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PERFORMANCE OF SPATIAL FILTERING OF RF INTERFERENCE IN RADIO ASTRONOMY

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ABSTRACT

The contamination of radio astronomical measurements by man-made Radio Frequency Interference (RFI) is becoming an increasingly serious problem and therefore the application of interference mitigation techniques is essential. Most current techniques address impulsive or intermittent interference and are based on time-frequency detection and blanking. Continually present interferers cannot be cut out in the time-frequency plane and have to be removed using spatial filtering. This technique is based on the estimation of the spatial signature vector of the interferer from short-term spatial covariance matrices followed by a subspace projection to remove that dimension from the covariance matrix, and by further averaging. The projections will also modify the astronomical data, and hence a correction has to be applied to the long-term average to compensate for this. In this paper we analyse the performance of this spatial filtering algorithm.

1. INTRODUCTION

In interferometric radio astronomy the distribution of the intensity of radiation is measured by cross-correlating the signals from a number of radio telescopes. The astronomical signals usually have a signal to noise ratio (SNR) of -20dB or less. Integration over a 10-60s period of the correlated astronomical signal and the uncorrelated noise will improve the SNR several orders of magnitude. Due to the rotation of the earth the orientation of the telescopes with respect to the stars is changing. After several hours of measurement, enough samples have been obtained to construct an image of the observed field.

The signal from an interferer is spatially correlated and will therefore not average out completely. If the interferer is continuously present, it is not possible to filter out its contribution by detection and blanking of contaminated samples. Spatial filtering can null the energy received from the direction of the interferer. The projections will also modify the astronomical data, and hence a correction has to be applied to the long-term average to compensate for this. This algorithm was introduced in [1]. In this paper we summarize the algorithm and analyse its performance.

2. DATA MODEL

Assume we have a telescope array with p elements. For the interference free case the array output vector $\mathbf{x}_0(t)$ is modeled in complex baseband form as

$$\mathbf{x}_0(t) = \mathbf{v}(t) + \mathbf{n}(t)$$

where $\mathbf{x}_0(t) = [x_{0,1}(t), \ldots, x_{0,p}(t)]^{\mathrm{T}}$ is the $p \times 1$ vector of output signals at time t, $\mathbf{v}(t)$ is the received sky signal, assumed a stationary Gaussian vector with covariance matrix $\mathbf{R}_v$, and $\mathbf{n}(t)$ is the $p \times 1$ noise vector with independent identically distributed Gaussian entries and covariance matrix $\sigma^2\mathbf{I}$. If an interferer is present the array output vector is modeled as

$$\mathbf{x}(t) = \mathbf{x}_0(t) + \mathbf{a}(t)s(t)$$

where $s(t)$ is the interferer signal with spatial signature vector $\mathbf{a}(t)$ which is assumed stationary only over short time intervals. The astronomer is interested in $\mathbf{R}_v$. We assume that σ^2 is known from calibration and $\mathbf{R}_v \ll \sigma^2\mathbf{I}$.

3. ALGORITHM

Given observations $\mathbf{x}_n := \mathbf{x}(nT_s)$, where T_s is the sampling period, the objective is to estimate $\mathbf{R}_0 = \mathbf{R}_v + \sigma^2\mathbf{I}$. We first construct short-term covariance estimates $\hat{\mathbf{R}}_k$,

$$\hat{\mathbf{R}}_k = \frac{1}{M}\sum_{n=kM}^{(k+1)M} \mathbf{x}_n\mathbf{x}_n^{\mathrm{H}}$$

where M is the number of samples per short-term average. MT_s is in the order of 1-100 millisecond. Suppose that the spatial signature $\mathbf{a}_k$ of the interferer is known. We can then form a spatial filter $\mathbf{P}_k$,

$$\mathbf{P}_k := \mathbf{I} - \mathbf{a}_k(\mathbf{a}_k^{\mathrm{H}}\mathbf{a}_k)^{-1}\mathbf{a}_k^{\mathrm{H}}$$

which is such that $\mathbf{P}_k\mathbf{a}_k = 0$. Thus, when this spatial filter is applied to the data covariance matrix all the energy due to the interferer will be nulled:

$$\hat{\mathbf{Q}}_k := \mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k = \mathbf{P}_k\hat{\mathbf{R}}_{0,k}\mathbf{P}_k$$

where

$$\hat{\mathbf{R}}_{0,k} := \frac{1}{M}\sum_{n=kM}^{(k+1)M} \mathbf{x}_{0,n}\mathbf{x}_{0,n}^{\mathrm{H}}$$

When we average the modified covariance matrices $\hat{\mathbf{Q}}_k$, we obtain a long-term (say $T_{int} = NMT_s = 10$ seconds) estimate

$$\hat{\mathbf{Q}} := \frac{1}{N}\sum_{k=1}^{N}\hat{\mathbf{Q}}_k = \frac{1}{N}\sum_{k=1}^{N}\mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k\ .$$

$\hat{\mathbf{Q}}$ is an estimate of $\mathbf{R}_0$, but it is biased due to the projection. To correct for this we first write the two-sided multiplication as a single-sided multiplication employing the matrix identity $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^{\text{T}} \otimes \mathbf{A})\text{vec}(\mathbf{B})$, where $\text{vec}(\cdot)$ denotes the stacking of the columns of a matrix in a vector and $\otimes$ the Kronecker product. This gives

$$\text{vec}(\hat{\mathbf{Q}}) := \frac{1}{N}\sum_{k=1}^{N} \mathbf{C}_k \text{vec}(\hat{\mathbf{R}}_k) \quad (1)$$

where

$$\mathbf{C}_k := (\mathbf{P}_k^{\text{T}} \otimes \mathbf{P}_k)$$

Note that, if the interference is completely removed then

$$\text{E}\left[\text{vec}(\hat{\mathbf{Q}})\right] = \frac{1}{N}\sum_{k=1}^{N} \mathbf{C}_k \text{E}\left[\text{vec}(\hat{\mathbf{R}}_{k,0})\right] = \mathbf{C}\text{vec}(\mathbf{R}_0) \quad (2)$$

where

$$\mathbf{C} := \frac{1}{N}\sum_{k=1}^{N} \mathbf{C}_k$$

Now we can apply a correction $\mathbf{C}^{-1}$ to $\hat{\mathbf{Q}}$ to obtain estimate $\hat{\mathbf{R}}$

$$\hat{\mathbf{R}} := \text{unvec}(\,\mathbf{C}^{-1}\text{vec}(\hat{\mathbf{Q}}))$$

This is the estimate of $\mathbf{R}_0$ produced by the algorithm. If the $\mathbf{a}_k$ are known and completely projected out then $\hat{\mathbf{R}}$ is an unbiased estimate of $\mathbf{R}_0$:

$$\text{E}\left[\hat{\mathbf{R}}\right] = \mathbf{R}_0$$

More in general, when the spatial signatures of the interferer are unknown, they can be estimated by an eigenanalysis of the sample covariance matrices $\hat{\mathbf{R}}_k$. assuming that the noise is white and the astronomical contribution is small, it is well known that the number of interferers can be detected from the eigenvalues of $\hat{\mathbf{R}}_k$, and that the subspace spanned by the spatial signatures of the interferers can be estimated by the corresponding eigenvectors. This allows us to construct the projection matrix $\hat{\mathbf{P}}_k$ [2]. However, the interference is not completely removed and (2) does not hold.
A second issue is the invertibility of $\mathbf{C}$ and the noise enhancement of $\mathbf{C}^{-1}$.

4. PERFORMANCE ANALYSIS

The result of the algorithm is $\hat{\mathbf{R}}$, an estimate of the true covariance matrix $\mathbf{R}_0$. The quality of an estimator is determined by its covariance. In the following sections we will determine the covariance of $\hat{\mathbf{R}}$ in three cases: 1) interference free case 2) the spatial signatures $\mathbf{a}_k$ are known and 3) the spatial signatures $\mathbf{a}_k$ are estimated. We use the following notation. With $\hat{\mathbf{X}}$ we denote an estimate, with $\mathbf{X} = \text{E}\left[\hat{\mathbf{X}}\right]$ the expected value of $\hat{\mathbf{X}}$ and with $\mathbf{X}' = \hat{\mathbf{X}} - \mathbf{X}$ the estimation error. The covariance of an estimate is defined as:

$$\text{cov}\{\hat{\mathbf{X}}\} := \text{E}\left[\text{vec}(\mathbf{X}')\text{vec}(\mathbf{X}')^{\text{H}}\right]$$

The variance is defined as

$$\text{var}\{\hat{\mathbf{X}}\} := \text{E}\left[\mathbf{X}' \odot \overline{\mathbf{X}'}\right] = \text{unvec}(\text{diag}(\text{cov}\{\hat{\mathbf{X}}\}))$$

where $\odot$ denotes entrywise multiplication of two matrices of equal size.

4.1. Case I: The variance of $\hat{\mathbf{R}}$ for the interference free case

Let

$$\hat{\mathbf{R}}_0 = \frac{1}{N}\sum_{k=1}^{N} \hat{\mathbf{R}}_{0,k}$$

be the long-term average of interference free samples $\hat{\mathbf{R}}_{0,k}$, then the standard result for Gaussian sources applies:

$$\text{cov}\{\hat{\mathbf{R}}_0\} = \frac{1}{MN}\overline{\mathbf{R}}_0 \otimes \mathbf{R}_0$$

Because $\mathbf{R}_0 \approx \sigma^2\mathbf{I}$

$$\text{cov}\{\hat{\mathbf{R}}_0\} \approx \frac{\sigma^4}{MN}\mathbf{I} \quad (3)$$

This defines the best performance that we can have for $\hat{\mathbf{R}}$.

4.2. Case II: The variance of $\hat{\mathbf{R}}$ for interference with known spatial signatures

Suppose the spatial signatures $\mathbf{a}_k$ of the interferers are known. In that case the algorithm is unbiased by design. The covariance of the estimate is

$$\begin{aligned} \text{cov}\{\hat{\mathbf{R}}\} &:= \text{E}\left[\text{vec}(\hat{\mathbf{R}}')\text{vec}(\hat{\mathbf{R}}')^{\text{H}}\right] \\ &= \mathbf{C}^{-1}\,\text{cov}\{\hat{\mathbf{Q}}\}(\mathbf{C}^{-1})^{\text{H}} \quad (4) \end{aligned}$$

where, using (1)

$$\text{cov}\{\hat{\mathbf{Q}}\} = \text{E}\left[\frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N} \mathbf{C}_k \text{vec}(\mathbf{R}'_k)\text{vec}(\mathbf{R}'_l)^{\text{H}}\mathbf{C}_l^{\text{H}}\right] \quad (5)$$

The estimation errors $\mathbf{R}'_k$ and $\mathbf{R}'_l$ are uncorrelated for $k \neq l$. Multiplication with $\mathbf{C}_k$ projects out the contribution of the interferers, so that $\mathbf{C}\text{vec}(\mathbf{R}'_k) = \mathbf{C}\text{vec}(\mathbf{R}'_{k,0})$

$$\begin{aligned} \text{cov}\{\hat{\mathbf{Q}}\} &= \text{E}\left[\frac{1}{N^2}\sum_{k=1}^{N} \mathbf{C}_k \text{vec}(\mathbf{R}'_{0,k})\text{vec}(\mathbf{R}'_{0,k})^{\text{H}}\mathbf{C}_k^{\text{H}}\right] \quad (6) \\ &= \frac{1}{N^2}\sum_{k=1}^{N} \mathbf{C}_k\,\text{cov}\{\hat{\mathbf{R}}_{0,k}\}\mathbf{C}_k \quad (7) \end{aligned}$$

$\hat{\mathbf{R}}_{0,k}$ is the covariance matrix of a complex Gaussian signal vector, so

$$\text{cov}\{\hat{\mathbf{R}}_{0,k}\} = \frac{1}{M}\overline{\mathbf{R}}_0 \otimes \mathbf{R}_0$$

and

$$\begin{aligned} \text{cov}\{\hat{\mathbf{Q}}\} &= \frac{1}{MN^2}\sum_{k=1}^{N} \mathbf{C}_k(\overline{\mathbf{R}}_0 \otimes \mathbf{R}_0)\mathbf{C}_k^{\text{H}} \quad (8) \\ &= \frac{1}{MN^2}\sum_{k=1}^{N} \overline{\mathbf{P}_k\mathbf{R}_0\mathbf{P}}_k \otimes \mathbf{P}_k\mathbf{R}_0\mathbf{P}_k \quad (9) \end{aligned}$$

Because $\mathbf{R}_0 \approx \sigma^2\mathbf{I}$ and $\mathbf{P}_k$ is a projection, $\mathbf{P}_k^2 = \mathbf{P}_k$,

$$\text{cov}\{\hat{\mathbf{Q}}\} \approx \frac{\sigma^4}{MN^2}\sum_{k=1}^{N}(\mathbf{P}_k^{\text{T}} \otimes \mathbf{P}_k) = \frac{\sigma^4}{MN}\mathbf{C}$$

$$\text{cov}\{\hat{\mathbf{R}}\} \approx \frac{\sigma^4}{MN}\mathbf{C}^{-1}\mathbf{C}(\mathbf{C}^{-1})^{\text{H}} = \frac{\sigma^4}{MN}\mathbf{C}^{-1}$$

The value of $\mathbf{C}^{-1}$ depends on $\mathbf{a}_k$, the spatial signatures of the interferer. Compared to (3), this indicates that $\mathbf{C}^{-1}$ determines the relative performance of the spatial filtering algorithm.

4.3. Case III: The variance of $\hat{\mathbf{R}}$ for interference with deterministic spatial signatures

If the spatial signatures are unknown, they need to be estimated, and hence the projection matrices are estimates too. $\mathbf{P}_k$, $\mathbf{C}_k$ and $\mathbf{C}$ are substituted by their estimates $\hat{\mathbf{P}}_k$, $\hat{\mathbf{C}}_k$ and $\hat{\mathbf{C}}$. In that case equation (2) is not true because $\hat{\mathbf{C}}$ and $\hat{\mathbf{R}}_k$ are not independent. The algorithm is not unbiased anymore, but it can be shown that the bias of $\hat{\mathbf{R}}$ is $O(M^{-1})$. This bias can be neglected because the standard deviation is $O(M^{-1/2})$.
Recall that

$$\text{cov}\{\hat{\mathbf{R}}\} = \text{E}\left[\text{vec}(\mathbf{R}')\text{vec}(\mathbf{R}')^{\text{H}}\right] \tag{10}$$

The first order approximation of $\mathbf{R}'$ is

$$\text{vec}(\hat{\mathbf{R}}') = (\mathbf{C}^{-1})'\text{vec}(\mathbf{Q}) + \mathbf{C}^{-1}\text{vec}(\mathbf{Q}')$$

where, in first order approximation

$$\begin{aligned}
(\mathbf{C}^{-1})' &= -\mathbf{C}^{-1}\mathbf{C}'\mathbf{C}^{-1} \\
\mathbf{C}' &= \frac{1}{N}\sum_{k=1}^{N}\mathbf{C}'_k \\
\mathbf{C}'_k &= (\mathbf{P}'^{\text{T}}_k \otimes \mathbf{P}_k) + (\mathbf{P}^{\text{T}}_k \otimes \mathbf{P}'_k) \\
\text{vec}(\mathbf{Q}') &= \frac{1}{N}\sum_{k=1}^{N}\text{vec}(\mathbf{Q}'_k) \\
\text{vec}(\mathbf{Q}'_k) &= \mathbf{C}'_k\text{vec}(\mathbf{R}) + \mathbf{C}\text{vec}(\mathbf{R}'). \qquad (11)
\end{aligned}$$

Working this out gives

$$\text{vec}(\mathbf{R}') = \mathbf{C}^{-1}(-\mathbf{C}'\mathbf{C}^{-1}\text{vec}(\mathbf{Q}) + \text{vec}(\mathbf{Q}')). \tag{12}$$

Because $\mathbf{C}^{-1}\text{vec}(\mathbf{Q}) = \text{vec}(\mathbf{R}_0)$ we obtain

$$\text{vec}(\mathbf{R}') = \mathbf{C}^{-1}(-\mathbf{C}'\text{vec}(\mathbf{R}) + \text{vec}(\mathbf{Q}'))$$

Substituting equation (11) gives

$$\text{vec}(\mathbf{R}') = \mathbf{C}^{-1}\frac{1}{N}\sum_{k=1}^{N}\mathbf{C}_k\text{vec}(\mathbf{R}'_k) \tag{13}$$

and hence

$$\text{cov}\{\hat{\mathbf{R}}\} = \mathbf{C}^{-1}\,\text{cov}\{\hat{\mathbf{Q}}\}(\mathbf{C}^{-1})^{\text{H}} \tag{14}$$

where $\text{cov}\{\hat{\mathbf{Q}}\}$ is as given in (5). Equation (14) is equal to (4) so in first order approximation, replacing the true projections $\mathbf{P}_k$ by the estimated projections $\hat{\mathbf{P}}_k$ does not change the covariance. Also in this case it follows that

$$\text{cov}\{\hat{\mathbf{R}}\} \approx \frac{\sigma^4}{MN}\mathbf{C}^{-1}$$

5. THE EXPECTED VALUE OF $\mathbf{C}^{-1}$

$\mathbf{C}^{-1}$ determines the penalty due to spatial filtering. The main diagonal of $\mathbf{C}^{-1}$ contains the factors by which the variance is multiplied compared to the interference free case. To describe the penalty in a single number we introduce the "quality factor" κ

$$\kappa = \max(\text{diag}(\mathbf{C}^{-1})) \tag{15}$$

κ is the worst case amplification of the variance. The value of κ is a function of $\mathbf{a}_k$. We will determine the asymptotic value of κ for two cases: 1) $\mathbf{a}_k$ are normally distributed and 2) $\mathbf{a}_k$ are the spatial signatures of a stationary interferer.

5.1. Case I: The variance of $\hat{\mathbf{R}}$ for normally distributed spatial signatures

If we choose a temporally i.i.d. statistical model for $\mathbf{a}_k$ we can determine $\text{E}[\mathbf{C}_k]$. When $N \to \infty$ $\mathbf{C}$ will converge to $\text{E}[\mathbf{C}_k]$, and $\mathbf{C}^{-1}$ to $\text{E}[\mathbf{C}_k]^{-1}$. Let $\mathbf{a}_k \sim \mathcal{CN}(0, \mathbf{I})$ and i.i.d. for different k, let

$$\mathbf{u} = \frac{\mathbf{a}_k}{|\mathbf{a}_k|}$$

then $\mathbf{u}$ is uniformly distributed over the unit-sphere in $\mathbb{C}^p$ and

$$\mathbf{P} = \mathbf{I} - \mathbf{u}\mathbf{u}^{\text{H}}$$

It follows that

$$\begin{aligned}
\text{E}[\mathbf{C}] &= \text{E}\left[\mathbf{P}^{\text{T}} \otimes \mathbf{P}\right] \\
&= \text{E}\left[(\mathbf{I} - \mathbf{u}\mathbf{u}^{\text{H}})^{\text{T}} \otimes (\mathbf{I} - \mathbf{u}\mathbf{u}^{\text{H}})\right] \\
&= \text{E}\left[\mathbf{I} \otimes \mathbf{I} - \mathbf{I} \otimes \mathbf{u}\mathbf{u}^{\text{H}} - \overline{\mathbf{u}\mathbf{u}^{\text{H}}} \otimes \mathbf{I} + \overline{\mathbf{u}\mathbf{u}^{\text{H}}} \otimes \mathbf{u}\mathbf{u}^{\text{H}}\right]
\end{aligned}$$

To evaluate this expression we need to know the second and fourth order moments. These can be found by integrating the probability density function (pdf) over the whole unit-sphere. The pdf on the unit-sphere is:

$$f(\mathbf{u}) = \frac{1}{A}$$

where A is a normalization constant, equal to the surface area of S_p, the unit sphere in $\mathbb{C}^p$, given by

$$A = \int_{S_p} ds = \frac{2\pi^p}{(p-1)!}.$$

Due to the symmetry of the pdf, $\text{E}[u_i\overline{u}_j]$ is zero except when $i = j$, and

$$\text{E}[u_i\overline{u}_i] = \frac{1}{A}\int_{S_p} u_i\overline{u}_i ds = \frac{1}{p}$$

$\text{E}[u_i\overline{u}_j u_k\overline{u}_l]$ is zero, except for:

$$\text{E}[u_i\overline{u_i}u_j\overline{u_j}] = \frac{1}{A}\int_{S_p} u_i\overline{u}_i u_j\overline{u}_j ds = \begin{cases} \frac{2}{p(p+1)} & (i = j) \\ \frac{1}{p(p+1)} & (i \neq j) \end{cases}$$

Hence,

$$\text{E}[\mathbf{C}] = \mathbf{I} - \frac{2}{p}\mathbf{I} + \frac{1}{p(p+1)}(\mathbf{I} + \mathbf{\Delta})$$

where $\mathbf{\Delta} = \text{vec}(\mathbf{I})\text{vec}(\mathbf{I})^{\text{T}}$. The inverse is given by:

$$\text{E}[\mathbf{C}]^{-1} = \frac{p(p+1)}{p^2 - p - 1}(\mathbf{I} - \frac{1}{p^2 - 1}\mathbf{\Delta})$$

So, for large N

$$\text{var}\{\hat{\mathbf{R}}\} = \frac{\sigma^4}{MN}\frac{p(p+1)}{p^2 - p - 1}(\mathbf{1}\mathbf{1}^{\text{T}} - \frac{1}{p^2 - 1}\mathbf{I})$$

and

$$\kappa = \frac{p(p+1)}{p^2 - p - 1}$$

If for example $p = 8$, then $\kappa = 72/55 \approx 1.3$, so the variance of the entries of $\hat{\mathbf{R}}$ increases with 30% in the worst case.

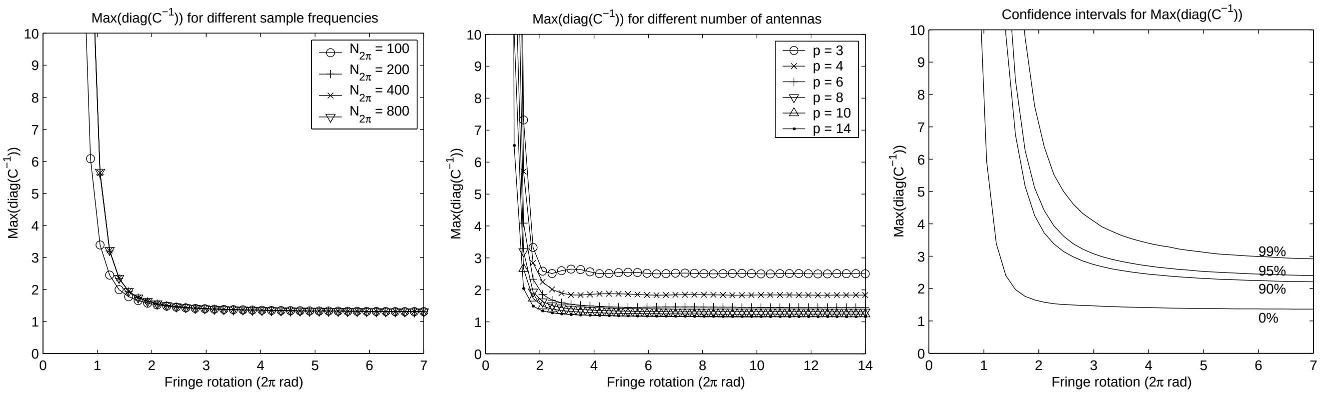

Figure 1: a) Quality factor κ for different $N_{2\pi}$, and b) for different numbers of antennas. c) Confidence intervals for κ based on a simulation with 10^4 randomly selected $\mathbf{a}_0$

5.2. Case II: The variance of $\hat{\mathbf{R}}$ for stationary interferers

For matrix $\mathbf{C}$ to be invertible the spatial signatures $\mathbf{a}_k$ need to be sufficiently variable. For stationary interferers (no own movement, no multi-path) the only source of variability is the geometric delay compensation. The geometric delay compensation is a delay placed between each telescope and the correlator to correct for the different path lengths of the astronomical signal to each of the telescopes. The geometric delay depends on the position of the observed field in the sky. Due to the daily rotation of the earth the stars are moving along the sky and hence the geometric delay is time varying. For a signal received from an interferer fixed on earth, the narrow band approximation of the geometric delay compensation is a time varying phase-shift, named fringe correction. For a linear array of telescopes the effect of the fringe correction on the spatial signature can be modeled as

$$\mathbf{a}(t) = \begin{bmatrix} a_1 \\ a_2 e^{j\varphi} \\ a_3 e^{2j\varphi} \\ \vdots \\ a_p e^{(p-1)j\varphi} \end{bmatrix}, \mathbf{a}_0 = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_p \end{bmatrix}, \quad \varphi = \frac{2\pi f_F t}{p-1}$$

where f_F is the fringe frequency, given by

$$f_F = \frac{2\pi}{24 \times 3600} D_\lambda \cos\delta \cos h$$

where D_λ is the longest baseline length in wavelengths, δ is the declination of the source and h is the hour angle of the source, which is time varying and has a period of 24 hours. For a stationary interferer $\mathbf{C}^{-1}$ depends on:

- The fringe rotation per short term sample $MT_s f_F$, where T_s is is the sampling time, M the number of samples per short-term average and f_F the fringe frequency.
- N, the number of short-term averages per long-term average.
- p, the number of antennas
- $\mathbf{a}_0$, the spatial signature without fringe correction

The total fringe correction over the long term integration period, is probably the parameter that has the most influence. The first two parameters can be converted to the total fringe rotation during the long term integration period,

$$\varphi_{tot} = MT_s 2\pi f_F N = T_{int} 2\pi f_F$$

and the number of samples per fringe cycle,

$$N_{2\pi} = N/(T_{int} f_F)$$

The lowest possible $N_{2\pi}$ is reached when f_F reaches its maximum value. If we choose $MT_s = 10ms$, $D_\lambda = 3000m/30cm$ then the mimimum value for $N_{2\pi}$ is 135. The results of simulations (figure 1a) show that within the range of possible values for $N_{2\pi}$ the effect on the quality factor κ is neglectable. Simulations with different numbers of antennas (figure 1b) show that although the asymptotic value of κ depends on p, the global character of the function is independent of p. All curves show a transition from poor to reasonably good performance in the range from 1 to 2 fringe cycles. Further simulations are carried out with the parameters $N_{2\pi} = 200$ and $p = 8$. The curves in figure 1c show how the performance increases with increasing fringe rotation For x% of randomly selected $\mathbf{a}_0$ the quality factor κ is below the x%-curve. From this graph a minimum φ for acceptable performance can be read. This condition on φ can be translated a division of the the sky in an observable and an unobservable area. The unobservable area is band from the East over the celestial pole to the West. The width of this band is given by:

$$\alpha = 2 \arcsin \frac{\varphi_{min} \times 24 \times 3600}{D_\lambda \times T_{int} \times 2\pi}$$

For example if $\varphi_{min} = 3$, $T_{int} = 30s$, $\lambda = 30cm$ and $D = 3000m$ then the width of this band is 16°.

6. REFERENCES

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