

Multi-Rate Block Transmission Over Wideband Multi-Scale Multi-Lag Channels

Tao Xu, Zijian Tang, Geert Leus, *Fellow, IEEE*, and Urbashi Mitra, *Fellow, IEEE*

Abstract—Many linear time-varying (LTV) channels of interest can be well-modeled by a multi-scale multi-lag (MSML) assumption, especially when wideband transmissions with a large bandwidth/carrier-frequency ratio are employed. In this paper, communications over wideband MSML channels are considered, where conventional communication methods often fail. A new parameterized data model is proposed, where the continuous MSML channel is approximated by discrete channel coefficients. It is argued that this parameterized data model is always subject to discretization errors in the baseband. However, by designing the transmit/receive pulse intelligently and imposing a multi-branch structure on the receiver, one can eliminate the impact of the discretization errors on equalization. In addition, to enhance the bandwidth efficiency, a novel multi-layer transmit signaling is proposed, which is characterized by multiple data rates on different layers. The inter-layer interference, induced by the multi-layer transmitter, can also be minimized by the same design of the transmit/receive pulse. As a result, the effective channel experienced by the receiver can be described by a block diagonal matrix, with each diagonal block being strictly banded. Such a channel matrix structure enables the use of several existing low-complexity equalizers viable.

Index Terms—Equalization, multi-rate transmission, underwater communications, wideband time-varying channels.

I. INTRODUCTION

WIDEBAND LINEAR TIME-VARYING (WLTV) channels arise in a variety of wireless communication scenarios such as underwater acoustic (UWA) systems and wideband terrestrial radio frequency systems utilizing spread-spectrum or ultra-wideband signaling. Compared to the more commonly considered narrowband channels that are experienced in many wireless systems such as cellular and WiFi, WLTV channels exhibit some key fundamental differences [1]. Wideband channels have a large fractional bandwidth, i.e., the transmission bandwidth is of the same order as the employed carrier

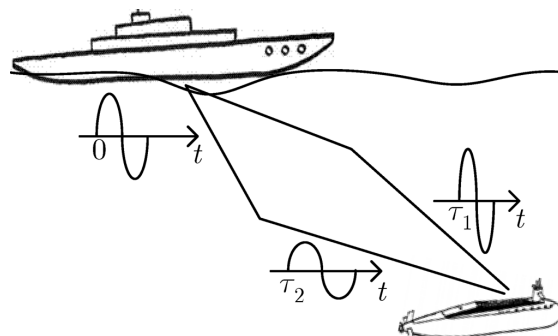


Fig. 1. An illustration of the MSML propagation generally encountered in underwater acoustic communications.

frequency¹. If the relative velocity between the transmitter and receiver is non-negligible relative to the speed of transmission over the medium, the resulting Doppler effects cannot be approximated by Doppler shifts. Rather, signals are compressed or dilated measurably due to Doppler scaling. Furthermore, due to distinct angles of arrival in a multipath environment, each component of the multipath channel might experience a different Doppler scale. These effects give rise to what we denote as a *multi-scale multi-lag* (MSML) [4]–[6] channel model, which is schematically depicted in Fig. 1.

One common signaling scheme proposed for multipath channels is based on orthogonal frequency division multiplexing (OFDM); however, it is well-known that significant intercarrier interference (ICI) will be induced when employing OFDM over time-varying channels [7], [8]. In this case, a more sophisticated equalizer is required versus the time-invariant scenario. The success of OFDM over narrowband channels is due to the fact that the transmission admits a uniform sampling in the lag and Doppler shift domains, which aligns with the uniform time-frequency (T-F) lattice of narrowband time-varying channels. In contrast, the wideband channel is characterized by a nonuniform T-F lattice [6], [9], [10]. To counteract this mismatch, a multi-band OFDM scheme is proposed in [11], wherein the WLTV channel is split into sub-channels with a sufficiently small bandwidth such that each of the sub-channels can be modeled as a narrowband LTV channel. Other often adopted approaches are based on a *single-scale multi-lag* (SSML) assumption for WLTV channels (see e.g., [9], [10], [12]). Such a SSML channel can be converted to a narrowband channel subject to a single carrier frequency offset (CFO) by means of resampling. However, we observe that this assumption is suboptimal in the presence of multiple scales [13], [14]. In this paper, we consider MSML models appropriate for WLTV channels, signaling

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T. Xu, Z. Tang, and G. Leus are with Delft University of Technology, Delft 2628CD, The Netherlands (e-mail: t.xu@tudelft.nl; z.tang@tudelft.nl; g.j.t.leus@tudelft.nl).

U. Mitra is with the Ming Hsieh Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089 USA (e-mail: ubli@usc.edu).

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¹Herein, ‘wideband’ refers to the relative bandwidth that is often used in acoustics and radar [2], rather than absolute bandwidth. Thus, some broadband systems, e.g., in [3], would not qualify as wideband in this work.

tailored to this model, and equalizers for this joint design of a channel model and signaling scheme.

The concept of an MSML model has been previously presented in [4]–[6]. These works exploit the transmission of a *single pulse/symbol in isolation*, develop the MSML model, and typically consider the associated matched-filter for the demodulation of this single pulse/symbol. In particular, in [4], information symbols are modulated onto a single-scale orthogonal wavelet-based pulse at the transmitter, and the channel is mathematically described by a discretized time-scale model based on the characteristics of the adopted wavelet. A crucial assumption adopted in [4] is that the time-scale channel model should not corrupt the scale-orthogonality of the transmit pulse, but it is not clear under which conditions this assumption remains valid. As an improvement, [5] combines direct-sequence spread-spectrum (DSSS) modulation with a wavelet-based pulse to enforce the scale-orthogonality of the transmit pulse. Common to these works is that both channel modeling and signaling is assumed to occur in baseband, but on the other hand, a special (wavelet-based) pulse is employed that has a bandpass property. In our own work [15], we consider a more general system, where we use a low-pass pulse, which is up-converted to a carrier frequency before transmission over an MSML channel. The challenge is that, at the receiver, the passband to baseband conversion must be carefully treated in MSML channels.

A unique feature of the MSML channel is that one can increase the spectral efficiency by communicating simultaneously over multiple scales ([4]–[6], [15] employ single-scale signaling); for clarity, we shall refer to such signaling as multi-layered. In fact, multi-layered signaling for narrowband channels has been considered in [16] with variants of orthogonal wavelet division multiplexing provided in [17]–[20]. In [19], [20], it is shown that such a multi-layered transmission scheme based on a wavelet modulation can achieve the same spectral efficiency as that of a traditional method, e.g., OFDM. A challenge with these signaling schemes is that wavelet orthogonality is not maintained after transmission over the MSML channel. In [21], we designed a multi-layer signal for MSML channels. The resulting channel was banded in nature, allowing for the use of low-complexity equalizers for banded narrowband channels [22]–[24]. However, it is not clear how to adapt the all-baseband processing scheme in [21] to a passband transmission. In the current work, we endeavor to fill this gap.

The main contributions of this work are 1) a novel parameterization of the continuous MSML passband channel; we show that the associated discrete baseband data model is subject to inter-scale interference without proper transmit signal design; 2) proposing a transmit and receive pulse design which aims to eliminate this inter-scale interference and induces a multi-branch receiver structure which can leverage channel diversity; 3) a multi-layer signal design matched to the parameterized channel model which increases spectral efficiency; 4) a new block transmission scheme with a guard interval to eliminate inter-block interference, enabling the use of low-complexity equalizers due to the resulting signal structure.

This paper is organized as follows. In Section II, we give a parameterized channel model in the passband, and show that a corresponding baseband model is always subject to nuisance. In Section III, we provide design constraints for transmit and re-

ceive pulses which eliminate inter-scale interference in the resulting baseband signal. A block-wise transceiver design is introduced in Section IV, and a low-complexity equalizer is presented in Section V. In Section VI, we provide simulation results, and conclude the paper in Section VII.

Notation: Upper (lower) bold-face letters stand for matrices (vectors); Superscript H denotes Hermitian, $*$ conjugate, T transpose, and $\dagger$ matrix pseudo-inverse. The notation $\otimes$ represents Kronecker product, and $\circledast$ linear convolution. We reserve j for the imaginary unit, and use $\Re\{\cdot\}$ for the real part, $\lceil \cdot \rceil$ for the integer ceiling, and $\mathcal{E}\{\cdot\}$ for the mean. $[\mathbf{A}]_{k,m}$ stands for the (k, m) th entry of the matrix $\mathbf{A}$, and δ_k for a delta function which is equal to one only if $k = 0$ and zero otherwise.

II. WIDEBAND LTV SYSTEMS

Suppose a transmitted signal $x(t)$ travels along a specific path, which is associated with a delay τ due to a non-negligible propagation time, which is normally considered as positive, and a radial velocity u uniquely determined by the incident angle of this path to the receiver. The received signal $r(t)$ resulting from this path can then be modeled as $r(t) = h(\alpha, \tau)\sqrt{\alpha}x(\alpha(t - \tau))$, where $\alpha = \frac{c+u}{c-u} \approx 1 + \frac{2u}{c}$ is the scaling factor with c the speed of the communication medium (normally $c \gg u$), $\sqrt{\alpha}$ is added as a normalization factor, and $h(\alpha, \tau)$ denotes the attenuation factor of this path, which is also known as the wideband spreading function [25].

In an environment where a rich number of scatterers exist, the channel can be viewed as a collection of fast moving scatterers that are continuously distributed in range and velocity [25]. As a result, the input-output (I/O) relationship should be formulated as

$$r(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\alpha, \tau)\sqrt{\alpha}x(\alpha(t - \tau))d\alpha d\tau + w(t), \quad (1)$$

where $w(t)$ stands for the additive noise, which is assumed to be a white Gaussian process with mean zero and variance σ^2 . This model reflects the fact that the received signal $r(t)$ can then be represented as a superposition of differently delayed (by τ) and scaled (by α) versions of the transmitted signal, each weighted by $h(\alpha, \tau)\sqrt{\alpha}$, and therefore has a multi-scale multi-lag (MSML) feature.

For a practical channel, without loss of generality, it is reasonable to assume that τ and α are limited to $\tau \in [0, \tau_{\max}]$ and $\alpha \in [1, \alpha_{\max}]$ due to physical restrictions, where the parameters $\tau_{\max} > 0$ and $\alpha_{\max} > 1^2$ represent the *maximal delay spread* and *maximal scale spread*, respectively. Further, the transmitted signal considered in this paper is assumed to be a passband signal with carrier frequency f_c and effective bandwidth W_* , and therefore

$$x(t) = \Re\{\bar{x}(t)e^{j2\pi f_c t}\}, \quad (2)$$

where $\bar{x}(t)$ denotes the baseband counterpart of $x(t)$, which is hence band-limited within $[-\frac{W_*}{2}, \frac{W_*}{2}]$.

²As a matter of fact, the case of dilation with $\alpha_{\max} < 1$ can be converted to a case of compression by means of proper resampling at the receiver. This justifies the consideration of a compressive scenario without loss of generality.

In the remainder of the section, we will first seek a parameterized representation for the I/O relationship in (1) in passband, and then try to derive a discrete data model in baseband. In light of the passband feature of $x(t)$, we cannot directly follow the approaches in e.g., [4], [6] to derive a discrete baseband data model. Due to the scaling effect, the original signal can have several differently scaled versions simultaneously at the receiver, each having a disparate effective carrier frequency deviating from f_c as well as a distinct bandwidth. As a result, conversion to baseband is not straightforward, and sampling in baseband can induce an error in the discrete data model. This subtlety is not explicitly treated in [4], [6].

A. Parameterized Passband Data Model

Our first step is to parameterize the continuous channel $h(\alpha, \tau)$ in (1) along the scale dimension. This can be achieved by employing the results of [4], giving rise to the following approximation

$$r(t) \approx r^S(t) = \sum_{r=0}^{R_*} \int_0^{\tau_{\max}} h_r(\tau) a_*^{r/2} x(a_*^r(t - \tau)) d\tau, \quad (3)$$

where a_* is referred to as the *basic scaling factor* in [4], or *dilation spacing* in [5], [6], whose physical interpretation will be discussed in detail in Remark 1; further, $R_* = \lceil \ln \alpha_{\max} / \ln a_* \rceil$, and

$$h_r(\tau) = \int_{-\infty}^{\infty} h(\alpha', \tau) \text{sinc} \left(\frac{\ln a_*^r - \ln \alpha'}{\ln a_*} \right) d\alpha', \quad (4)$$

represents the scale-smoothed version of $h(\alpha, \tau)$ that is evaluated at the scale a_*^r .

Note that in (3), we have used a superscript “S” to underscore that, so far, only the scale parameter is discretized (later, the superscript “L” will refer to lag discretization, and superscript “SL” for joint scale and lag discretization). In light of the finite summation in (3), we can interpret $r^S(t)$ as resulting from a *time-invariant* multiple-input single-output (MISO) system, where the signal transmitted via the r th virtual channel is $a_*^{r/2} x(a_*^r t)$; the effective associated channel is $h_r(t)$, and the r th component of the received signal can be denoted as

$$r_r^S(t) = \int_0^{\tau_{\max}} h_r(\tau) a_*^{r/2} x(a_*^r(t - \tau)) d\tau. \quad (5)$$

Equation (3) represents a passband data signal, and our objective is eventually to establish a baseband model. Towards this end, we first find an expression for the r th component of the signal in terms of its baseband counterpart of the transmit signal $a_*^{r/2} x(a_*^r t) = \Re \{ a_*^{r/2} \bar{x}(a_*^r t) e^{j2\pi f_c a_*^r t} \}$. It is clear from this expression that the baseband signal $a_*^{r/2} \bar{x}(a_*^r t)$ is up-converted to an effective carrier frequency $a_*^r f_c$ and has an effective bandwidth $a_*^r W_*$. Accordingly, we can also obtain the baseband version for $r_r^S(t)$ by observing that

$$\begin{aligned} r_r^S(t) &= \int_0^{\tau_{\max}} h_r(\tau) a_*^{r/2} \Re \left\{ \bar{x}(a_*^r(t - \tau)) e^{j2\pi f_c a_*^r(t - \tau)} \right\} d\tau, \\ &= \Re \left\{ e^{j2\pi f_c a_*^r t} \int_0^{\tau_{\max}} \left(h_r(\tau) e^{-j2\pi f_c a_*^r \tau} \right) \right. \end{aligned}$$

$$\begin{aligned} &\left. \times a_*^{r/2} \bar{x}(a_*^r(t - \tau)) d\tau \right\}, \\ &= \Re \left\{ e^{j2\pi f_c a_*^r t} \int_0^{\tau_{\max}} \bar{h}_r(\tau) a_*^{r/2} \bar{x}(a_*^r(t - \tau)) d\tau \right\}, \quad (6) \end{aligned}$$

where we have introduced the notation $\bar{h}_r(t)$ in the last equality to represent

$$\bar{h}_r(t) = h_r(t) e^{-j2\pi f_c a_*^r t}, \quad (7)$$

which can be interpreted as the continuous baseband channel for the r th component signal. Let $\bar{r}_r^S(t)$ represent the baseband counterpart of $r_r^S(t)$, i.e.,

$$r_r^S(t) = \Re \left\{ \bar{r}_r^S(t) e^{j2\pi f_c a_*^r t} \right\}.$$

From (6), it then follows that

$$\bar{r}_r^S(t) = \int_0^{\tau_{\max}} \bar{h}_r(\tau) a_*^{r/2} \bar{x}(a_*^r(t - \tau)) d\tau. \quad (8)$$

Now, we are able to exploit the results of [4] again to seek a discrete approximation of $\bar{h}_r(t)$ in (7). Due to the fact that the r th scaled version $\bar{x}(a_*^r t)$ is band-limited to $a_*^r W_*$, we can approximate (8) as

$$\bar{r}_r^S(t) \approx \bar{r}_r^{SL}(t) = \sum_{l=0}^{L_*(r)} \bar{h}_{r,l} a_*^{r/2} \bar{x}(a_*^r t - lT_*), \quad (9)$$

where T_* is referred to as the *translation spacing* in [5], [6], and $L_*(r) = \lceil a_*^r \tau_{\max} / T_* \rceil$ denotes the number of channel taps, which is clearly dependent on the component index r ; further,

$$\bar{h}_{r,l} = \bar{h}_r^L(lT_*/a_*^r), \quad (10)$$

with $\bar{h}_r^L(\tau)$ being the lag-smoothed version of $\bar{h}_r(\tau)$:

$$\bar{h}_r^L(\tau) = \int_0^{\tau_{\max}} \bar{h}_r(\tau') \text{sinc} \left(a_*^r \frac{\tau - \tau'}{T_*} \right) d\tau'. \quad (11)$$

Substituting (9) into (3) yields

$$\begin{aligned} r^{SL}(t) &= \Re \left\{ \sum_{r=0}^{R_*} e^{j2\pi f_c a_*^r t} \bar{r}_r^{SL}(t) \right\} \\ &= \Re \left\{ \sum_{r=0}^{R_*} e^{j2\pi f_c a_*^r t} \sum_{l=0}^{L_*(r)} \bar{h}_{r,l} a_*^{r/2} \bar{x}(a_*^r t - lT_*) \right\}, \quad (12) \end{aligned}$$

where the continuous channel $h(\alpha, \tau)$ in passband is expressed in terms of the baseband channel parameters $\bar{h}_{r,l}$ that are discretized in both the scale and lag dimension. Combining (4) and (11), we obtain

$$\begin{aligned} \bar{h}_{r,l} &= \int_0^{\tau_{\max}} \int_1^{\alpha_{\max}} h(\alpha, \tau) e^{-j2\pi f_c a_*^r \tau} \\ &\quad \times \text{sinc} \left(r - \frac{\ln \alpha}{\ln a_*} \right) \text{sinc} \left(l - \frac{a_*^r \tau}{T_*} \right) d\alpha d\tau. \quad (13) \end{aligned}$$

A schematic overview of the passband model in (12) is given in Fig. 2.

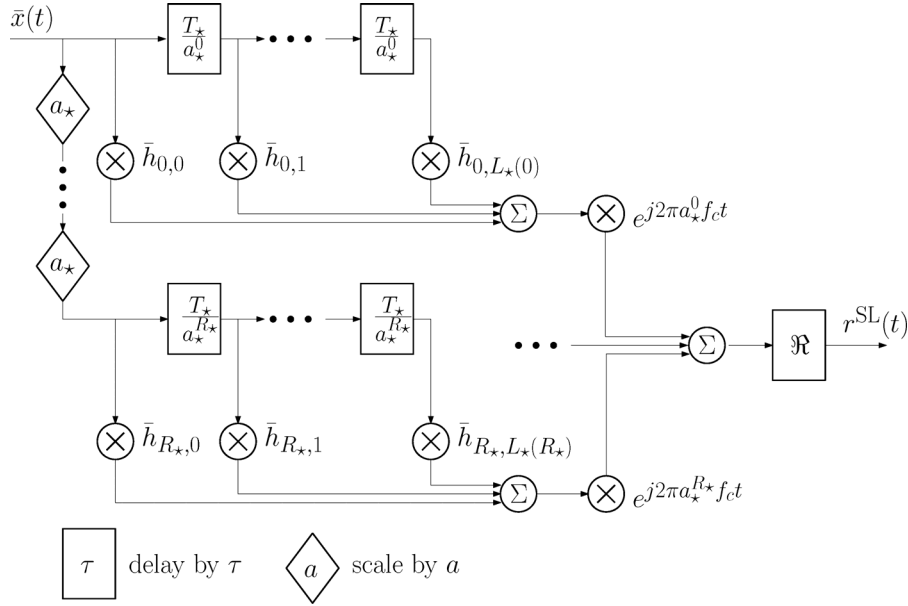


Fig. 2. A parameterized passband data model.

Remark 1: In the above data model, the continuous channel is approximated by a finite number of discrete channel coefficients, which inevitably induces an approximation error. To enable a good fit, it is desired that the scale and lag resolution should be as high as possible. These resolutions are determined, respectively, by the *dilation spacing* a_* and the *translation spacing* T_* . On the other hand, too high of a resolution will give rise to a channel model with a large order, which is undesirable from a receiver design point of view.

In practice, one approach to seek a proper a_* is linked to the wideband ambiguity function (WAF) of $x(t)$ in the passband [5], [6]:

$$\chi(\alpha, \tau) = \int x(t) \sqrt{\alpha} x(\alpha(t - \tau)) dt; \quad (14)$$

similarly, T_* is linked to the WAF of $\bar{x}(t)$ in baseband

$$\bar{\chi}(\alpha, \tau) = \int \bar{x}(t) \sqrt{\alpha} \bar{x}(\alpha(t - \tau)) dt. \quad (15)$$

Under the assumption that $\chi(\alpha, \tau)$ decays rapidly in the scale dimension, a_* is defined as the first zero-crossing of $\chi(\alpha, 0)$. Likewise, under the assumption that $\bar{\chi}(\alpha, \tau)$ decays rapidly in the lag dimension, T_* is defined as the first zero-crossing of $\bar{\chi}(1, \tau)$. An alternative approach [4] assumes that $x(t)$ has a limited effective bandwidth W_* and Mellin support M_* ³. It is well-known that in the Fourier domain the Nyquist sampling theorem dictates that $T_* = 1/W_*$ to ensure perfect signal reconstruction. We can apply an adapted Nyquist sampling result in the Mellin domain to obtain $a_* = e^{1/M_*}$.

That these two approaches render a good approximation is derived and motivated in [5], [6] and [4], respectively. We will show, in a subsequent numerical example, that these two approaches produce similar values of T_* and a_* . The first approach

is easier to use, but relies on the rapid decay assumption of the WAFs. In this sense, the second approach is more robust.

B. Related Works

A comparison between the parameterization of wideband LTV channels and that of narrowband LTV channels (see for the latter e.g., [28], [29]) has been thoroughly treated in [4], [6]. Here, we just recall the fact that the parameterized narrowband LTV channel is arithmetically uniform in both the lag (time) and frequency dimension, while the parameterized wideband LTV channel is arithmetically uniform in the lag (time) dimension, but geometrically uniform in the scale (frequency) dimension, resulting in a different T-F tiling diagram. Thus, in the presence of channel selectivity, a transmitted symbol will disperse differently over a narrowband LTV channel than over a wideband LTV channel. This fact is schematically depicted in Fig. 3, where the circles indicate the positions where the channel is sampled in the time-frequency (T-F) plane. In the figure, we assume a single symbol is transmitted at time 0 and carrier frequency f_c , whose location is represented by a dark circle, while the open circles show the locations of signal leakage. The symbol f_* in Fig. 3(b) denotes the frequency spacing (analogous to the *dilation spacing* for wideband systems) used to sample the channel in the Doppler (frequency) dimension.

Compared to the wideband scale-lag canonical models in [4]–[6], in the derivation towards our channel model, we first parameterize the channel in the scale dimension in passband, and then convert the channel to baseband where it is further parameterized in the lag dimension. Such a conversion between passband and baseband is not taken into account by [4]–[6] in the parameterization process.

The fundamental difference lies in the choice of the transmit (and receive) pulse $p(t)$. This paper follows the convention of most communication systems by assuming a general low-pass waveform for $p(t)$. To make it suitable for transmission, $p(t)$ is converted to passband by multiplying it with $e^{j2\pi f_c t}$ prior to transmission. In comparison, [4] uses a Haar wavelet and [5] uses a second-order derivative passband Gaussian chip (a

³The Mellin support is the scale analogy of the Doppler spread for narrowband LTV channels. Specifically, the Mellin support of a signal $x(t)$ is the support of the Mellin transform of $x(t)$ which is given by $\int_0^\infty x(t) t^{s-1} dt$. More details about the Mellin transform can be found in [26], [27], and we provide in Appendix C a numerical example to show how the Mellin transform can be implemented.

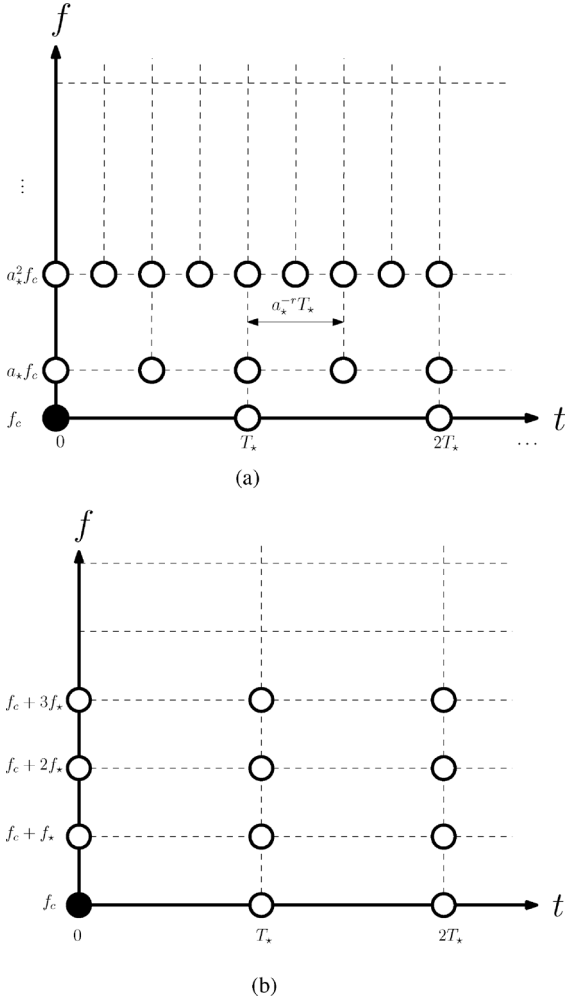


Fig. 3. Time-frequency (T-F) tile diagram of a discretized channel model. (a) Wideband. (b) Narrowband.

Ricker wavelet) for $p(t)$, which are bandpass signals in nature. The pulse $p(t)$ can therefore be directly transmitted without an extra step of conversion to passband. The transmit pulse and the data model in this paper will have a more general application than those in [4], [5].

In light of our MISO view, each component of the received signal in our model, denoted as $r_r^S(t)$ in (3), can be represented in the T-F plane by a block centered around a distinctive carrier frequency $a_*^r f_c$ as illustrated in Fig. 4(a). Because there lacks an explicit conversion between passband and baseband, the data models in [4], [5] are, strictly speaking, derived in baseband for a general definition of $p(t)$. Therefore, the T-F representation of the received signal in [4], [5] is depicted by Fig. 4(b), where each component, $r_r^S(t)$, is represented by a block around DC in a nested manner.

C. Parameterized Baseband Data Model

The passband signal model (12) clearly establishes the challenges of deriving a baseband signal representation. As shown in Fig. 4(a), each component of the received signal, $r_r^S(t)$, is characterized by a unique carrier frequency $a_*^r f_c$. There exists no universal carrier frequency for down-conversion of all the components. Similarly, since each component of the received signal has a distinct bandwidth $a_*^r W_*$, which is dependent on

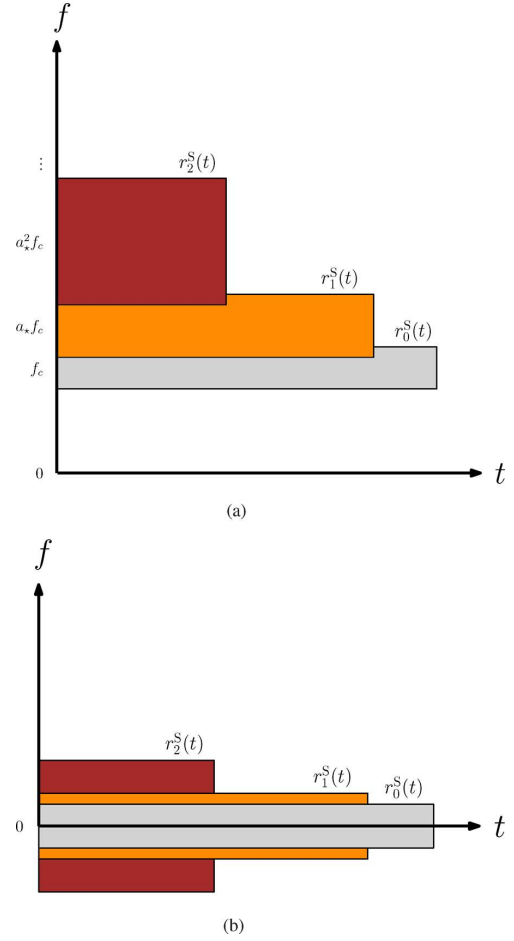


Fig. 4. Decomposition of the received signal. (a) In the proposed model. (b) In the model of [4].

the component index r , this invites the question of which sampling rate we should adopt to discretize the received signal⁴.

In particular, suppose we let the receiver be synchronized with the k th component of the received signal. After down-conversion, the resulting baseband signal can be expressed as

$$\begin{aligned} \tilde{z}_k^{\text{SL}}(t) &= r^{\text{SL}}(t) e^{-j2\pi f_c a_*^k t} \\ &= \sum_{l=0}^{L_*(k)} \bar{h}_{k,l} a_*^{k/2} \bar{x}(a_*^k t - lT_*) \\ &\quad + \sum_{r=0, r \neq k}^{R_*} e^{j2\pi f_c (a_*^r - a_*^k) t} \sum_{l=0}^{L_*(r)} \bar{h}_{r,l} a_*^{r/2} \bar{x}(a_*^r t - lT_*). \end{aligned} \quad (16)$$

For this baseband signal, if we choose a sampling period T_*/a_*^k for discretization, it is only optimal for the k th component (the first summand above). In addition, the other channel coefficients $\bar{h}_{r,l}$, for $r \neq k$, are obtained by sampling the channel in the lag domain with T_*/a_*^r rather than T_*/a_*^k [c.f. (13)]. This means that once the signal in (16) is discretized, the resulting discrete baseband model will be subject to a nuisance embedded in the second term on its right-hand side, which will inevitably give rise to a performance penalty on a practical receiver.

⁴We notice that a similar problem (finding an optimal single sampling rate) is considered in [13].

In this paper, we will tackle the above problem through the design of the transmit and receive pulse. As will become evident soon, if the transmit and receive pulse can smartly be designed, we are able to annihilate the nuisance from the discrete baseband model.

III. TRANSMIT SIGNAL DESIGN

Prior to proceeding, we first assume that there exists a real pulse $p(t)$ of unit energy that is strictly band-limited with baseband bandwidth W_* . In other words, if $P(f)$ denotes the Fourier transform of $p(t)$, then $P(f)$ has nonzero elements only within $[-W_*/2, W_*/2]$.

For a pulse $p(t)$, we denote its scaled version as

$$p_{k'}(t) = a^{k'/2} p(a^{k'} t), \quad (17)$$

where a is referred to as the *base scale*. The effective bandwidth of $p_{k'}(t)$ equals $a^{k'} W_*$. If we use $p_{k'}(t)$ as a transmit pulse to modulate symbols $s_{k',n}$, then the baseband transmit signal $\bar{x}_{k'}(t)$ can be written as

$$\bar{x}_{k'}(t) = \sum_n s_{k',n} p_{k'}(t - nT/a^{k'}), \quad (18)$$

where T is referred to as the *base lag*. The above expression suggests that $\bar{x}_{k'}(t)$ has symbol period $T/a^{k'}$. The value of a and T will be soon determined in Section III-A.

For the sake of clarity, we first derive a *single-layer* signaling scheme, where a single-rate pulse $p_{k'}(t)$ is used to modulate the transmit symbols, and then generalize it to a *multi-layer* signaling scheme.

A. Single-Layer Signaling

In the *single-layer* signaling scheme, the transmit signal is $\bar{x}_{k'}(t)$, which is next up-converted to the carrier frequency $a^{k'} f_c$ resulting in the passband signal

$$x_{k'}(t) = \Re \left\{ \bar{x}_{k'}(t) e^{j2\pi a^{k'} f_c t} \right\}.$$

A critical element of our design is the assumption that we can properly match the scales and delays of our signaling to that of the channel. This boils down to matching the parameters as follows:

$$a = a_*, \text{ and } T = a^{k'} T_*, \quad (19)$$

which corresponds to a Nyquist sampling scheme using a and T in the Mellin domain and in the Fourier domain, respectively, for the received signal on the k' th layer (see [4] for more details). Note that the above requirements are not always easy to satisfy

because a_* and T_* themselves are in turn determined by $p_{k'}(t)$. We will return to this issue in Section III-B, but for now assume that (19) is perfectly achieved.

At the receiver, we down-convert the received signal using a center frequency $a^k f_c$ (note that k is not necessarily equal to k'). After down-conversion, the baseband representation given by (16) can be rewritten as shown in (20) at the bottom of the page. Comparing (20) to (16), we have added k' in the subscript of $\bar{z}_{k,k'}^{\text{SL}}(t)$ to emphasize the dependence of this signal on the specific carrier frequency, $a^{k'} f_c$, used for up-conversion. Later, we will see that in a multi-layer signaling scheme, k' represents the k' th transmission layer. In (20), the number of scales $R_{k'}$ is equal to

$$R_{k'} = \lceil \ln \alpha_{\max} / \ln a \rceil \equiv R. \quad (21)$$

Because $R_{k'}$ is independent of k' , we will drop this subscript in the sequel for the sake of notational ease. The number of lags $L_{k'}(r)$ in (20) is determined by

$$L_{k'}(r) = \lceil a^{r+k'} \tau_{\max} / T \rceil = L(r + k'), \quad (22)$$

with $L(r) = \lceil a^r \tau_{\max} / T \rceil$; $\bar{h}_{r,l}^{(k')}$ is similarly defined as in (13), but taking (19) into account:

$$\begin{aligned} \bar{h}_{r,l}^{(k')} &= \int_0^{\tau_{\max}} \int_1^{\alpha_{\max}} h(\alpha, \tau) e^{-j2\pi f_c a^{k'+r} \tau} \\ &\quad \times \text{sinc} \left(r - \frac{\ln \alpha}{\ln a} \right) \text{sinc} \left(l - \frac{a^{k'+r} \tau}{T} \right) d\alpha d\tau. \end{aligned} \quad (23)$$

We next seek to nullify the crosstalk term in (20) by taking the following steps. We first deploy a receive filter $p_k(t)$ on $\bar{z}_{k,k'}^{\text{SL}}(t)$, and then discretize the resulting signal by sampling at rate T/a^k . The resulting sample obtained at the m th sampling instant, denoted as $\bar{y}_{k,k',m}$, can be expressed as

$$\begin{aligned} \bar{y}_{k,k',m} &= p_k^*(-t) \otimes \bar{z}_{k,k'}^{\text{SL}}(t) \big|_{t=mT/a^k} \\ &= \int p_k \left(t - \frac{mT}{a^k} \right) \bar{z}_{k,k'}^{\text{SL}}(t) dt \end{aligned} \quad (24)$$

$$\begin{aligned} &= \int p_k \left(t - \frac{mT}{a^k} \right) \sum_{r=0}^R \delta_{k-r-k'} \\ &\quad \times \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} a^{r/2} \bar{x}_{k-r}(a^r t - lT/a^{k-r}) dt \\ &\quad + \int p_k \left(t - \frac{mT}{a^k} \right) \sum_{r=0, r \neq k-k'}^R C_{r+k'}(t) dt. \end{aligned} \quad (25)$$

$$\begin{aligned} \bar{z}_{k,k'}^{\text{SL}}(t) &= \sum_{r=0}^{R_{k'}} e^{j2\pi f_c (a^{k'+r} - a^k) t} \sum_{l=0}^{L_{k'}(r)} \bar{h}_{r,l}^{(k')} a^{r/2} \bar{x}_{k'}(a^r t - lT/a^{k'}) = \sum_{r=0}^{R_{k'}} \delta_{k-r-k'} \sum_{l=0}^{L(k'+r)} \bar{h}_{r,l}^{(k')} a^{r/2} \bar{x}_{k'}(a^r t - lT/a^{k'}) \\ &\quad + \underbrace{\sum_{r=0, r \neq k-k'}^{R_{k'}} e^{j2\pi f_c (a^{k'+r} - a^k) t} \sum_{l=0}^{L(k'+r)} \bar{h}_{r,l}^{(k')} a^{r/2} \bar{x}_{k'}(a^r t - lT/a^{k'})}_{C_{r+k'}(t): \text{CROSSTALK}}. \end{aligned} \quad (20)$$

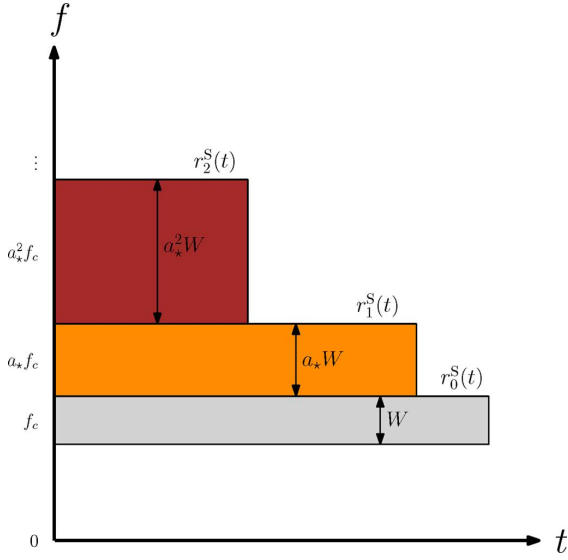


Fig. 5. Components of the received signal are non-overlapping in the frequency domain thanks to Theorem 1.

The following theorem will be useful to the ensuing derivations (see Appendix A for a proof).

Theorem 1: If the base scale a satisfies both (19) and

$$a \geq \frac{2f_c + W_*}{2f_c - W_*}, \quad (26)$$

then

$$\int_{-\infty}^{\infty} \sqrt{a^k a^{k'}} p(a^k t - mT) e^{j2\pi f_c a^k t} \times p(a^{k'} t - nT) e^{-j2\pi f_c a^{k'} t} dt = \delta_{k-k'} g_{n-m}, \quad (27)$$

where

$$g_n = \int p(t)p(t - nT) dt. \quad (28)$$

With the aid of Theorem 1, we are able to eliminate the crosstalk term in (20) since (see Appendix B for a proof)

$$\int p_k \left(t - \frac{mT}{a^k} \right) \sum_{r=0, r \neq k-k'}^R C_{r+k'}(t) dt = 0. \quad (29)$$

As a result of (29), we can simplify (25) to

$$\begin{aligned} \bar{y}_{k,k',m} &= \sum_{r=0}^R \delta_{k-r-k'} \int p_k \left(t - \frac{mT}{a^k} \right) \\ &\quad \times \sum_n s_{k-r,n} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} a^{k/2} p(a^k t - (n+l)T) dt \\ &= \sum_{r=0}^R \delta_{k-r-k'} \sum_n s_{k-r,n} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} \\ &\quad \times \int a^k p(a^k t - (n+l)T) p(a^k t - mT) dt \\ &= \sum_{r=0}^R \delta_{k-r-k'} \sum_n s_{k-r,n} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} g_{m-n-l} \end{aligned} \quad (30)$$

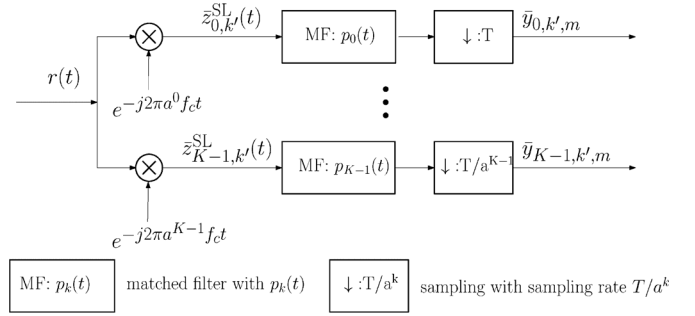


Fig. 6. The proposed receiver architecture with K receive branches.

where g_{m-n-l} in the last equality is defined in (28).

To avoid information loss, we will repeat the above operations for $k = 0, \dots, K-1$ with $K \geq R+1$ in the single-layer transmission. This means that a multi-branch structure is imposed on the receiver, where each branch is aimed at processing one component of the received signal. Such a receiver structure is schematically depicted in Fig. 6. In Section IV, we will show how to combine the results from each branch optimally to estimate the data symbols.

Remark 2: As mentioned earlier, corresponding to the parameterized channel model, we have effectively decomposed the received signal into several components, each one occupying a different position in the frequency domain. As a matter of fact, Theorem 1 ensures that these components will not be overlapping with each other. This idea is suggested by Fig. 5, where the equality in (26) is assumed. Accordingly, the receive filter $p_k(t)$ serves as a low-pass filter eliminating the crosstalk term.

In comparison, the components of the received signal in [4], [5] are nested within each other in the frequency domain [see Fig. 4(b)]. To eliminate the crosstalk term, [4], [5] resort to the scale-orthogonality of the transmit waveform, i.e.,

$$\int p_k(t)p_{k'}(t) dt = \delta_{k-k'}. \quad (31)$$

It is not specified by [4] how to guarantee the above equality. A more solid treatment is given by [5], which, however, relies on a particular direct-sequence spread-spectrum construction of the signal.

B. Pulse Design

In this subsection, we give a heuristic illustration of the design of the pulse $p(t)$. Without loss of generality, we consider the case of $k' = 0$, for which the transmit pulse in passband admits an expression of $p(t)e^{j2\pi f_c t}$. Usually, the carrier frequency f_c is a system parameter, and therefore our design freedom is the pulse type and its effective bandwidth W_* .

For a given pulse type, once we have chosen a certain bandwidth W_* for the baseband pulse $p(t)$, resulting in its passband pulse $\Re\{p(t)e^{j2\pi f_c t}\}$, the dilation spacing a_* and translation spacing T_* are accordingly determined (see Remark 1 for more details). Then, matching these parameters with the base scale a and base lag T of the transmit signal means setting $a = a_*$ and $T = T_*$ [c.f. (19)]. We next determine whether the resulting a satisfies (26) in Theorem 1. If so, the design is complete. Otherwise, one should select a different bandwidth for the pulse or even a different pulse type to repeat the above steps.

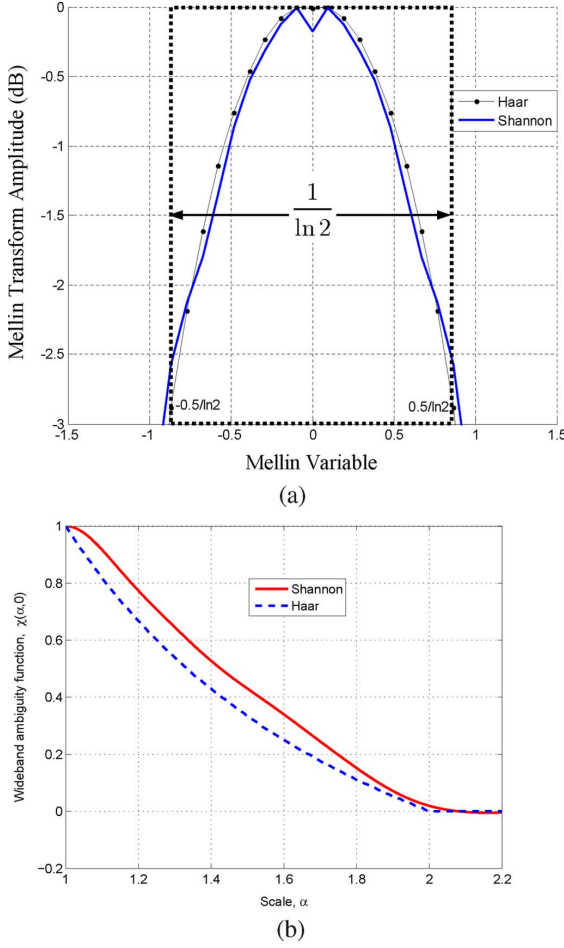


Fig. 7. Two approaches for solving a . (a) Mellin support. (b) Wideband ambiguity function.

Here, we give a specific example of $p(t)$, which is a sinc function defined as

$$p(t) = W^{1/2} \text{sinc}(Wt), \quad (32)$$

whose effective bandwidth is exactly $W_\star = W$. It is known that $\Re\{p(t)e^{j2\pi f_c t}\}$ belongs to the Shannon wavelets [30] if we choose $W = \frac{2}{3}f_c$ in (32), and in this case a *dilation spacing* of $a_\star = 2$ is yielded. This is corroborated by Fig. 7(a) and (b), which depicts the results based on a Mellin approach and a WAF approach, respectively. Additionally, the corresponding *translation spacing* is given by $T_\star = 1/W_\star = 1/W$. For the parameter matching, we have $a = a_\star$ and $T = T_\star$. In this case, $a = 2$ and $W_\star = W = \frac{2}{3}f_c$ satisfy the equality of (26). In this manner, this specific example is a suitable pulse design.

As a comparison, we note that the Haar wavelet used in [4] as the passband pulse, for which $p(t)$ corresponds to a rectangular function, is not a suitable pulse design for our purposes. Although it yields the same $a_\star = 2$ as shown in Fig. 7, it has a much larger effective bandwidth than the Shannon wavelet due to the spectrum leakage as shown in Fig. 8. As a result, after parameter matching, the (in)equality in (26) cannot hold, which implies that the cross-talk in (20) is non-negligible. This effect is further studied in Section VI.

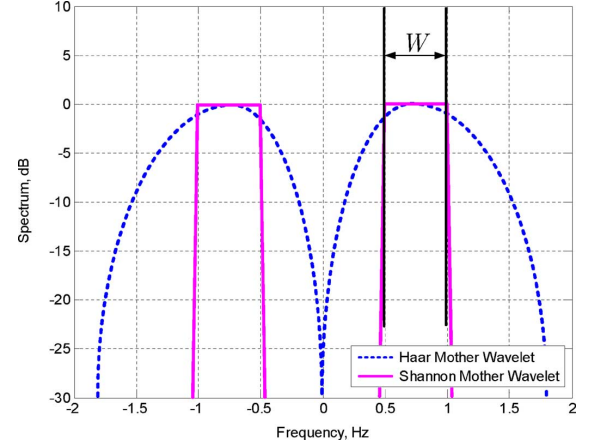


Fig. 8. Spectrum of the Haar and Shannon mother wavelets.

Another interesting consequence of using a Shannon wavelet is that the resulting sampled correlation function g_n defined in (28) is

$$g_n = W_\star \int \text{sinc}(W_\star t) \text{sinc}(W_\star t - n) dt = \text{sinc}(n) = \delta_n. \quad (33)$$

As a result, we are able to simplify (30) further to

$$\begin{aligned} \bar{y}_{k,k',m} &= \sum_{r=0}^R \delta_{k-r-k'} \sum_n s_{k-r,n} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} \delta(m-n-l) \\ &= \sum_{r=0}^R \delta_{k-r-k'} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} s_{k-r,m-l}, \end{aligned} \quad (34)$$

which enables the design of a low-complexity equalizer in the sequel.

C. Multi-Layer Signaling

Recall that in OFDM, the maximum spectral efficiency can be achieved by partitioning the available bandwidth into several orthogonal sub-bands. Analogously, we can also design a multi-layer transmission scheme, where in the k' th layer, the transmit data symbols are modulated by a different pulse $p_{k'}(t)$, and up-converted to a carrier frequency $a^{k'}f_c$ for $k' = 0, \dots, K' - 1$. Thanks to Theorem 1, the sub-bands occupied by each layer will not overlap with each other. When (26) holds, these sub-bands will be contiguous, resulting in a maximum spectral efficiency. In contrast to OFDM, the sub-bands have unequal bandwidth. The proof of the above ideas is rather trivial by straightforwardly applying Theorem 1. Here, we can just reuse Fig. 5 to illustrate the idea schematically.

With multiple layers, the actually transmitted signal $x(t)$ in passband can be expressed as

$$\begin{aligned} x(t) &= \sum_{k'=0}^{K'-1} x_{k'}(t) \\ &= \sum_{k'=0}^{K'-1} \sum_n \Re \left\{ a^{k'/2} s_{k',n} p(a^{k'}t - nT) e^{j2\pi f_c a^{k'}t} \right\}. \end{aligned} \quad (35)$$

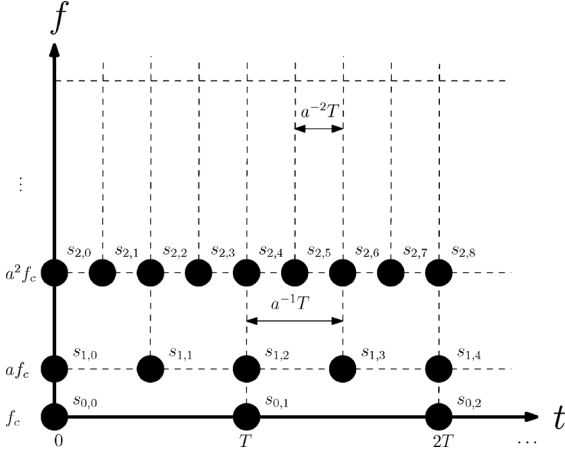


Fig. 9. T-F tiling diagram of a multi-layer transmission signaling scheme.

Accordingly, at the k th receive branch, the resulting sample obtained at the m th time-instant, denoted as $\tilde{y}_{k,m}$, is just a superposition of $\tilde{y}_{k,k',m}$ derived in (34) for $k' = 0, \dots, K' - 1$, i.e.,

$$\begin{aligned} \tilde{y}_{k,m} &= \sum_{k'=0}^{K'-1} \tilde{y}_{k,k',m} \\ &= \sum_{k'=0}^{K'-1} \sum_{r=0}^R \delta_{k-r-k'} \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} s_{k-r,m-l} \\ &= \sum_{r=0}^R \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} s_{k-r,m-l}. \end{aligned} \quad (36)$$

The above indicates that the received signal at each branch is subject to both inter-symbol interference (ISI) and inter-layer interference (ILI) as a consequence of the MSML channel model.

We conclude this subsection with the following remark.

Remark 3: With W_* and a obtained as indicated in the previous subsection, we can impose an upper-bound on the number of transmit layers K' . Like the base frequency f_c , usually the total available transmission bandwidth of a communication system B is fixed, and therefore

$$B \geq \sum_{k'=0}^{K'-1} a^{k'} W_*, \quad (37)$$

from which an upper-bound for K' can be attained.

Remark 4: The T-F tiling diagram of the proposed multi-layer transmission scheme is shown in Fig. 9, where each black circle indicates the T-F position where one transmit data symbol is located. One can immediately observe the resemblance to the T-F tiling diagram of the parameterized channel plotted in Fig. 3(a). By this means, we match the transmit signal to the channel in the T-F plane.

Remark 5: The transmit signal described in (35) belongs to the multi-scale wavelet modulation (MSWM) family proposed in [19], [20] if $p(t)e^{j\pi f_c t}$ is an orthogonal wavelet. One difference between this paper and [19], [20] is that the latter works only examine a wavelet signal over a flat fading channel, while we tailor our signal by intelligently designing the pulse to the MSML channel model. Despite this difference, one can still

use the same arguments in [19], [20] to show that the transmit signal given in (35) will have the same spectral efficiency⁵ as traditional transmission schemes such as OFDM if the equality in (26) is satisfied (we refer readers to [19], [20] for the detailed proof). If only the inequality in (26) is satisfied, there will be some frequency gap between adjacent transmit layers, and the bandwidth efficiency will be reduced. Similarly, such a frequency gap can also emerge in practical multi-carrier systems, where spectrum gaps are introduced to reduce the inter-carrier interference induced by Doppler, e.g., in [29], [32], [33].

IV. BLOCK-WISE TRANSCEIVER DESIGN

For the sake of clarity, we recap the results in (36) here

$$\bar{y}_{k,m} = \sum_{r=0}^R \sum_{l=0}^{L(k)} \bar{h}_{r,l}^{(k-r)} s_{k-r,m-l} + \bar{v}_{k,m}, \quad (38)$$

where we have also added the noise term $\bar{v}_{k,m}$, whose expression can be obtained by

$$\bar{v}_{k,m} = \int_{-\infty}^{+\infty} a^{k/2} w(t) e^{-j2\pi f_c a^k t} p(mT - a^k t) dt, \quad (39)$$

where the continuous time noise $w(t)$ is introduced in (1). Equation (38) shows that the discrete samples at the k th receiver branch are related to the transmitted information symbols via a 2-D time-varying discrete finite impulse response (FIR) filter. This feature will be exploited by considering a block-wise transmission, where the transmitted symbols on each layer are partitioned into successive blocks, each containing $N + Z$ data symbols. The data symbols contained in such a block from all the K' layers can be collectively expressed as

$$\sum_{k'=0}^{K'-1} \sum_{n=0}^{N+Z-1} \Re \left\{ a^{k'/2} s_{k',n} p(a^{k'} t - nT) e^{j2\pi f_c a^{k'} t} \right\}. \quad (40)$$

To avoid inter-block interference (IBI), we introduce a cyclic prefix (CP) of a length of Z symbols along each layer, such that

$$s_{k',n} = \begin{cases} b_{k',n-Z}, & \text{for } Z \leq n < N+Z \\ b_{k',N+n-Z}, & \text{for } 0 \leq n < Z \end{cases}, \quad (41)$$

where $b_{k',n}$ stands for the n th information symbol transmitted at the k' th transmit layer.

At the receiver, we will consider a filter bank with $K = R + K'$ branches, whose structure is depicted in Fig. 6, with the received samples on the k th branch given by (38). Obviously, IBI can be completely annihilated if $Z \geq L(k)$ for all $k \in \{0, 1, \dots, R + K' - 1\}$, or in other words,

$$Z \geq \lceil a^{R+K'-1} \tau_{\max} / T \rceil = \lceil a^{R+K'-1} L(0) \rceil. \quad (42)$$

All the data blocks are treated in this way. Here, it is interesting to note that because of the disparate scale at each transmit layer, the representations of the different blocks in the T-F plane are not parallel to each other as for OFDM. This effect is schematically illustrated in Fig. 10(a), where the shaded area indicates the area in the T-F plane where information symbols reside, and the blank area represents that

⁵Spectral efficiency refers to the available information rate for a given transmission bandwidth [31].

of the CPs. It is noteworthy that the use of these CP symbols is another difference distinguishing our work from that of [19], [20], where it is not clear how to add a guard interval to the MSWM signal. We show that adding these CPs is not trivial as shown in Fig. 10(a). For comparison, the case of OFDM block transmission is sketched in Fig. 10(b).

To design a block equalizer, we stack the information symbols sent through the k' th transmission layer in a vector $\mathbf{b}_{k'} = [b_{k',0}, \dots, b_{k',N-1}]^T$, and $\mathbf{b}_k = \mathbf{0}$ if $k < 0$ or $k > K' - 1$. Likewise, we stack the received samples from the k th receiver branch, with CP stripped off, in a vector $\mathbf{y}_k = [\bar{y}_{k,Z}, \dots, \bar{y}_{k,Z+N-1}]^T$. It follows from (38) that

$$\mathbf{y}_k = \sum_{r=0}^R \mathbf{H}_r^{(k-r)} \mathbf{b}_{k-r} + \mathbf{v}_k, \quad (43)$$

where $\mathbf{v}_k$ is similarly defined as $\mathbf{y}_k$, and $\mathbf{H}_r^{(k-r)}$ denotes an $N \times N$ circulant matrix with first column $[h_{r,0}^{(k-r)}, h_{r,1}^{(k-r)}, \dots, h_{r,L(k)}^{(k-r)}, 0, \dots, 0]^T$. If we next stack the $\mathbf{b}_{k'}$'s from all the transmit layers into one vector $\mathbf{b} = [\mathbf{b}_0^T, \dots, \mathbf{b}_{K'-1}^T]^T$, and the $\mathbf{y}_k$'s from all receive branches into one vector $\mathbf{y} = [\mathbf{y}_0^T, \dots, \mathbf{y}_{K'+R-1}^T]^T$, it then follows from (43) that

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{v} \quad (44)$$

where $\mathbf{v}$ is similarly defined as $\mathbf{y}$, and $\mathbf{H}$ stands for the $(K' + R)N \times K'N$ matrix specified as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0^{(0)} & & & \mathbf{0} \\ \vdots & \ddots & & \\ \mathbf{H}_R^{(0)} & & \ddots & \\ & \ddots & & \mathbf{H}_0^{(K'-1)} \\ & & \ddots & \vdots \\ & \mathbf{0} & & \mathbf{H}_R^{(K'-1)} \end{bmatrix} \quad (45)$$

We conclude this section with the following remarks.

Remark 6: The 2-D FIR filter structure is clearly revealed in (45), where the block element $\mathbf{H}_r^{(k)}$ can be viewed as the block tap of a time-varying outer FIR filter (note the varying superscript). Each $\mathbf{H}_r^{(k)}$ yields an FIR filter with scalar tap $h_{r,l}^k$, which is time-invariant inducing the circulant structure of $\mathbf{H}_r^{(k)}$.

Remark 7: With $K' = 1$, the proposed transceiver scheme reduces to a single-layer approach. We can then interpret the I/O relationship in (44) as a SIMO-OFDM system with $R + 1$ receive antennas. Further, if the Doppler effect is absent with $R = 0$, then the I/O relationship in (44) can be interpreted as a multi-band OFDM system [11] with $K' - 1$ bands.

V. FREQUENCY-DOMAIN EQUALIZATION

The circulant structure of $\mathbf{H}_r^{(k)}$ suggests that it is possible to equalize the channel in the frequency domain, as in traditional OFDM systems for narrowband time-invariant channels, to lower the equalization complexity. This is achieved in two steps.

In the first step, let us transform the received signal to the frequency domain by $\tilde{\mathbf{y}} = (\mathbf{I}_{K'+R} \otimes \mathbf{F})\mathbf{y}$, where $[\mathbf{F}]_{n,m} = \frac{1}{\sqrt{N}} e^{-j2\pi \frac{nm}{N}}$ denotes the normalized discrete Fourier transform

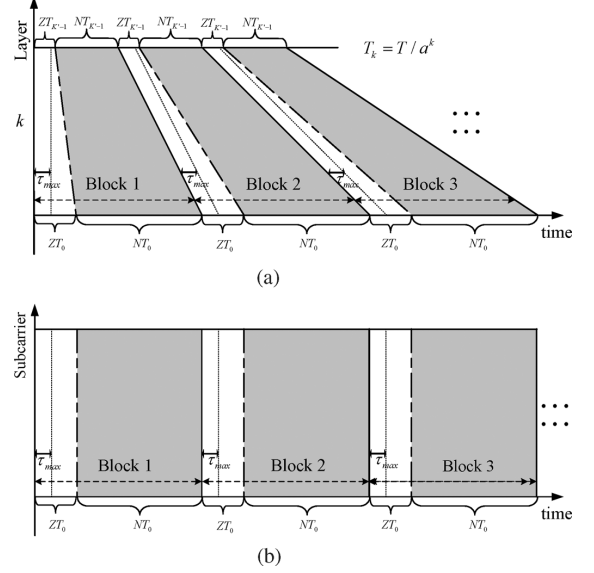


Fig. 10. Structure of transmitted data blocks in the T-F plane. (a) Multi-layer transmission. (b) OFDM transmission.

(DFT) matrix. The frequency-domain expression of (44) can then be expressed as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{b}} + \tilde{\mathbf{v}}, \quad (46)$$

where $\tilde{\mathbf{b}} = (\mathbf{I}_{K'} \otimes \mathbf{F})\mathbf{b}$, and $\tilde{\mathbf{v}}$ is similarly defined as $\tilde{\mathbf{b}}$. Furthermore,

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{\mathbf{H}}_0^{(0)} & & & \mathbf{0} \\ \vdots & \ddots & & \\ \tilde{\mathbf{H}}_R^{(0)} & & \ddots & \\ & \ddots & & \tilde{\mathbf{H}}_0^{(K'-1)} \\ & & \ddots & \vdots \\ & \mathbf{0} & & \tilde{\mathbf{H}}_R^{(K'-1)} \end{bmatrix} \quad (47)$$

where $\tilde{\mathbf{H}}_r^{(k)} = \mathbf{F}\mathbf{H}_r^{(k)}\mathbf{F}^{-1}$ denotes an $N \times N$ diagonal matrix whose n th diagonal is

$$\tilde{h}_{r,n}^{(k)} = \sum_{l=0}^{L(k)} h_{r,l}^{(k)} e^{j\frac{2\pi}{N}nl}. \quad (48)$$

Observe that $\tilde{\mathbf{H}}$ has a banded structure on the block level with each block entry being a diagonal matrix. There exists a $(K' + R)N \times (K' + R)N$ permutation matrix $\mathbf{P}_{K'+R}$ and a $K'N \times K'N$ permutation matrix $\mathbf{P}_{K'}$ matrix⁶, such that we can permute (46) to

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{b}} + \tilde{\mathbf{v}}, \quad (49)$$

where $\tilde{\mathbf{y}} = \mathbf{P}_{K'+R}\tilde{\mathbf{y}}$; $\tilde{\mathbf{b}} = \mathbf{P}_{K'}\tilde{\mathbf{b}}$; $\tilde{\mathbf{v}} = \mathbf{P}_{K'+R}\tilde{\mathbf{v}}$, and $\tilde{\mathbf{H}} = \mathbf{P}_{K'+R}\tilde{\mathbf{H}}\mathbf{P}_{K'}^T$. It is straightforward to show that $\tilde{\mathbf{H}}$ is a *block diagonal* matrix, where each diagonal block is a $(K' + R) \times K'$

⁶We use $\mathbf{P}_K$ to represent a permutation matrix of a proper dimension with depth K . Specifically, consider a vector $\mathbf{a} = [a_0, a_1, \dots, a_{NK-1}]^T$, then $\mathbf{P}_K\mathbf{a} = [\mathbf{a}_0^T, \mathbf{a}_1^T, \dots, \mathbf{a}_{K-1}^T]^T$ with $\mathbf{a}_k = [a_k, a_{K+k}, a_{2K+k}, \dots, a_{(N-1)K+k}]^T$ for $k = 0, 1, \dots, K-1$.

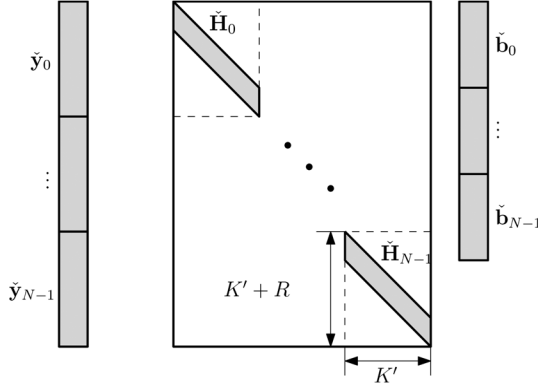


Fig. 11. Illustration of the matrix-vector form for (49).

strictly banded matrix with a bandwidth of $R + 1$. The structure of $\tilde{\mathbf{H}}$ is illustrated in Fig. 11. Denoting the k th diagonal block as $\tilde{\mathbf{H}}_n$, for $n \in \{0, \dots, N - 1\}$, we can split $\tilde{\mathbf{y}}$ into N subvectors, where the n th subvector $\tilde{\mathbf{y}}_n$, which is comprised of the nK' th through $[(n + 1)K' - 1]$ st entries of $\tilde{\mathbf{y}}$, is given by

$$\tilde{\mathbf{y}}_n = \tilde{\mathbf{H}}_n \tilde{\mathbf{b}}_n + \tilde{\mathbf{v}}_n, \quad (50)$$

where $\tilde{\mathbf{b}}_n$ and $\tilde{\mathbf{v}}_n$ are defined similarly to $\tilde{\mathbf{y}}_n$. The strictly banded structure of $\tilde{\mathbf{H}}_n$ enables us to employ the low-complexity LMMSE equalizer designed in [22] or the low-complexity turbo equalizer in [23] to equalize each $\tilde{\mathbf{H}}_n$, one by one.

Remark 8: The derivations throughout the paper do not exploit any assumption about the noise statistics of $\tilde{\mathbf{v}}_{k,m}$. For the low-complexity LMMSE equalizer of [22] or the low-complexity turbo equalizer of [23], it is desirable that the noise should be zero mean and uncorrelated. In Appendix D, we show that this is guaranteed if the continuous-time noise $w(t)$ is white and zero mean, and if an ideal pulse $p(t)$ can be designed as in Section III-B.

VI. NUMERICAL RESULTS

In this section, we provide simulation results to demonstrate the performance of the proposed wideband system. We will use a discrete path model to emulate the real wideband LTV channel

$$h(\alpha, \tau) = \sum_{p=0}^P h_p \delta(\alpha - \alpha_p) \delta(\tau - \tau_p), \quad (51)$$

with $P = 10$; h_p is modeled as an i.i.d. Gaussian variable with zero mean and unit variance. Without loss of generality, we assume that τ_p is equal to 0 if $p = 0$; otherwise it is modeled to have a uniform distribution over $[0, \tau_{\max})$. Likewise, we assume that α_p is equal to 1 if $p = 0$; otherwise it is modeled to have a uniform distribution over $[1, \alpha_{\max})$. Although the values of h_p , τ_p and α_p are assumed to stay constant during several transmitted blocks, they result in a wideband channel whose channel response varies with time. Consequently, the I/O relationship in (1) can be written as

$$r(t) = \sum_{p=0}^P h_p \sqrt{\alpha_p} x(\alpha_p(t - \tau_p)), \quad (52)$$

For the transmission, we use

$$p(t) = \text{sinc}(t/T)/\sqrt{T}, \quad (53)$$

as the transmission waveform with the *base lag* T equal to 10^{-3} s ($W = 1$ kHz). The carrier frequency f_c is chosen to be 1.5 kHz. As a result, the *base scale* a of $p(t)e^{j2\pi f_c t}$ is equal to 2. Refer to Section III-B for more details about these parameters.

A. Channel Model Validation

To examine the accuracy of the proposed channel model, we follow a similar channel sounding approach as used in [4]: we send a single information symbol $s_{0,0}$ modulated on $p(t)$ in order to examine the channel in terms of the impulse response function. The normalized mean squared-error (NMSE) between $r(t)$ in (52) and $r^{\text{SL}}(t)$ evaluated at the output of the receiver branches is computed as follows:

$$\begin{aligned} \text{NMSE}_{\text{MSML}} &= \frac{\sum_{k=0}^R \sum_{l=0}^{L(k)} \left| \int p_k \left(t - \frac{lT}{a^k} \right) (r(t) - r^{\text{SL}}(t)) e^{-j2\pi f_c a^k t} dt \right|^2}{\sum_{r=0}^R \sum_{l=0}^{L(k)} \left| \int p_k \left(t - \frac{lT}{a^k} \right) r(t) e^{-j2\pi f_c a^k t} dt \right|^2}. \end{aligned} \quad (54)$$

We now compare three NMSEs in Fig. 12, corresponding to the following situations: a MSML model using a pulse design with parameter matching (“Shannon, MSML”), a MSML model using a pulse design without parameter matching (“Haar, MSML”), and a SSML model (“Shannon, SSML”). We underscore that the transmit pulse given in (53) satisfies the equality in Theorem 1 (“Shannon, MSML”). The second curve (“Haar, MSML”) corresponds to the case where a Haar wavelet is used as the transmit pulse, which is characterized by the same parameters T , a and f_c . We derive a channel model following the approach of [4], and calculate the NMSE of this channel model in the same way as (54). Note that because the Haar wavelet has a considerable power leakage outside the considered bandwidth [see Fig. 8], Theorem 1 is violated, implying that the crosstalk in (16) cannot be entirely eliminated. The resulting cross-talk, which can be viewed as a modeling error, results in the performance degradation seen in Fig. 12 (“Haar, MSML”). The third NMSE curve (“Shannon, SSML”) is motivated by the fact that the WLTV channel is often modeled using an SSML assumption [9], [10], [12], or assuming a single rate to sample the channel [13], [14]. In these works, a single-scale signal, denoted as $r_{\text{single}}(t)$, is coined to approximate the received signal. This signal $r_{\text{single}}(t)$ can be expressed as

$$r_{\text{single}}(t) = \sum_{p=0}^P h_p \sqrt{a_{\text{single}}} x(a_{\text{single}}(t - \tau_p)), \quad (55)$$

where a_{single} can be found by e.g., [13]

$$a_{\text{single}} = \arg \min_{\alpha} \left| r(t) - \sum_{p=0}^P h_p \sqrt{\alpha} x(\alpha(t - \tau_p)) \right|^2. \quad (56)$$

The corresponding channel modeling error is computed by adapting (54) as shown in (57) at the bottom of the next page, where $p_{\text{single}}(t) = a_{\text{single}}^{1/2} p(a_{\text{single}} t)$. It can be seen that the modeling performance yielded by the SSML channel model is similar to the proposed MSML model for a low-to-moderate Doppler spread $\alpha_{\max}$, but deteriorates fast when the Doppler spread gets higher.

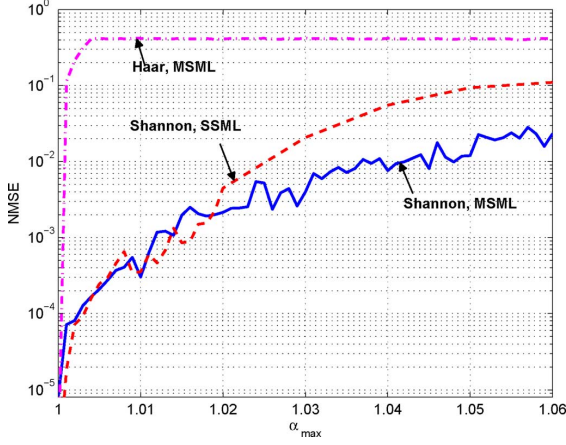


Fig. 12. Channel modeling performance. The solid line corresponds to the NMSE of the proposed model; the dash-dot line to the NMSE of the channel model in [4], and the dash line to the NMSE of the channel model based on an SSML assumption.

TABLE I
PARAMETERS FOR THE ADOPTED WIDEBAND CHANNELS

Channel	τ_{max}	α_{max}	$L(0)$	R	the maximal data rate of a single-layer transmission *	the data rate of our multi-layer transmission *
A	0.6 ms	1.00	1	0	3.76×10^3 bps	6.59×10^3 bps
B	1.2 ms	1.02	2	1	3.76×10^3 bps	6.59×10^3 bps
C	1.8 ms	1.04	2	1	3.76×10^3 bps	6.59×10^3 bps

* using BPSK, $N = 128$, $K' = 3$, $Z = 16$ and $T = 1.0$ ms. The data rate is given by $\frac{1}{L(0)T}$ for the work of [4], $\frac{N}{N+Z} \frac{a^{K-1}}{T}$ for a single-layer transmission, and $\frac{N}{N+Z} \frac{a^{K-1}}{(a-1)T}$ for a multi-layer transmission, where $a = 2$.

B. Equalization Performance

Supported by the results in Fig. 12, we will assume in the ensuing simulations that our model (12) has negligible errors and therefore, $r^{\text{SL}}(t) \approx r(t)$. For equalization, three types of channels are tested, whose channel parameters are specified in Table I. A multi-layer transmission is deployed with $K' = 3$ transmit layers. Accordingly, $K = R + K'$ receiver branches are employed at the receiver. Each transmit block contains $N = 128$ binary phase shift keying (BPSK) symbols, and is preceded by a CP of length $Z = 16$.

Fig. 13 shows the bit-error-rate (BER) performance of the proposed transceiver architecture using an LMMSE equalizer. As can be seen, the LMMSE equalizer renders a similar performance irrespective of the channel spread in delay and scale. As a comparison, we have also provided the performance of a matched-filter (MF) equalizer, which is used in [4], which is inferior

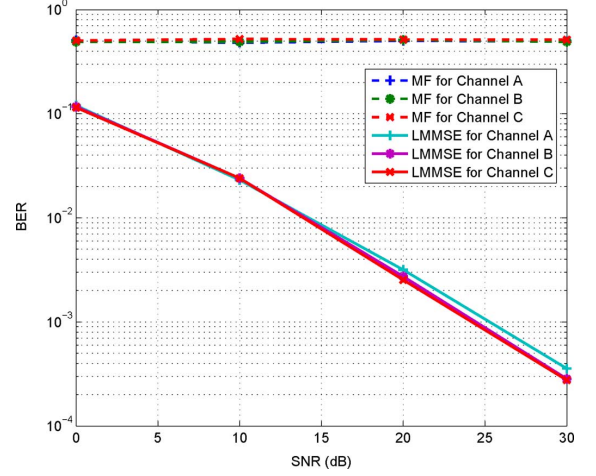


Fig. 13. BER performance using an LMMSE equalizer.

due to a high modeling error and indicates the necessity of channel equalization in the presence of the inter-symbol/inter-scale interference.

As mentioned before, since the proposed transceiver architecture results in a banded channel matrix (see (49)), many techniques designed for narrowband systems, with suitable adaptation, can be employed for our transmission scheme over wideband MSML channels. For instance, the matrix inversion required for the LMMSE equalizer can be achieved using the low-complexity algorithm given in [22]. Further, we can employ the banded turbo equalizers proposed in [23], which rely also on the banded structure of the channel matrix, to improve the BER performance even further via more equalization iterations. The results of the banded turbo equalizers for channel B are illustrated in Fig. 14. These simulation results indicate the suitability to our system of these low-complexity algorithms designed in [22], [23] for narrowband systems.

C. Single-Layer or Multi-Layer

In this subsection, we compare the multi-layer transmission scheme with respect to the single-layer transmission scheme, where we use the parameters of Channel C that are summarized in Table I, and the multi-layer transmitter consists of $K' = 3$ layers. The BER performance is compared with the results given in Fig. 15. One can see that the single-layer transmission results in better equalization performance. This is not a surprise, since the receiver for the multi-layer transmission has the more demanding task of resolving the interference among the different layers sent from the transmitter. On the other hand, the multi-layer transmitter results in a much higher spectral efficiency. To make a more fair comparison, we utilize the “goodput ratio” as a criterion, which is defined as

$$\text{NMSE}_{\text{SSML}} = \frac{\sum_{l=0}^{L(k)} \left| \int p_{\text{single}} \left(t - \frac{lT}{\alpha_{\text{single}}} \right) (r(t) - r_{\text{single}}(t)) e^{-j2\pi f_c \alpha_{\text{single}} t} dt \right|^2}{\sum_{l=0}^{L(k)} \left| \int p_{\text{single}} \left(t - \frac{lT}{\alpha_{\text{single}}} \right) r(t) e^{-j2\pi f_c \alpha_{\text{single}} t} dt \right|^2}, \quad (57)$$

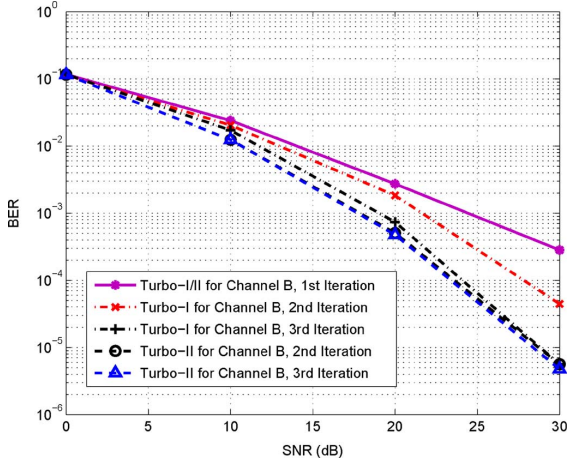


Fig. 14. BER performance using banded turbo equalizers [23].

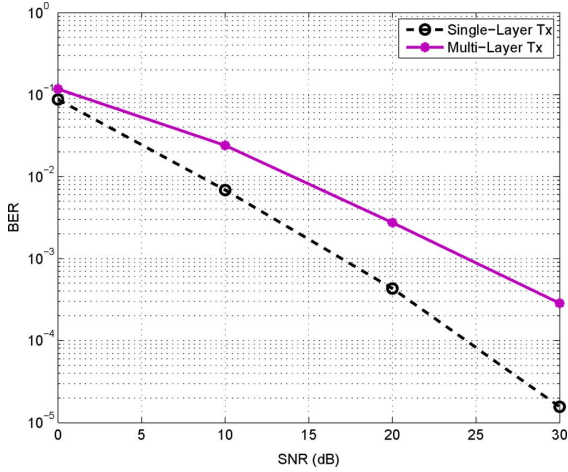


Fig. 15. BER comparison between the multi-layer transmission and the single-layer transmission.

$$\text{Goodput Ratio} = \frac{(1 - \text{BER}_M)\beta_M}{(1 - \text{BER}_S)\beta_S},$$

where β_M and β_S denote the maximal data rate of the multi-layer transmission and the single-layer transmission, respectively, and BER_M and BER_S denote the BER of the multi-layer transmission and the single-layer transmission, respectively. The goodput gives an index of the effective throughput of a system. The goodput ratio is plotted in Fig. 16, where we observe that the multi-layer transmission always has a larger goodput than the single-layer transmission, and this advantage is even more pronounced when the number of layers increases.

D. OFDM vs. Multi-Layer Block Transmission

In this subsection, we compare the performance of the multi-layer block transmission (MLBT) scheme with respect to the traditional OFDM transmission scheme over a wideband channel (i.e., Channel C in Table I). The multi-layer scheme consists of $K' = 3$ layers, with the blocks on each layer containing $N = 128$ symbols. Accordingly, we let the OFDM scheme employ 224 subcarriers, within a duration of 128 ms, to fill the same effective transmission bandwidth as our multi-layer scheme. In order to allow for a fair uncoded

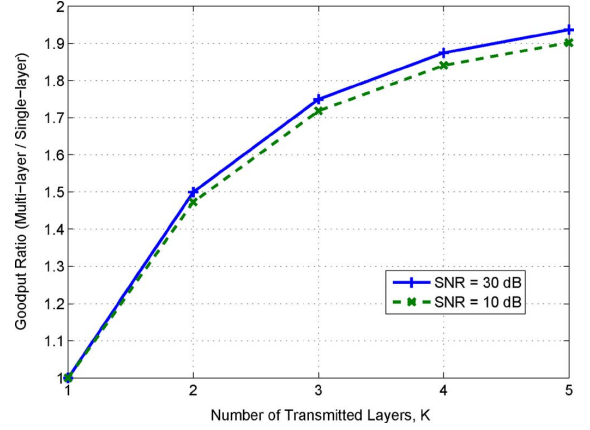


Fig. 16. Goodput ratio between the multi-layer transmission and the single-layer transmission.

performance comparison, we precode OFDM with a discrete Fourier transform at the transmitter, and use BSPK modulation as in our MLBT scheme. In addition, both schemes are equipped with the same guard interval of 16 ms (i.e., $Z = 16$ for our MLBT or 28 samples for OFDM), such that the spectral efficiencies are identical (i.e., $\frac{16}{128+16} \approx 0.89$). To equalize such an OFDM channel, we follow the widely used approach in practical OFDM systems, by first obtaining a uniform sampling rate [13] and then performing a banded channel equalization [11], [22], [23] in the frequency domain. The adopted matrix bandwidth here is 3. Note that, in this manner, the equalization of the OFDM channel has the same complexity as the frequency-domain equalization of our MLBT scheme, since they both induce a banded channel matrix with the same bandwidth. As shown in Fig. 17, the MLBT schemes yield a better performance than OFDM, because the transmit signal in the MLBT schemes is specially designed for MSML channels while the OFDM transmit signal is only optimized for SSML channels. By assuming an SSML model to approximate the actual MSML channel, a large channel modeling error is inevitable in the presence of a profound Doppler scale spread as shown in Fig. 12. Note that in Fig. 17, we have depicted the performance of the multi-layer scheme based on two pulses for $p(t)$: one is the sinc function as given in (32) that has been used so far, and the other is the root-raised cosine (RRC) function given by

$$p(t) = \frac{\sin(\pi(1 - \beta)Wt) + 4\beta Wt \cos(\pi(1 + \beta)Wt)}{\pi\sqrt{Wt}(1 - (4\beta Wt)^2)}$$

with $\beta = 0.25$ being the roll-off factor. For both pulses, the same *base scale* $a = 2$ and *base lag* $T = 1$ ms is applied. We have argued in Section III-B that these parameters are chosen to match the *dilation spacing* a_* and *translation spacing* T_* of the sinc function. For the RRC function, it can be computed that the corresponding a_* is larger than 2 and the corresponding T_* is less than 1 ms (because its effective bandwidth $(1 + \beta)W$ is more than $W = 1$ kHz). It indicates that the use of $a = 2$ and $T = 1$ ms does not match the channel parameters tightly, which inflicts a performance penalty on the multi-layer scheme based on the RRC pulse.

VII. CONCLUSION

Multi-scale multi-lag (MSML) channel models are appropriate for a variety of wideband time-varying channels such as

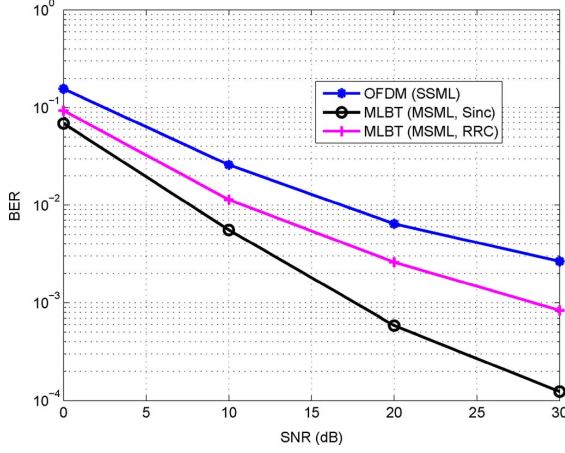


Fig. 17. BER comparison between the multi-layer transmission and OFDM.

underwater acoustic systems or terrestrial ultra-wideband radio systems. In this work, we have provided a novel parameterization of the continuous time multi-scale multi-lag (MSML) channel by taking the passband nature of the propagating signal explicitly into account. The associated baseband signal is evaluated and shown to result in inter-scale and inter-symbol interference. We have proposed a novel multi-layer transceiver for such MSML channels. At the transmitter, the information symbols are placed at different non-overlapping sub-bands to enhance the spectral efficiency, where each sub-band has a distinctive bandwidth, and therefore, the transmission in each sub-band is characterized by a different data rate. Our multi-layer transmission is a special case of the known multi-scale wavelet modulation (MSWM), and can thus achieve the same spectral efficiency as traditional transmissions, e.g., OFDM. Different from a traditional MSWM signal, we have built a block transmission scheme and introduced a guard interval to eliminate inter-block interference. To combat the multi-scale multi-lag effect of the channel, a filterbank is deployed at the receiver, where each branch of the filterbank will resample the received signal in a different way. By selecting a proper transmitter pulse, we have shown that the effective I/O relationship in the discrete domain can be captured by a block-diagonal channel, with each diagonal block being a banded matrix. As a result, the low-complexity equalizers that have been intensively used for narrow-band systems become also applicable here. For a comparison, without a proper pulse design, the multi-layer transmission is subject to inter-layer interference and a performance loss is thus inevitable. We have argued that the key to the success of the proposed scheme lies in a proper choice of the transmit pulse such that the channel parameters will have a tight match with the parameters of the transmit pulse. Optimal transmit pulse designs remains an interesting topic for future work.

APPENDIX A PROOF OF THEOREM 1

Let

$$a(t) = \sqrt{\frac{a^k}{T}} p(a^k t - mT) e^{j2\pi f_c a^k t},$$

$$b(t) = \sqrt{\frac{a^{k'}}{T}} p(a^{k'} t - nT) e^{j2\pi f_c a^{k'} t},$$

whose Fourier transform is denoted as $\mathcal{A}(f)$ and $\mathcal{B}(f)$, respectively. With these notations, the left-hand side of (27) can be rewritten as

$$\int_{-\infty}^{\infty} \frac{\sqrt{a^k a^{k'}}}{T} p(a^k t - mT) e^{j2\pi f_c a^k t} p(a^{k'} t - nT) e^{-j2\pi f_c a^{k'} t} dt$$

$$= \int_{-\infty}^{\infty} a(t) b^*(t) dt = \int_{-\infty}^{\infty} \mathcal{A}(f) \mathcal{B}^*(f) df,$$

where the last equality holds due to Parseval's theorem. We understand that $\mathcal{A}(f)$ is defined within the range

$$\mathcal{S}_a = \left[a^k f_c - a^k \frac{W}{2}, a^k f_c + a^k \frac{W}{2} \right)$$

$$\cup \left[-a^k f_c - a^k \frac{W^*}{2}, -a^k f_c + a^k \frac{W^*}{2} \right),$$

and $\mathcal{B}(f)$ is defined within the range

$$\mathcal{S}_b = \left[a^{k'} f_c - a^{k'} \frac{W^*}{2}, a^{k'} f_c + a^{k'} \frac{W^*}{2} \right)$$

$$\cup \left[-a^{k'} f_c - a^{k'} \frac{W^*}{2}, -a^{k'} f_c + a^{k'} \frac{W^*}{2} \right). \quad (58)$$

With a chosen as in (26), $\mathcal{S}_a \cap \mathcal{S}_b = \emptyset$ if $k \neq k'$. When $k = k'$, we have

$$g_{n-m} = \int a^k p(a^k t - mT) p^*(a^k t - nT) dt, \quad (59)$$

$$= \int p(t) p^*(t - (n-m)T) dt. \quad (60)$$

Because $p(t)$ is real, we obtain (28).

APPENDIX B PROOF OF (29)

The crosstalk term in (25) can be fully written as

$$\int p_k \left(t - \frac{mT}{a^k} \right) \sum_{r=0, r \neq k-k'}^R C_{r+k'}(t) dt$$

$$= \int p_k \left(t - \frac{mT}{a^k} \right) \sum_{r=0, r \neq k-k'}^R e^{j2\pi f_c (a^{r+k'} - a^k) t}$$

$$\times \sum_{l=0}^{L(k'+r)} \bar{h}_{r,l}^{(k')} a^{r/2} \bar{x}_{k'}(a^r t - lT) dt$$

$$= \sum_{r=0, r \neq k-k'}^R \sum_{l=0}^{L(k'+r)} \sum_n \bar{h}_{r,l}^{(k')} s_{k',n} T$$

$$\times \int \frac{\sqrt{a^k a^{r+k'}}}{T} e^{j2\pi f_c (a^{r+k'} - a^k) t}$$

$$\times p \left(t - \frac{mT}{a^k} \right) p \left(a^{k+r} t - \frac{(l+n)T}{a^{k'}} \right) dt.$$

It is then sufficient to prove that

$$\int \frac{\sqrt{a^k a^{r+k'}}}{T} e^{j2\pi f_c (a^{r+k'} - a^k) t}$$

$$\times p_k \left(t - \frac{mT}{a^k} \right) p \left(a^r t - \frac{(l+n)T}{a^{k'}} \right) dt = 0, \quad (61)$$

for $r \neq k - k'$. Note that $p_k(t - \frac{mT}{a^k}) = a^{k/2}p(a^k t - mT)$. This enables us to rewrite (61) as

$$\begin{aligned} & \int \frac{\sqrt{a^k a^{r+k'}}}{T} e^{j2\pi f_c(a^{r+k'} - a^k)t} \\ & \times p_k\left(t - \frac{mT}{a^k}\right) p(a^r t - (l+n)T_{k'}) dt \\ & = \int \left(\sqrt{\frac{a^k}{T}} p(a^k t - mT) e^{j2\pi f_c a^k t} \right)^* \\ & \times \sqrt{\frac{a^{r+k'}}{T}} p(a^{r+k'} t - (l+n)T) e^{j2\pi f_c a^{r+k'} t} dt, \\ & = \delta_{r+k'-k} g_{k,m-l-n}, \end{aligned}$$

where the last equality is due to Theorem 1. This concludes the proof.

APPENDIX C

THE BASIC SCALING FACTOR OF THE SHANNON WAVELET

Here, we examine the signal $x(t) = \sqrt{W} \text{sinc}(Wt) e^{j2\pi f_c t}$, which yields a Shannon wavelet with $f_c = 1.5W$. We resort to two approaches to determine the *basic scaling factor* of the Shannon wavelet. The first approach, which is adopted in [4], utilizes the Mellin transform, while the second approach, which is adopted in [5], utilizes the wideband ambiguity function.

For the first approach, we use a general Mellin variable $s = c - j2\pi\beta$, where c and β are both a real number. It can be derived that the Mellin transform of $x(t)$ can be expressed as

$$\begin{aligned} \mathcal{M}(\beta) &= \int_0^\infty x(t) t^{s-1} dt = \int_0^\infty t^{c-j2\pi\beta} x(t) \frac{dt}{t}, \\ &= \int_0^\infty x(t) e^{c \ln(t)} e^{-j2\pi\beta \ln(t)} \frac{dt}{t}. \end{aligned}$$

If we take a geometrically time-warped version of $x(t)$, i.e., $y(t) := x(e^t) e^{ct}$, we can rewrite the above equation as

$$\mathcal{M}(\beta) = \int_{-\infty}^\infty y(u) e^{-j2\pi\beta u} du,$$

which actually shows that the Mellin transform is inherently a logarithmic-time Fourier transform. Consequently, the discrete (inverse) Mellin transform can also be implemented by an inverse discrete Fourier transform (IDFT) but in the geometric sampling domain, which is obtained by interpolating the uniform domain [34]. In this paper, we follow the scale-representation [27] for the Mellin transform and use $c = 1/2$ instead of $c = 1$. The latter is used for the discrete Mellin transform in [26]. Therefore, we can adopt the DFT on the geometric samples to examine the Mellin bandwidth of $p(t)$, which is shown in Fig. 7(a), indicating that $M_* \approx \ln(1/2)$, and thus $a_* \approx 2$.

For the second approach, we use the wideband ambiguity function $\chi_p(\alpha, \tau) = \int p(t) \sqrt{\alpha} p(\alpha(t - \tau)) dt$ and select a_* according to $a_* = \min|\alpha|$ subject to $\chi_p(\alpha, 0) = 0$. This yields also $a_* = 2$ as suggested by Fig. 7(b).

APPENDIX D

NOISE STATISTICS

From (39), it is easy to show that $\mathcal{E}(\bar{v}_{k,m}) = 0$ if $\mathcal{E}(w(t)) = 0$.

For the second-order moment of $\bar{v}_{k,m}$, it follows that

$$\begin{aligned} & \mathcal{E}(\bar{v}_{k,m} \bar{v}_{k',m'}) \\ & = \mathcal{E} \left(\int a^{k/2} w(t) e^{-j2\pi f_c a^k t} p(a^k t - mT) dt \right. \\ & \quad \times \left. \int a^{k'/2} w^*(t') e^{j2\pi f_c a^{k'} t'} p(a^{k'} t' - m'T) dt' \right), \\ & = T \iint \mathcal{E}(w(t) w^*(t')) \frac{\sqrt{a^k a^{k'}}}{T} e^{-j2\pi f_c a^k t} \\ & \quad \times p(a^k t - mT) e^{j2\pi f_c a^{k'} t'} p(a^{k'} t' - m'T) dt dt'. \end{aligned}$$

With the assumption that $\mathcal{E}(w(t) w^*(t')) = \sigma^2 \delta(t - t')$, the above can be further simplified as

$$\begin{aligned} & \mathcal{E}(\bar{v}_{k,m} \bar{v}_{k',m'}) \\ & = \sigma^2 T \int \frac{\sqrt{a^k a^{k'}}}{T} e^{-j2\pi f_c a^k t} \\ & \quad \times p(a^k t - mT) e^{j2\pi f_c a^{k'} t} p(a^{k'} t - m'T) dt, \\ & \stackrel{a}{=} \sigma^2 T \delta_{k-k'} g_{k,m-m'} \\ & \stackrel{b}{=} \sigma^2 T \delta_{k-k'} \delta_{m-m'}, \end{aligned}$$

where $\stackrel{a}{=}$ holds as per Theorem 1, and $\stackrel{b}{=}$ holds due to (33).

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Tao Xu received the Bachelor's degree and the Master's degree in electronic science and technology from the National University of Defense Technology (NUDT), Changsha, China, in 2005 and 2007, respectively.

Since December 2008, he has been working towards the Ph.D. degree in the Circuits and Systems (CAS) Group at the Faculty of Electrical Engineering, Mathematics and Computer Science of the Delft University of Technology (TU Delft), The Netherlands. During his Bachelor's and Master's

studies, he used to work on the fields of hardware system designs, employing FPGAs and VLSI circuits. His research work in TU Delft focused on the area

of signal processing for communications (SP-COM) and also electronic system level (ESL) design, employing tools from linear algebra and statistical signal processing as well as high-level synthesis techniques.

Mr. Xu won a four-year scholarship from the China Scholarship Council (CSC) as the financial support for overseas studies as a Ph.D. student at NUDT in 2008.



Zijian Tang received the M.Sc. and Ph.D. degrees, both in electrical engineering, from the Delft University of Technology (TUD), Delft, The Netherlands, in 2003 and 2007, respectively.

He worked with Mathworks in 2008 and afterwards with the Netherlands Organization of Applied Scientific Research (TNO) from 2008 to 2012. Between 2011 and 2012, he also held a part-time position with TUD. In the beginning of 2012, he visited the University of Southern California. Since September 2012, he has been working the Shell

Global Solutions as a research geophysicist.



Geert Leus (M'01–SM'05–F'12) received the Electrical Engineering degree and the Ph.D. degree in applied sciences from the Katholieke Universiteit Leuven, Belgium, in 1996 and 2000, respectively.

Currently, he is an "Antoni van Leeuwenhoek" Full Professor at the Faculty of Electrical Engineering, Mathematics and Computer Science of the Delft University of Technology, Delft, The Netherlands. His research interests are in the area of signal processing for communications.

Dr. Leus received a 2002 IEEE Signal Processing Society Young Author Best Paper Award and a 2005 IEEE Signal Processing Society Best Paper Award. He was the Chair of the IEEE Signal Processing Society for Communications and Networking Technical Committee, and an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, and the IEEE SIGNAL PROCESSING LETTERS. Currently, he is a member of the IEEE Sensor Array and Multichannel Technical Committee and serves as the Editor in Chief of the *EURASIP Journal on Advances in Signal Processing*.

Urbashi Mitra (F'07) received the B.S. and the M.S. degrees from the University of California at Berkeley and the Ph.D. degree from Princeton University, Princeton, NJ.

After a six-year stint at the Ohio State University, she joined the Department of Electrical Engineering at the University of Southern California, Los Angeles, where she is currently a Professor. He has held visiting appointments at the Delft University of Technology, Delft, The Netherlands; Stanford University, Stanford, CA; Rice University, Houston, TX; and the Eurecom Institute. She served as Co-Director of the Communication Sciences Institute at the University of Southern California from 2004 to 2007. Her research interests are in wireless communications, communication and sensor networks, detection and estimation and the interface of communication, sensing and control.

Dr. Mitra has been/is an Associate Editor for the following IEEE publications: the IEEE TRANSACTION ON SIGNAL PROCESSING since 2012 to present, the IEEE TRANSACTIONS ON INFORMATION THEORY from 2007 to 2011, the IEEE JOURNAL OF OCEANIC ENGINEERING from 2006 to 2011, and the IEEE TRANSACTIONS ON COMMUNICATIONS from 1996 to 2001. She was a member of the IEEE Information Theory Society's Board of Governors from 2002 to 2007 and began a third term in 2012. She is a member of the IEEE Signal Processing Society's Technical Committee for Signal Processing in Communications and Networks from 2012 to 2014. She is the recipient of the 2012 GLOBECOM Signal Processing for Communications Symposium Best Paper Award, the 2012 NAE Lillian Gilbreth Lectureship, the USC Center for Excellence in Research Fellowship from 2010 to 2013, the Viterbi School of Engineering Dean's Faculty Service Award in 2009, the USC Mellon Mentoring Award in 2008, the Texas Instruments Visiting Professor (Fall 2002, Rice University), the 2001 Okawa Foundation Award, the 2000 Lumley Award for Research (OSU College of Engineering), the 1997 MacQuigg Award for Teaching (OSU College of Engineering), and the 1996 National Science Foundation (NSF) CAREER Award.