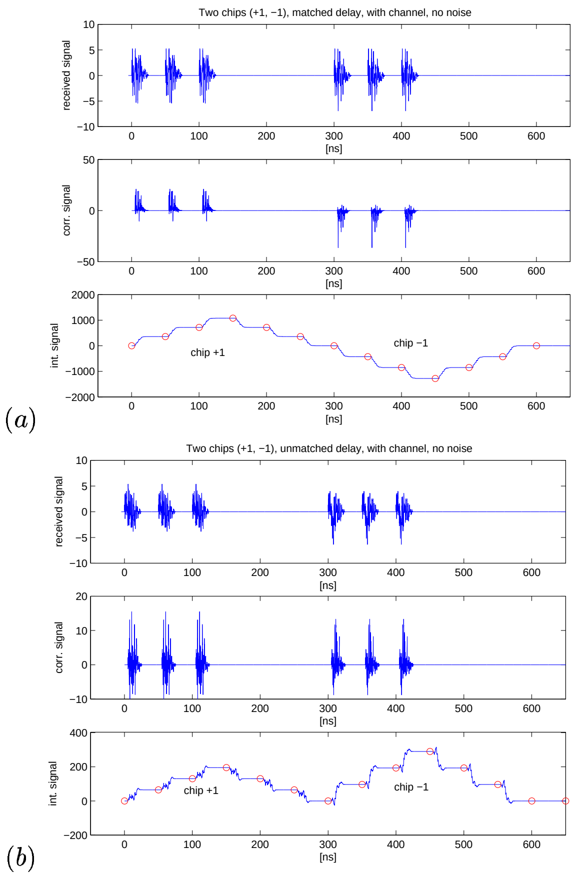


Fig. 4. Transmission of two chips (spaced wider than in actual systems): (a) with matched delay correlation and integration; (b) after unmatched delay correlation and integration.

In general, however, we cannot assume that a specific channel satisfies its expected value in (2). A more accurate expression is obtained if we model $x_m(t)$ in (1) as

$$x_m(t) = p(t)(\alpha_{mi}c + \beta_{mi}) \,, \tag{3}$$

where the gain α_{mi} depends on the correlation $\kappa(t, D_m - D_i)$, and the offset β_{mi} depends on the correlation $\kappa(t, D_m)$. For matched delays ($D_m = D_i$), Fig. 4(a) illustrates that the gain is much greater than the offset, i.e. $\alpha_{mi} \gg \beta_{mi}$. For unmatched delays ($D_m \neq D_i$), Fig. 4$(b)$ shows that the offset β_{mi}, positive, can be comparable or even greater than the gain α_{mi}, negative in this example. In the proposed receiver algorithm, the α_{mi} and β_{mi} will be estimated.

2.2. Matrix formulation

Now, we consider the data model after transmission of N_c consecutive chips $\mathbf{c} = [c_1 \cdots c_{N_c}]^T$ for a single symbol s. Suppose we transmit a chip using one of the delays $D_1, \cdots, D_M$ and receive with a bank of receivers with delays $D_1, \cdots, D_M$. The next chip may be transmitted with a different delay.

Let α_{ij} be the gain coefficient of the effective channel $p(t)$ for a transmitter delay D_j and a receiver delay D_i, and β_{ij} the corresponding gain offset. We also define matrices $\mathbf{A} = [\alpha_{ij}]$, $\mathbf{B} = [\beta_{ij}]$ of size $M \times M$. If a channel does not have temporal correlations, then $\mathbf{A} = \alpha\mathbf{I}$ (only a response at matching delays) and $\mathbf{B} = 0$, but in general the matrices can be arbitrary, although $\mathbf{A}$ is expected to be diagonally dominant.

To model the transmitter, define a 'code delay' matrix $\mathbf{J} = [J_{ij}] : M \times N_c$, where

$$J_{ij} = \begin{cases} 1, \text{if transmit at delay } D_i \text{ for chip } j \\ 0, \text{elsewhere.} \end{cases} \tag{4}$$

The matrix $\mathbf{J}$ has for each column only one nonzero entry, corresponding to the transmitted delay index.

In terms of these coefficients, the model of the received data at the output of the integrator with delay D_m becomes

$$x_m(t) = \sum_{i=1}^{M}\sum_{j=1}^{N_c} p(t - jT_c)(\alpha_{mi}J_{ij}c_j + \beta_{mi}J_{ij}) \,. \tag{5}$$

This data is sampled at instances $t = k\frac{T_c}{P}$, where the integer P is the oversampling factor. Define a channel matrix $\mathbf{P} = [p_{ij}]$ of size $N \times N_c$, where $p_{ij} = p(i\frac{T_c}{P} - jT_c)$, for $i = 0, \cdots, N-1$ samples and $j = 1, \cdots, N_c$ chips. The structure of $\mathbf{P}$ is illustrated in Fig. 5. The sampled data $x_{mk} = x_m(k\frac{T_c}{P})$ has the model

$$x_{mk} = \sum_{i=1}^{M}\sum_{j=1}^{N_c} p_{kj}(\alpha_{mi}J_{ij}c_j + \beta_{mi}J_{ij}) \,.$$

Collecting the samples x_{mk} into a vector, we have

$$\begin{aligned}
\mathbf{x}_m &= \begin{bmatrix} x_{m0} \\ \vdots \\ x_{m,N-1} \end{bmatrix} \\
&= \sum_{i=1}^{M}\sum_{j=1}^{N_c} \mathbf{p}_j(\alpha_{mi}J_{ij}c_j + \beta_{mi}J_{ij}) \\
&= \sum_{j=1}^{N_c} \mathbf{p}_j[\mathbf{a}_m^T\mathbf{j}_jc_j + \mathbf{b}_m^T\mathbf{j}_j] \\
&= \sum_{j=1}^{N_c} (\mathbf{a}_m^T\mathbf{j}_j) \otimes (\mathbf{P}\cdot\mathbf{e}_j)c_j + (\mathbf{b}_m^T\mathbf{j}_j) \otimes (\mathbf{P}\cdot\mathbf{e}_j) \\
&= \sum_{j=1}^{N_c} (\mathbf{a}_m^T \otimes \mathbf{P})(\mathbf{j}_j \otimes \mathbf{e}_j)c_j + (\mathbf{b}_m^T \otimes \mathbf{P})(\mathbf{j}_j \otimes \mathbf{e}_j) \\
&= (\mathbf{a}_m^T \otimes \mathbf{P})(\mathbf{J} \circ \mathbf{I}_{N_c})\mathbf{c} + (\mathbf{b}_m^T \otimes \mathbf{P})(\mathbf{J} \circ \mathbf{I}_{N_c})\mathbf{1} \,,
\end{aligned}$$

where $\mathbf{a}_m^T$ and $\mathbf{b}_m^T$ are the m-th rows of $\mathbf{A}$ and $\mathbf{B}$ respectively, $\mathbf{p}_j$ and $\mathbf{j}_j$ are the j-th columns of $\mathbf{P}$ and $\mathbf{J}$ respectively, and $\mathbf{e}_j$ is the j-th column of the identity matrix. Here,

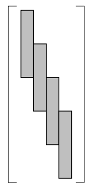
$\mathbf{P}_{N\times N_c} =$ 

Fig. 5. Structure of the matrix $\mathbf{P}$.

we denote by $\otimes$ the Kronecker product, and by $\circ$ the Khatri-Rao product (column-wise Kronecker product).

If we stack the received MN samples in a vector $\mathbf{x}$, we obtain

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T \otimes \mathbf{P} \\ \vdots \\ \mathbf{a}_M^T \otimes \mathbf{P} \end{bmatrix} (\mathbf{J}\circ\mathbf{I}_{N_c})\mathbf{c} + \begin{bmatrix} \mathbf{b}_1^T \otimes \mathbf{P} \\ \vdots \\ \mathbf{b}_M^T \otimes \mathbf{P} \end{bmatrix} (\mathbf{J}\circ\mathbf{I}_{N_c})\mathbf{1}\,.$$

This can be written compactly as

$$\begin{aligned} \mathbf{x} &= (\mathbf{A}\otimes\mathbf{P})(\mathbf{J}\circ\mathbf{I}_{N_c})\mathbf{c} + (\mathbf{B}\otimes\mathbf{P})(\mathbf{J}\circ\mathbf{I}_{N_c})\mathbf{1} \\ &= (\mathbf{AJ}\circ\mathbf{P})\mathbf{c} + (\mathbf{BJ}\circ\mathbf{P})\mathbf{1}\,. \end{aligned}$$

Taking the symbol value s into account, we have the final data model for a single symbol

$$\mathbf{x} = (\mathbf{AJ}\circ\mathbf{P})\mathbf{c}s + (\mathbf{BJ}\circ\mathbf{P})\mathbf{1}\,. \tag{6}$$

In this model, $\mathbf{J}$, $\mathbf{P}$ and $\mathbf{c}$ are known, while $\mathbf{A}$ and $\mathbf{B}$ are unknown $M \times M$ matrices, and s is the unknown data symbol. The model is easily extended for a burst of N_s symbols, we omit the details.

3. RECEIVER ALGORITHMS

In section 2, we have obtained a matrix representation for the received signal $\mathbf{x}$, in a multiple symbol transmission. In this model, $\mathbf{A}$ and $\mathbf{B}$ are unknown $M \times M$ matrices that represent the channel effects, after correlation and integration at reception. In this section, we propose a method of estimating $\mathbf{A}$ and $\mathbf{B}$ from a single received symbol, and a corresponding matched filter receiver for estimating all transmitted symbols.

3.1. Hoctor-Tomlinson receiver

The receiver proposed by Hoctor and Tomlinson in [2] can be expressed by (6) if we assume $\mathbf{A} = \alpha\mathbf{I}$ and $\mathbf{B} = 0$. In that case, we can write the simplified model as

$$\mathbf{x} = \alpha(\mathbf{J}\circ\mathbf{P})\mathbf{c}s\,.$$

Based on this model, a matched filter receiver is

$$\hat{s} = [\alpha(\mathbf{J}\circ\mathbf{P})\mathbf{c}]^T\mathbf{x}\,.$$

Subsequently, the symbol is detected as $\text{sign}(\hat{s})$. Since $\alpha > 0$ does not change the result, it does not have to be estimated. Alternatively, this receiver can be written as (using properties of Kronecker products)

$$\hat{s} = \text{tr}[\text{diag}(c)\mathbf{P}^T\mathbf{XJ}]\,,$$

where tr is the trace operator, matrix $\mathbf{X} : N \times M$ is the restacking of the received samples $\mathbf{x}$.

This receiver can be interpreted as follows. Each column of $\mathbf{X}$ is the output of the integrator for a specific delay. Premultiplication of the data $\mathbf{X}$ by $\mathbf{P}^T$ constitutes a matched filter with the pulse shape $p(t)$. From the output of this, the j-th column corresponds to the j-th transmitted chip. For each transmitted chip, the corresponding delay is selected by $\mathbf{J}$, and the result is multiplied by the corresponding chip value. The trace operator sums the results. Except for the pulse-matched filtering, this is precisely the same as the receiver proposed by Hoctor and Tomlinson [2].

3.2. Estimating $\mathbf{A}$ and $\mathbf{B}$; improved TR receiver

An improved TR receiver extends the Hoctor-Tomlinson receiver to deal with correlation mismatches, i.e., $\mathbf{B}$ has non-zero entries. We first show how $\mathbf{A}$, $\mathbf{B}$ can be estimated based on a limited data set, e.g., the received samples of the first data symbol.

Consider the model for the first symbol s_1, (with some abuse of notation)

$$\mathbf{x}_1 = (\mathbf{AJ}\circ\mathbf{P})\mathbf{c}s_1 + (\mathbf{BJ}\circ\mathbf{P})\mathbf{1}\,.$$

Restack the received signal $\mathbf{x}_1$ given by Eq. (6) into a matrix $\mathbf{X}_1 : N \times M$, such that $\text{vec}(\mathbf{X}_1) = \mathbf{x}_1$, then

$$\begin{aligned} \mathbf{X}_1 &= \mathbf{P}\text{diag}(\mathbf{c})\mathbf{J}^T\mathbf{A}^T s_1 + \mathbf{PIJ}^T\mathbf{B}^T \\ &= \mathbf{P}[\text{diag}(\mathbf{c})\mathbf{J}^T \quad \mathbf{J}^T][\mathbf{A}s_1 \quad \mathbf{B}]^T\,. \end{aligned}$$

Define $\mathbf{Z} = \mathbf{P}[\text{diag}(\mathbf{c})\mathbf{J}^T \quad \mathbf{J}^T]$. It is a known matrix of size $N \times 2M$. If $\mathbf{Z}$ is tall and both its factors ($\mathbf{P}$ and $[\text{diag}(\mathbf{c})\mathbf{J}^T \quad \mathbf{J}^T]$) are tall, then generically it is left invertible: this requires $N \geq N_c$ and $N_c \geq 2M$. The first condition is always satisfied, the latter requires that the number of chips per symbol is larger than twice the number of possible delays.

Let $\mathbf{Z}^\dagger$ be a left inverse (pseudo-inverse) of $\mathbf{Z}$, then we can estimate $[\mathbf{A}s_1 \;\; \mathbf{B}]$ as

$$[\widehat{\mathbf{A}s_1} \quad \hat{\mathbf{B}}] = (\mathbf{Z}^\dagger\mathbf{X}_1)^T\,.$$

Since the diagonal elements of $\mathbf{A}$ are dominant and positive (matched delays), we can easily estimate $\mathbf{A}$ and s_1 from $\widehat{\mathbf{A}s_1}$, e.g., $\hat{s}_1 = \text{sign}\{\text{tr}(\widehat{\mathbf{A}s_1})\}$. Once $\mathbf{A}$, $\mathbf{B}$ are known, it is straightforward to estimate remaining symbols. From (6), we can write down a matched filter receiver

$$\hat{s} = \text{sign}\{((\mathbf{AJ}\circ\mathbf{P})\mathbf{c})^H(\mathbf{x} - (\mathbf{BJ}\circ\mathbf{P})\mathbf{1})\}\,.$$

Many other receivers are possible, this just serves as an illustration. Note that $\mathbf{A}, \mathbf{B}$ can also be estimated from several symbols, with less constraints on N_c.

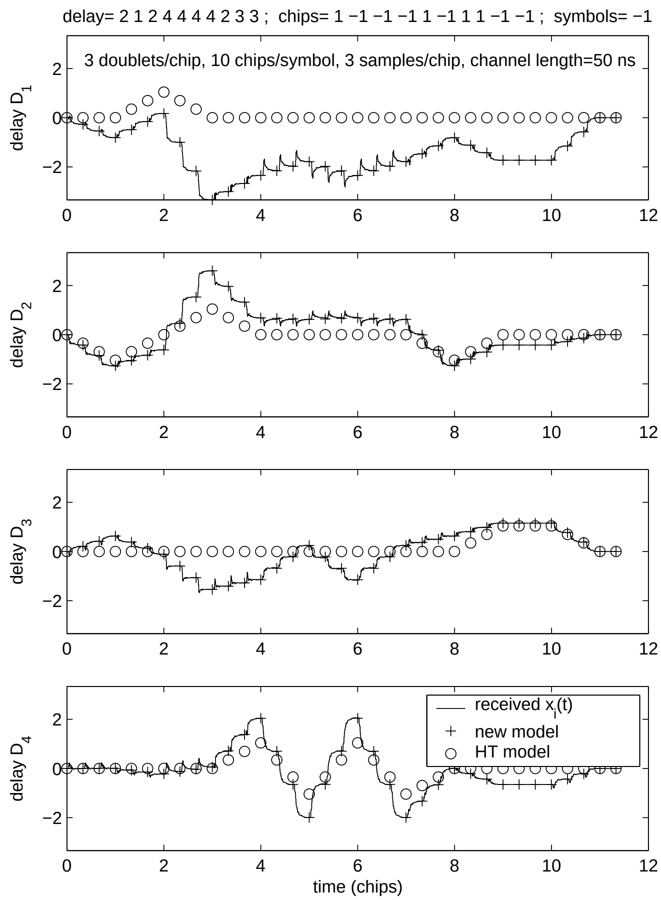


Fig. 6. The correspondence of the actually received data to 'o' the simple model and '+' the proposed data model in (6).

4. SIMULATION RESULTS

To demonstrate the accuracy of the data model, we have simulated a symbol transmission over an exponentiallyi-decaying channel, with $M = 4$ delay positions, $N_c = 10$ chips, $N_d = 3$ doublets per chip, and $P = 3$ times oversampling at the output of the integrators. The transmitted pulse is a first derivative of a Gaussian pulse with duration 0.5 ns, the two pulses in a doublet are separated by 1, 2, 3 or 4 ns, and the doublets are spaced by $T_d = 50$ ns. An important parameter to consider is the channel length. Simulations were performed with a channel length $T_h = 50$ ns, where the channel coefficients were selected randomly, but piecewise constant over periods of 1 ns, and with amplitude exponentially tapering down in time. The effective channel thus has significant cross-correlations for neighboring delay lags, but most of the transmitted signals energy is concentrated in the first arrival paths.

The result of the simulation is shown in Fig. 6. The solid lines in each panel show the received data $x_i(t)$ for the corresponding correlation lag (1–4 ns). The transmitted chip values and delay lags are shown at the top. It is clear that, due to the cross-correlations in the channel, the received chips do not only have a response at the matching delays, but also at other delays. The simple data model used by Hoctor-Tomlinson (shown as 'o') does not take the effect of the channel into account, hence assumes a response only at the matching delay. For the simulated channel, the deviations can be significant. The new model (shown as '+') is almost indistinguishable from the actually received data, hence provides a very good match. The values of $\mathbf{A}$ and $\mathbf{B}$ were estimated from the received data as described in the previous section.

5. CONCLUSIONS AND RESEARCH DIRECTIONS

We have proposed an accurate signal processing model for the transmit-reference UWB system proposed by Hoctor-Tomlinson, taking into account the channel and receiver characteristics. The model considers the channel correlation coefficients, which can be estimated from a single symbol and used in a simple matched filter receiver. Although we were not able yet to test the effect of the improved data model on the estimation of the symbols, it is reasonable to expect that a more accurate model can provide much better detection results. This will translate in a smaller number of bit errors in the presence of a large number of users and/or large noise.

6. REFERENCES

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