Time Delay Estimation in Dense Multipath with Matched Subspace Filters

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***Abstract*— Efficient multipath time delay estimation is of great importance for positioning with Ultra-Wide-Band signals in indoor environments. In dense multipath environments a simple assumption is to model the multipath terms as attenuated and delayed copies of a known waveform. A more realistic model is however the scenario where the pulse shape is different for every multipath term due to scattering effects and the directionality of the antennas. For the purposes of positioning we face three problems. Signal detection, time delay estimation of the strongest path, and leading edge detection. Leading edge detection is necessary since the strongest path may not be the first.**

We apply Maximum Likelihood Estimation and Generalized Likelihood Ratio Tests to these problems. In the case of a known pulse shape this leads to the well known matched filter. In the case of unknown pulse shape we show that the matched subspace filter is the optimal solution. Another significant property of the matched subspace filter is that it does not require Nyquist rate sampling. Beyond these advantages, the matched subspace filter is not computationally demanding.

Finally we discuss various leading edge detection methods like Generalized Likelihood Rule (GLR) and energy detectors.

I. Introduction

Positioning is usually based on two steps. In the first step parameters like time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA), and signal-strength (SS) are estimated from the received signal. Later these parameters are used to obtain the estimate of the location. In the first step we usually take the problem as a joint detection and estimation problem in the sense that first the presence of a signal has to be detected and later the parameters must be estimated. These two steps are usually coupled. In this paper we focus on time delay estimation and detection in dense multipath.

Ultra-Wide-Band (UWB) technology has become a strong candidate for positioning in indoor environments. The reason for this is mainly the fact that UWB signals can easily penetrate through materials (e.g., walls) and have very high time resolution to combat multipath. A common assumption for the propagation of UWB signals is to model the multipath environment as simple delayed and attenuated replicas of a known waveform. To this problem then many high resolution time delay estimation algorithms can be applied. For localizing a radiating source we are interested only in the first arriving path which may not be the strongest. Under this assumption the matched filter approach has some very attractive features. First it is the Maximum Likelihood (ML) estimate of time delay in the case of a single reflected pulse. Furthermore it has a simple detection interpretation in the case where only the amplitude of the pulse is not known. For the case of multipath the matched filter approach is still efficient if the pulses are well separated. Iterative modifications of the matched filter approach were developed in [1] to separate two close pulses. Because of these properties the matched filter is a strong candidate for detection and estimation purposes. It has also been successfully applied to practical UWB data [2], [3].

However there are some issues that need to be evaluated more carefully. The first is that for UWB the exact pulse shape may not be accurately known and due to frequency dependent scattering effects it is realistic to assume that every received pulse is moderately different. The second factor is the fact that antennas are directional hence pulses arriving from different directions will be distorted differently. Then the problem becomes more difficult since we do not know the time delays and we do not know the pulses. However we must mention again that for positioning we only need to find the delay of the peak and to decide whether there is an early arriving pulse which is not the strongest. In this paper, we develop algorithms to handle that situation where we pose some *regularity conditions* similar to conditions that are necessary for the matched filter approach. In other words we treat the multipath delay profile uniquely and try to find the location of the strongest peak and the first arriving peak. A simple approach to that problem would be to use energy detectors or thresholding however these methods are not suitable for signals embedded in noise.

In this paper we base our development of the problem as a detection-estimation-detection procedure. In other words, we need first to detect the presence of the signal. Later we have to estimate the time delay. In the case of Generalized Likelihood Ratio Tests (GLRT), the estimated time delay is already used in the detection procedure. Hence the first two steps are coupled. And finally we have to search backwards in order to decide whether there is path which is not the strongest since the Maximum Likelihood (ML) estimation of time delay finds the strongest path. This final step can be called a leading edge detection procedure. All these problems can be solved efficiently with the theory of Maximum Likelihood (ML) and Generalized Likelihood Ratio Tests (GLRT). We treat the problem both in the case of known pulse shape and the case of unknown pulse shape from an ML and GLRT perspective.

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It turns out that the proposed method for the case of unknown pulse shape is a simple generalization of the matched filter based approach to higher dimensions since matched filtering is a rank-1 estimation and detection procedure. The matched subspace filter does not require Nyquist rate sampling in contrast to the matched filter. In summary, the matched subspace filter is more flexible and is not computationally costly.

Due to its importance, multipath time delay estimation [1], [4]–[8] and its performance analysis [9]–[11] received much interest. In the UWB literature the common approach to estimate time delays is matched filtering [3], [12]. Iterative extensions of the matched filtering method for time delay estimation in the classical model and in the context of UWB can be found in [2].

The organization of this paper is as follows. In the following section we provide the matched filter solution to the detection-estimation-detection problem we previously described based on ML and GLRTs. Later we approach the same detection-estimation-detection problem for the case where pulse shape is not known. Finally we consider two leading edge detection methods. The first is based on multiple hypothesis testing and the Bayesian Generalized Maximum Likelihood rule. Later we provide the energy detector and relate it to the previous cases. Finally we provide simulations comparing the estimation performance of the matched filter and the matched subspace filter.

II. Notation Summary

We will use the following notations throughout the paper.

$$\mathbf{A} : \text{Bold letters denote matrices and vectors} \quad (1)$$

$$\|.\| : \text{2-Norm of-vectors} \quad (2)$$

$$|.| : \text{Absolute value} \quad (3)$$

$$(.)^T : \text{Matrix transpose} \quad (4)$$

$$\mathbf{I} : \text{Identity Matrix} \quad (5)$$

$$\mathbf{P_H} = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T \quad (6)$$

$$\mathbf{P_H^\perp} = \mathbf{I} - \mathbf{P_H} \quad (7)$$

$$\mathbf{I}_p : p \times p \text{ Identity matrix} \quad (8)$$

$$\mathcal{N}(0, \sigma^2\mathbf{I}) : \text{Distribution os multivariate white Gaussian noise} \quad (9)$$

III. Matched Filter Approach

In this section our aim is to review the well known matched filtering based results for time delay estimation and first path detection. While our formulations are not based on any source the theory of the matched filter is well understood in the literature. Consider the following multipath propagation model.

$$r(t) = \sum_{\ell=1}^{L} \alpha_\ell s(t - \tau_\ell) \quad (10)$$

α_ℓ denote the real attenuation coefficients and τ_ℓ denotes the associated time delay. In this notation $s(t)$ is a coded waveform given as

$$s(t) = \sum_{m=1}^{M} c_m p(t - K_m) \quad (11)$$

where $c_m = \{1, -1\}$. Without loss of generality $K_1 = 0$. We assume that the coding sequence c_m and the time delays K_m are perfectly known and there is no inter symbol interference.

The sampled version of the received signal which is corrupted by zero mean white Gaussian noise can be expressed in matrix form is given as

$$\mathbf{y} = \sum_{\ell=1}^{L} \mathbf{h}_{\tau_\ell}\alpha_\ell + \mathbf{w} \quad (12)$$

where $\mathbf{w}$ is $\mathcal{N}(0, \sigma^2\mathbf{I})$. The vector $\mathbf{h}_{\tau_\ell}$ is nonzero only for the samples starting from the instant τ_ℓ and ranging in the duration of the samples of the waveform $s(t)$. We have normalized the sampling rates to unity to simplify development. Estimation of multipath parameters for closely spaced pulses is a challenging problem. It is a nonlinear optimization problem and has been widely addressed in the literature. However when the pulses are separated in time the vectors $\mathbf{h}_{\tau_\ell}$ become orthogonal hence a simplified algorithm can be used which is developed in the next section.

A. Time delay estimation

We will first consider the problem of time delay estimation for a single path case where the data model is given as follows:

$$\mathbf{y} = \mathbf{h}_\tau\alpha + \mathbf{w} \quad (13)$$

where α is a real attenuation coefficient, $\mathbf{w}$ is $\mathcal{N}(0, \sigma^2\mathbf{I})$, and $\mathbf{y}$ is an $N \times 1$ vector.

The ML estimate of unknown parameters is given as:

$$[\hat{\alpha}, \hat{\tau}, \hat{\sigma}^2] = \arg\max_{\alpha,\tau,\sigma^2} \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\Big(- \frac{1}{2\sigma^2}\|\mathbf{y} - \mathbf{h}_\tau\alpha\|^2\Big) \quad (14)$$

Minimizing with respect to σ^2 yields the following

$$\hat{\sigma}^2 = \frac{1}{N}\|\mathbf{y} - \mathbf{h}_\tau\alpha\|^2 \quad (15)$$

Putting this expression back in to the previous ML formulation leads to

$$[\hat{\alpha}, \hat{\tau}] = \arg\min_{\alpha,\tau} \|\mathbf{y} - \mathbf{h}_\tau\alpha\|^2 \quad (16)$$

This is a separable optimization problem and can be solved as

$$\hat{\alpha} = (\mathbf{h}_\tau^T\mathbf{h}_\tau)^{-1}\mathbf{h}_\tau^T\mathbf{y} \quad (17)$$

Putting this back we obtain

$$[\hat{\tau}] = \arg\min_{\tau} \|\mathbf{y} - \mathbf{P}_{\mathbf{h}_\tau}\mathbf{y}\|^2 \quad (18)$$

In summary the ML estimate of time delay is given as:

$$[\hat{\tau}] = \arg\max_{\tau} \mathbf{y}^T\mathbf{P}_{\mathbf{h}_\tau}\mathbf{y} \quad (19)$$

which can further be simplified due to the fact that the matrix $\mathbf{h}_\tau$ is rank-1. And $(\mathbf{h}_\tau^T\mathbf{h}_\tau)^{-1}$ is a constant independent of the value of τ. From here we obtain the simplified matched filter expression as

$$[\hat{\tau}] = \arg\max_{\tau} \|\mathbf{y}^T\mathbf{h}_\tau\|^2 \quad (20)$$

$$= \arg\max_{\tau} |\mathbf{y}^T\mathbf{h}_\tau|^2 \quad (21)$$

We must note that we would obtain the same expression even if the noise variance was known. Hence the matched filter is the ML estimation of time delays regardless of the knowledge of the noise variance or signal attenuation coefficient.

There is another intuitive way of viewing that result. First note that we are taking an inner product of the observations $\mathbf{y}$ with a vector $\mathbf{h}_\tau$ which is non zero only in a specified manner. Let the matrix $\mathbf{A}_\tau$ denote the resulting selection matrix that takes only the values of the vector in accordance with the vector $\mathbf{h}_\tau$ which is shaped due to the signal coding $s(t) = \sum_{m=1}^{M} c_m p(t - K_m)$. Then we can form the following expression

$$\mathbf{x}_\tau = \mathbf{A}_\tau\mathbf{y} \quad (22)$$

where $\mathbf{x}_\tau$ is now a $D \times 1$ vector and $\mathbf{A}_\tau$ is a $D \times N$ matrix consisting only of 0 and 1. Then the matched filter expression can be written as

$$[\hat{\tau}] = \arg\max_{\tau} \mathbf{y}^T\mathbf{P}_{\mathbf{h}_\tau}\mathbf{y} \quad (23)$$

$$= \arg\max_{\tau} \mathbf{x}_\tau^T\mathbf{P}_{\mathbf{h}}\mathbf{x}_\tau \quad (24)$$

$$= \arg\max_{\tau} \mathbf{y}^T\mathbf{A}_\tau^T\mathbf{P}_{\mathbf{h}}\mathbf{A}_\tau\mathbf{y} \quad (25)$$

where

$$\mathbf{h} = \begin{bmatrix} c_1\mathbf{p} \\ c_2\mathbf{p} \\ \cdot \\ \cdot \\ \cdot \\ c_M\mathbf{p} \end{bmatrix}, \quad (26)$$

and $\mathbf{p}$ denotes the sampled version of the pulse shape $p(t)$. In that case $\mathbf{h}$ becomes a $D \times 1$ vector. We must note that $\mathbf{h}_\tau^T\mathbf{h}_\tau = \mathbf{h}^T\mathbf{h}$, $\mathbf{x}_\tau^T\mathbf{h} = \mathbf{y}^T\mathbf{h}_\tau$, and $\mathbf{h} = \mathbf{A}_\tau\mathbf{h}_\tau$. The point is that we are simply computing the energy of a projection to a rank-1 subspace and this can be represented in two ways since the rank-1 subspace is also described in a highly redundant manner.

This derivation is valid only for a single path case. However when two or more pulses are well separated in time then matched filter is still optimal since the vectors $\mathbf{h}_{\tau_\ell}$ become orthogonal. The only difficult arises when we attempt to estimate time delays for two closely spaced multipath terms. Then we may use alternating projections to iteratively estimate the the time delays. Nevertheless these steps are not necessary for localization purposes since we are interested only in the first path which may not be the strongest. Finally we must note that matched filter estimate is simply an energy computation operation prejected on a rank-1 subspace. The subspace is formed by the pulse shape and coding.

Now it is time to provide the derivation for the case we assume that there are several multipaths. We must note that th matched filter ramins the same for time delay estimation but estimation of noise variance has to be done more carefully. The ML estimation of unknown parameters can be written as follows

$$[\hat{\alpha}, \hat{\tau}, \hat{\sigma}^2] = \arg\max_{\alpha,\tau,\sigma^2} \frac{1}{(2\pi\sigma^2)^{D/2}} exp\Big(-\frac{1}{2\sigma^2}\|\mathbf{x}_\tau - \mathbf{h}\alpha\|^2\Big) \quad (27)$$

From here it follows easily that the estimate of noise variance is

$$\hat{\sigma}^2 = \frac{1}{D}\|\mathbf{x}_\tau - \mathbf{h}\alpha\|^2 \quad (28)$$

Putting this expression back in to the previous maximum likelyhood formulation leads to

$$[\hat{\alpha}, \hat{\tau}] = \arg\min_{\alpha,\tau} \|\mathbf{x}_\tau - \mathbf{h}\alpha\|^2 \quad (29)$$

This is a separable optimization problem and can be solved

$$\hat{\alpha} = (\mathbf{h}^T\mathbf{h})^{-1}\mathbf{h}^T\mathbf{x}_\tau \quad (30)$$

Putting this back we obtain

$$[\hat{\tau}] = \arg\min_{\tau} \|\mathbf{x}_\tau - \mathbf{P}_{\mathbf{h}}\mathbf{x}_\tau\|^2 \quad (31)$$

In summary this leads to the following

$$[\hat{\tau}] = \arg\max_{\tau} \mathbf{x}_\tau^T\mathbf{P}_{\mathbf{h}}\mathbf{x}_\tau \quad (32)$$

$$= \arg\max_{\tau} |\mathbf{x}_\tau^T\mathbf{h}|^2 \quad (33)$$

Now we again obtained the matched filter for the estimation of multipaths but the estimation of noise variance is based only on the time samples that the signal exists. This does not have any implications on time delay estimation but is significant for detection which will be the subject of the next section.

We must also note that the matched filter expression also follows from the following argument

$$[\hat{\tau}] = \arg\max_{\tau} \hat{\alpha}^2 \quad (34)$$

$$= \arg\max_{\tau} \mathbf{x}_\tau^T\mathbf{h}(\mathbf{h}^T\mathbf{h})^{-1}(\mathbf{h}^T\mathbf{h})^{-1}\mathbf{h}^T\mathbf{x}_\tau \quad (35)$$

$$= \arg\max_{\tau} \mathbf{x}_\tau^T\mathbf{P}_{\mathbf{h}}\mathbf{x}_\tau \quad (36)$$

This simply implies that in multipath case, matched filter finds the location of the strongest path.

B. Signal Detection

In this section we consider the problem of signal detection. As we mentioned earlier the exact ML computation of all the multipath delays is computationally prohibitive hence we must not formulate a GLRT on this basis. For this reason we will adopt an alternative approach. We will use GLRTs only based on the strongest multipath term.

The hypothesis testing problem for the single path case can be formulated as follows:

$$\mathcal{H}_0 : \alpha = 0 \tag{37}$$
$$\mathcal{H}_1 : \alpha \neq 0 \tag{38}$$

The GLRT is formulated as follows

$$L_1(\mathbf{y}) = \frac{p(\mathbf{y}, \hat{\alpha}, \hat{\tau}, \hat{\sigma}_1^2, \mathcal{H}_1)}{p(\mathbf{y}, \hat{\sigma}_0^2, \mathcal{H}_0)} > \eta \tag{39}$$

The estimated parameters in the GLRT are given as follows

$$[\hat{\tau}] = \arg\max_{\tau} \mathbf{y}^T \mathbf{P}_{\mathbf{h}_{\hat{\tau}}} \mathbf{y} \tag{40}$$
$$\hat{\alpha} = (\mathbf{h}_{\hat{\tau}}^T \mathbf{h}_{\hat{\tau}})^{-1} \mathbf{h}_{\hat{\tau}}^T \mathbf{y} \tag{41}$$
$$\hat{\sigma}_1^2 = \frac{1}{N} \|\mathbf{y} - \mathbf{h}_{\hat{\tau}} \hat{\alpha}\|^2 \tag{42}$$
$$\hat{\sigma}_0^2 = \frac{1}{N} \|\mathbf{y}\|^2 \tag{43}$$

We must note that the above formulation GLRT is simply a matched subspace detection problem where the signal subspace is replaced with an estimated version $\mathbf{h}_{\hat{\tau}}$. In that case it is known that the test statistics can be reduced to the following [13], [14]

$$L_1'(\mathbf{y}) = \frac{N-p}{p} \frac{\mathbf{y}^T \mathbf{P}_{\mathbf{h}_{\hat{\tau}}} \mathbf{y}}{\mathbf{y}^T \mathbf{P}_{\mathbf{h}_{\hat{\tau}}}^{\perp} \mathbf{y}} > \eta \tag{44}$$

where $p = 1$. Since this is simply $F_{1,N-1}$ distributed under the hypothesis $\mathcal{H}_0$ we can set an exact Constant False Alarm Rate (CFAR) rate. However under the alternative hypothesis we do not have anymore an exact description of the distribution.

When noise variance is known we obtain the following test for the single path case

$$L_2(\mathbf{y}) = \frac{p(\mathbf{y}, \hat{\alpha}, \hat{\tau}, \mathcal{H}_1)}{p(\mathbf{y}, \mathcal{H}_0)} > \eta \tag{45}$$

The test statistics $L_2(\mathbf{y})$ can be reduced to the following

$$L_2'(\mathbf{y}) = \frac{\mathbf{y}^T \mathbf{P}_{\mathbf{h}_{\hat{\tau}}} \mathbf{y}}{\sigma^2} > \eta \tag{46}$$

Under the hypothesis $\mathcal{H}_0$, $L_2'(\mathbf{y})$ is χ_1^2 distributed.

Until now we assumed that there is only single path. This does not cause problems when the noise variance is known but when noise variance is not known we need to estimate the noise variance more carefully. Consider the following hypothesis testing problem

$$\mathcal{H}_0 : \mathbf{x}_{\hat{\tau}} = \mathbf{w} \tag{47}$$
$$\mathcal{H}_1 : \mathbf{x}_{\hat{\tau}} = \mathbf{h}\alpha + \mathbf{w} \tag{48}$$

In this formulation noise variance is estimated only from the samples where the strongest path exists.

$$T_F(\mathbf{x}_{\hat{\tau}}) = \frac{D-p}{p} \frac{\mathbf{x}_{\hat{\tau}}^T \mathbf{P}_{\mathbf{h}} \mathbf{x}_{\hat{\tau}}}{\mathbf{x}_{\hat{\tau}}^T \mathbf{P}_{\mathbf{h}}^{\perp} \mathbf{x}_{\hat{\tau}}} > \eta \tag{49}$$

where $p = 1$. Under the hypothesis $\mathcal{H}_0$ $T_F(\mathbf{x}_{\hat{\tau}})$ is $F_{1,D-1}$ distributed.

For the case where noise variance is known we obtain the following test

$$T_{\chi^2}(\mathbf{x}_{\hat{\tau}}) = \frac{\mathbf{x}_{\hat{\tau}}^T \mathbf{P}_{\mathbf{h}} \mathbf{x}_{\hat{\tau}}}{\sigma^2} > \eta \tag{50}$$

Under the hypothesis $\mathcal{H}_0$ $T_{\chi^2}(\mathbf{x}_{\hat{\tau}})$ is χ_1^2 distributed.

C. Leading Edge Detection

In this section we will consider the problem of leading edge detection. Until now we have located the strongest peak and detected the signal. Now we need to look back and search for signal for which the strongest path is not the first. As shown earlier matched filtering locates the strongest peak not the first. We have the following hypothesis testing problem ($\tau < \hat{\tau}$)

$$\mathcal{H}_0 : \mathbf{x}_{\tau} = \mathbf{w} \tag{51}$$
$$\mathcal{H}_1 : \mathbf{x}_{\tau} = \mathbf{h}\alpha + \mathbf{w} \tag{52}$$

Now depending on whether we know the noise variance or not we face with the following decision statistics

$$T_{\chi^2}(\mathbf{x}_{\tau}) = \frac{\mathbf{x}_{\tau}^T \mathbf{P}_{\mathbf{h}} \mathbf{x}_{\tau}}{\sigma^2} > \eta \tag{53}$$

where $T_{\chi^2}(\mathbf{x}_{\tau})$ is χ_1^2 distributed under the hypothesis $\mathcal{H}_0$ and $\chi_1^2(\lambda)$ distributed under the hypothesis $\mathcal{H}_1$. The desired constant false alarm rate can be obtained easily.

And if noise variance is not known then the GLRT reduces to the following

$$T_F(\mathbf{x}_{\tau}) = \frac{D-p}{p} \frac{\mathbf{x}_{\tau}^T \mathbf{P}_{\mathbf{h}} \mathbf{x}_{\tau}}{\mathbf{x}_{\tau}^T \mathbf{P}_{\mathbf{h}}^{\perp} \mathbf{x}_{\tau}} > \eta \tag{54}$$

where $p = 1$. $T_F(\mathbf{x}_{\tau})$ is $F_{1,N-1}$ distributed under the hypothesis $\mathcal{H}_0$ and $F_{1,N-1}(\lambda)$ distributed under the hypothesis $\mathcal{H}_1$.

IV. Matched Subspace Filters

Let $c_m s(t)$ denote the transmitted waveform where $c_m \in \{-1, 1\}$. We assume that the propagation environment is linear and frequency dependent hence the pulse gets distorted for every multipath differently. This can also be caused by the directionality of the receiver. In that case the received signal when only one pulse is transmitted can be described as follows

$$r_m(t) = c_m \sum_{\ell=1}^{L} \alpha_{\ell} s_{\ell}(t - \tau_{\ell}) \tag{55}$$

In this formulation multipath terms are different for every time delay. α_{ℓ} is the real attenuation coefficient associated with multipath delay waveform labeled by ℓ and $s_{\ell}(t - \tau_{\ell})$ is the associated waveform. We assume that the waveform $s_{\ell}(t - \tau_{\ell})$ is different for every multipath term. τ_{ℓ} is the time delay for the multipath term labeled by ℓ. Note that in general the total number of multipath terms L is not known.

While we assume that the pulse shapes for every multipath term are unknown we still impose some *regularity conditions* since they are distorted versions of UWB-IR pulses. We assume also that the multipath delay profile admits some *regularity conditions* in the sense that it is non-zero for a small time duration (p samples length) where it achieves its peak. It would be good to list the assumptions we make as follows:

Regularity Assumptions

- The tranmitted pulse has a subnanosecond time duration
- Multipath terms do not distort the location of the peak by destructive addition
- We assume that the received pulse has undergone moderate distortion
- The received signal $r(t)$ has p non-zero samples where it attains its maximum and the leading edge
- When $s(t)$ is transmitted the received waveform through multipah ℓ is $s_\ell(t-\tau_\ell)$ and when $-s(t)$ is transmitted the received waveform through multipath ℓ is $-s_\ell(t-\tau_\ell)$. In other words we have a linear frequncy selective environment.

We must note that most of these conditions are also necessary in order to apply the matched filter to the problem.

In addition to the above model we also assume that the pulse is sent M times with a known time structure

$$y(t) = \sum_{m=1}^{M} r_m(t - K_m). \tag{56}$$

$K_m - K_{m-1}$ is chosen much larger than the duration of the multipath delay profile. In other words there is no inter symbol interference. We assume that $K_1 = 0$ without loss of generality.

$$y(nT_s) = \sum_{m=1}^{M} r(nT_s - K_m) \tag{57}$$

Let $\mathbf{y} = [y(T_s)\ y(2T_s)\ ...\ y(NT_s)]^T$ where the observed sampled signal consists of N samples. In matrix form the sampled version of the received signal under white Gaussian noise can be written as

$$\mathbf{y} = \mathbf{H}_\tau \boldsymbol{\theta} + \mathbf{w} \tag{58}$$

We can construct a vector $\mathbf{x}_\tau$ of length pM from the vector $\mathbf{y}$ as follows.

$$\mathbf{x}_\tau = [\mathbf{y}_1^{(\tau)}\ \mathbf{y}_2^{(\tau)}\ ...\ \mathbf{y}_M^{(\tau)}]^T \tag{59}$$

where $\mathbf{y}_n^{(\tau)}$ denotes a $1 \times p$ vector obtained from the p successive samples of the received signal $y(t)$ after the time instant $\tau + K_n$. In matrix form this can be written as follows

$$\mathbf{x}_\tau = \mathbf{A}_\tau \mathbf{y} \tag{60}$$

where $\mathbf{A}_\tau$ is a selection matrix consisting of only 0 and 1.

Then the signal model for the true time delay case can be given as

$$\mathbf{x}_\tau = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{61}$$

where the vector $\boldsymbol{\theta} = [\theta_1\ \theta_2\ ...\ \theta_p]^T$ denotes the associated samples of the multipath delay profile stacked from left to right. And the noise is assumed to be white Gaussian.

$$\mathbf{H} = \begin{bmatrix} c_1\mathbf{I}_p \\ c_2\mathbf{I}_p \\ \cdot \\ \cdot \\ \cdot \\ c_M\mathbf{I}_p \end{bmatrix}, \tag{62}$$

and $\mathbf{I}_p$ is a $p \times p$ identity matrix. In general the matrix $\mathbf{H}$ is of size $Mp \times p$. In this formulation the matrices $\mathbf{H}$ and $\mathbf{H}_\tau$ are related as follows

$$\mathbf{H} = \mathbf{A}_\tau \mathbf{H}_\tau \tag{63}$$

This is a simple model in the sense that we ignore the possibilities that a signal may be partially appearing in the window. However from the point of view of of detection theory this has little implication. We will take this into account after a detection has been declared for the problem. Now we have reduced the problem to standard form for which an optimal solution is available [15].

In the following sections N denotes the length of the vector $\mathbf{x}_\tau$ and p is the rank of the matrix $\mathbf{H}$.

A. ML estimation of time delays and related coefficients

$$[\hat{\alpha}, \hat{\tau}, \hat{\sigma}^2] = \arg\max_{\alpha,\tau,\sigma^2} \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\Big(-\frac{1}{2\sigma^2}\|\mathbf{y} - \mathbf{H}_\tau\boldsymbol{\theta}\|^2\Big) \tag{64}$$

the derivation of this is exactly the same with the only difference that we have a rank-p system. the solution is then

$$\begin{aligned} [\hat{\tau}] &= \arg\max_\tau \mathbf{y}^T \mathbf{P}_{\mathbf{H}_\tau} \mathbf{y} &(65)\\ &= \arg\max_\tau \mathbf{x}_\tau^T \mathbf{P}_{\mathbf{H}} \mathbf{x}_\tau &(66)\\ &= \arg\max_\tau \mathbf{y}^T \mathbf{A}_\tau^T \mathbf{P}_{\mathbf{H}} \mathbf{A}_\tau \mathbf{y} &(67)\end{aligned}$$

by following similar arguments to the matched filter case

$$[\hat{\alpha}, \hat{\tau}, \hat{\sigma}^2] = \arg\max_{\alpha,\tau,\sigma^2} \frac{1}{(2\pi\sigma^2)^{Mp/2}} exp\Big(-\frac{1}{2\sigma^2}\|\mathbf{x}_\tau - \mathbf{H}\boldsymbol{\theta}\|^2\Big) \tag{68}$$

In summary this leads to the following

$$[\hat{\tau}] = \arg\max_\tau \mathbf{x}_\tau^T \mathbf{P}_{\mathbf{H}} \mathbf{x}_\tau \tag{69}$$

Now consider the case where we want to analyze the multipath delay profile with a small window in order to locate the peak. We can argue as follows

$$\begin{aligned} [\hat{\tau}] &= \arg\max_\tau \hat{\boldsymbol{\theta}}^T \hat{\boldsymbol{\theta}} &(70)\\ &= \arg\max_\tau \mathbf{x}_\tau^T \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}_\tau &(71)\\ &= \arg\max_\tau \mathbf{x}_\tau^T \mathbf{P}_{\mathbf{H}} \mathbf{x}_\tau &(72)\end{aligned}$$

where we have used the property that $\mathbf{H}^T\mathbf{H} = M\mathbf{I}$. For notational simplicity we will label the following operation as *matched subspace filter*

$$T(\mathbf{x}_\tau) = \mathbf{x}_\tau^T \mathbf{P_H} \mathbf{x}_\tau \tag{73}$$

This can be simply regarded as a high dimensional matched filter since the matrix $\mathbf{H}$ is of rank-p.

B. Signal Detection

In this section we consider the problem of signal detection with GLRTs. Since the purpose is efficient detection, we may choose the window of observation equal to the length of multipath delay profile to capture all the available energy. We can construct the following hypothesis testing problem

$$\mathcal{H}_0 : \boldsymbol{\theta} = 0 \tag{74}$$

$$\mathcal{H}_1 : \boldsymbol{\theta} \neq 0 \tag{75}$$

where

$$\mathbf{y} = \mathbf{H}_\tau \boldsymbol{\theta} + \mathbf{w} \tag{76}$$

This leads to the following GLRT

$$L_1(\mathbf{y}) = \frac{p(\mathbf{y}, \hat{\tau}, \hat{\boldsymbol{\theta}}, \hat{\sigma}_1^2, \mathcal{H}_1)}{p(\mathbf{y}, \hat{\sigma}_0^2, \mathcal{H}_0)} > \eta \tag{77}$$

where the detection performance is provided as follows. The parameters used in the LR are given as:

$$[\hat{\tau}] = \arg\max_\tau T(\mathbf{x}_\tau) \tag{78}$$

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}_{\hat{\tau}} \tag{79}$$

$$\hat{\sigma}_1^2 = \frac{1}{N}\|\mathbf{y} - \mathbf{H}_{\hat{\tau}}\hat{\boldsymbol{\theta}}\|^2 \tag{80}$$

$$\hat{\sigma}_0^2 = \frac{1}{N}\|\mathbf{y}\|^2 \tag{81}$$

After simple manipulations we obtain the following

$$L_1'(\mathbf{y}) = \frac{N-p}{p}\frac{\mathbf{y}^T\mathbf{P}_{\mathbf{H}_{\hat{\tau}}}\mathbf{y}}{\mathbf{y}^T\mathbf{P}_{\mathbf{H}_{\hat{\tau}}}^{\perp}\mathbf{y}} > \eta' \tag{82}$$

And the distribution is $F_{p,N-p}$ under the null hypothesis.

And when we do know the noise variance

$$L_2(\mathbf{y}) = \frac{p(\mathbf{y}, \hat{\tau}, \hat{\boldsymbol{\theta}}, \mathcal{H}_1)}{p(\mathbf{y}, \mathcal{H}_0)} > \eta \tag{83}$$

it can be shown that this expression reduces to

$$L_2'(\mathbf{y}) = \frac{\mathbf{y}^T\mathbf{P}_{\mathbf{H}_{\hat{\tau}}}\mathbf{y}}{\sigma^2} > \eta' \tag{84}$$

The distribution is χ_p^2 under the null hypothesis.

Until now we assumed that a single signal is present in an observation time. This may not be the case and we need a safer procedure for estimating the noise variance. In a similar way to the matched filter case we formulate the following hypothesis testing problem

$$\mathcal{H}_0 : \mathbf{x}_{\hat{\tau}} = \mathbf{w} \tag{85}$$

$$\mathcal{H}_1 : \mathbf{x}_{\hat{\tau}} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{86}$$

This again leads to the following tests

$$T_{\chi^2}(\mathbf{x}_{\hat{\tau}}) = \frac{\mathbf{x}_{\hat{\tau}}^T\mathbf{P_H}\mathbf{x}_{\hat{\tau}}}{\sigma^2} > \eta \tag{87}$$

where $T_{\chi^2}(\mathbf{x}_{\hat{\tau}}) \sim \chi_p^2$ under the null hypothesis. and the case where the noise variance is not known can be described as

$$T_F(\mathbf{x}_{\hat{\tau}}) = \frac{Mp-p}{p}\frac{\mathbf{x}_{\hat{\tau}}^T\mathbf{P_H}\mathbf{x}_{\hat{\tau}}}{\mathbf{x}_{\hat{\tau}}^T\mathbf{P_H^{\perp}}\mathbf{x}_{\hat{\tau}}} > \eta \tag{88}$$

where $T_F(\mathbf{x}_{\hat{\tau}}) \sim F_{p,Mp-p}$ under the null hypothesis.

C. Leading edge detection methods

We want to decide whether there is a an early arriving path which is not the strongest. This can be done by testing for various values of time delay instants τ satisfying $\hat{\tau} - T_c < \tau < \hat{\tau}$ where T_c is typically 50ns.

$$\mathcal{H}_0 : \mathbf{x}_\tau = \mathbf{w} \tag{89}$$

$$\mathcal{H}_1 : \mathbf{x}_\tau = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{90}$$

For the case where noise variance is known, the GLRT is given as

$$T_{\chi^2}(\mathbf{x}_\tau) = \frac{\mathbf{x}_\tau^T\mathbf{P_H}\mathbf{x}_\tau}{\sigma^2} > \eta \tag{91}$$

In that case $T_{\chi^2}(\mathbf{x})$ is χ_p^2 distributed under the hypothesis $\mathcal{H}_0$ and $\chi_p^2(\lambda)$ distributed under the hypothesis $\mathcal{H}_1$ where $\lambda = \boldsymbol{\theta}^T\mathbf{H}^T\mathbf{H}\boldsymbol{\theta}/\sigma^2$ for the true value of $\boldsymbol{\theta}$.

In this section we assume that the noise variance is not known. In that case, the GLRT is given as

$$T_F(\mathbf{x}_\tau) = \frac{Mp-p}{p}\frac{\mathbf{x}_\tau^T\mathbf{P_H}\mathbf{x}_\tau}{\mathbf{x}_\tau^T\mathbf{P_H^{\perp}}\mathbf{x}_\tau} > \eta \tag{92}$$

where $T_F(\mathbf{x})$ is $F_{p,N-p}$ distributed under the hypothesis $\mathcal{H}_0$ and $F_{p,N-p}(\lambda)$ distributed under the hypothesis $\mathcal{H}_1$ where $\lambda = \boldsymbol{\theta}^T\mathbf{H}^T\mathbf{H}\boldsymbol{\theta}/\sigma^2$ for the true value of $\boldsymbol{\theta}$.

D. Algorithm Summary

1. Detect the presence of the signal by either using $L_1'(\mathbf{y})$, $L_2'(\mathbf{y})$, $T_{\chi^2}(\mathbf{x}_{\hat{\tau}})$, $T_F(\mathbf{x}_{\hat{\tau}})$
2. Locate the peak by finding maximum of $T(\mathbf{x}_\tau)$
3. Search to the left of the peak to decide whether there is an early arriving path that is not the strongest by using $T_F(\mathbf{x}_\tau)$ or $T_{\chi^2}(\mathbf{x}_\tau)$. Pick the earliest time value for which $T_F(\mathbf{x}_\tau)$ or $T_{\chi^2}(\mathbf{x}_\tau)$ is above the specified threshold.

V. Leading Edge Detection Methods

In this section we will analyze various methods for the leading edge detection problem. We propose the Bayesian formalism for leading edge detection first and later review the energy detection approach.

A. Bayesian and Information Theoretic Approach

In the previous sections we applied a selection matrix to the received data and performed decisions and estimations on this basis. In the decision process our assumption was that either there is no signal present in the window or the signal appears completely but not partially. To take into account the cases where the signal appears partially has to be handled with multihypothesis decision theory. We can very well use sequential F tests or sequential χ^2 tests however these tests yield constant false alarms and the performance of true decision never improves even if we increase SNR. This is mainly due to the constant false alarm property. Instead we can use the following Bayesian method for the cases where the signal periodicity M is large. This ensures that the algorithms will converge to the true hypothesis with probability 1.

Lets consider the following multihypothesis problem where $M = 3$ and $p = 3$

$$\mathbf{H}_1 = \begin{bmatrix} \\ \\ 1 \\ \\ \\ 1 \\ \\ \\ 1 \end{bmatrix} \qquad \mathbf{H}_2 = \begin{bmatrix} & \\ & 1 \\ 1 & \\ & \\ & 1 \\ 1 & \\ & \\ & 1 \\ 1 & \end{bmatrix} \qquad \mathbf{H}_3 = \begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \\ & & 1 \\ & 1 & \\ 1 & & \\ & & 1 \\ & 1 & \\ 1 & & \end{bmatrix} \tag{93}$$

$$\mathcal{H}_0 : \mathbf{x}_\tau = \mathbf{w} \tag{94}$$

$$\mathcal{H}_1 : \mathbf{x}_\tau = \mathbf{H}_1\boldsymbol{\theta}_1 + \mathbf{w} \tag{95}$$

$$\mathcal{H}_2 : \mathbf{x}_\tau = \mathbf{H}_2\boldsymbol{\theta}_2 + \mathbf{w} \tag{96}$$

$$\mathcal{H}_3 : \mathbf{x}_\tau = \mathbf{H}_3\boldsymbol{\theta}_3 + \mathbf{w} \tag{97}$$

We need to test a nested hypothesis and for which variety of methods can be used. We will only use the Generalized Maximum Likelyhood (GML) rule which is based on Bayesian methods. In the following i denotes the model order and $\mathbf{H}_i$ is the matrix constructed from the first i columns of the matrix $\mathbf{H}$ by preserving the order. In this case the generalized ML rule for non-zero i is given as [15]

$$\begin{aligned} GML(i) = &-\frac{N}{2}\ln(2\pi\sigma^2) + \frac{i}{2}\ln(\sigma^2) - \frac{1}{2}\ln(\det(\mathbf{H}_i^T\mathbf{H}_i)) \\ &- \frac{1}{2\sigma^2}\mathbf{x}_\tau^T(\mathbf{I} - \mathbf{H}_i(\mathbf{H}_i^T\mathbf{H}_i)^{-1}\mathbf{H}_i^T)\mathbf{x}_\tau \end{aligned} \tag{98}$$

and

$$GML(0) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\mathbf{x}_\tau^T\mathbf{x}_\tau \tag{99}$$

And the true hypothesis is given by

$$\hat{i} = \arg\max_i GML(i) \tag{100}$$

We must note that this approach works only if the length of the vector $\mathbf{x}_\tau$ is very large. In that case the probability of true decision converges to 1. For small lengths this method should not be used.

B. Energy Detection

The idea behind energy detection is just to decide whether there is a strong component in a given window of the signal. There is however a trade-off between probability of detection and range estimation. In other words for good detection we require that the energy is large which can be achieved by a large window. For good ranging we require that the window be small.

First lets give the data model,

$$\mathbf{x}_\tau = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{101}$$

The maximum likelihood estimate of the coefficients is given as

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}_\tau \tag{102}$$

and we must note that $\hat{\boldsymbol{\theta}} \sim \mathcal{N}(\boldsymbol{\theta}, \sigma^2(\mathbf{H}^T\mathbf{H})^{-1})$ This can further be simplified to $\hat{\boldsymbol{\theta}} \sim \mathcal{N}(\boldsymbol{\theta}, \frac{\sigma^2}{M}\mathbf{I})$ Now we have a remarkable property that the columns of the matrix $\mathbf{H}$ are orthogonal hence after ML estimation there is no correlation in the noise structure

$$T_E(\mathbf{x}_\tau) = \frac{M}{\sigma^2}\hat{\boldsymbol{\theta}}^T\hat{\boldsymbol{\theta}} > \eta \tag{103}$$

The performance can again be obtained clearly and we can reason on the estimation of signal parameters. And we must note that $T_E(\mathbf{x}_\tau) \sim \chi_p^2$ and under the other hypothesis it is distributed as $T_E(\mathbf{x}_\tau) \sim \chi_p^2(\lambda)$ where $\lambda = M\boldsymbol{\theta}^T\boldsymbol{\theta}/\sigma^2$.

The resulting detection performance should be compared to other results especially when noise level is unknown. We must note that it takes the following form

$$T_E(\mathbf{x}_\tau) = \frac{M}{\sigma^2}\hat{\boldsymbol{\theta}}^T\hat{\boldsymbol{\theta}} \tag{104}$$

$$= \frac{1}{M\sigma^2}\mathbf{x}_\tau^T\mathbf{H}\mathbf{I}\mathbf{H}^T\mathbf{x}_\tau \tag{105}$$

which is identical to the previous notation of the T_{χ^2} detector. We must note that energy detection works only if noise variance is known.

VI. Simulations

In order to test the performance of the proposed algorithms we performed extensive computer simulations. In the first set of simulations we applied our results to a single rectangular pulse. This was done in order to compare the matched subspace filter with the matched filter. We must note that in general matched subspace filter is applicable to a wider class of problems. On the other hand we use less information in the matched subspace filter compared to the matched filter. The parameters were chosen as $M = 5$, $K_m = 20(m-1)$ and pulse duration is ($p = 5$). We chose the window size (p) equal to the pulse width. Figure 1 shows the deviation of the time delay estimate from the true value (151). As can be seen the mean values is quite small. However if we look at the standard deviation in Figure 2 we see that as the noise level increases performance of the matched subspace filter gets poorer compared to the matched filter. We must note that for high Signal-to-Noise-Ratio (SNR) the performance of the proposed method gets closer to the matched filter. This simple example demonstrates the relationship between the matched filter and the matched subspace filter in terms of estimation performace. It is known that the detection performance of the matched subspace filter will be poorer compared to the matched filter. This stems from the fact that there is more uncertainty in the case of the matched subspace filter.

In the second set of simulations we compare the error structure of the matched filter in the case of multipath propagation. The sampled version of the received signal was set to the following

$$r(n) = s(n-200) + 0.5s(n-206) + 0.2s(n-211) + 0.1s(n-216) \quad (106)$$

where $s(n)$ is a rectangular pulse of duration 5. This choice was again made in order to compare the performance of the matched filter and the matched subspace filter. We have also set the window size $p = 5$ and $M = 5$ and $K_m = 50(m-1)$. Figure 3. Shows the values of the estimated time delays of the peak for 1000 simulations. As can be seen from the figure there are outliers exactly at time delays 250 and 150. Figure 4. shows the same result for the matched filter. These graphics illustrate the structure of the errors. Due to the periodicity, and low SNR, we we make large errors. These errors stem from the periodic nature of the received signal. In all the simulations the noise standard deviation σ was set to 0.7. Such errors can easily be avoided if we choose a suitable coding waveform which has low sidelobes. Barker codes are a good example.

Finally we performed simulations to test the performance of the GML method. One interesting observation is that this method is not directly applicable for the process of locating the peak in multipath delay profile. The difficulty stems from the periodicity and the structure of the GML rule. However this method can be used for fine tuning when an approximate estimate of the strongest and the first arriving paths are available. Another limitation is the size of the observation matrix N. To work asymptotically efficient the value of N must be very large. This is another limitation of the GML rule. In spite of these restrictions we obtained good results for large N. Figure 5. shows the result of a local search and the true hypothesis is selected accurately.

VII. Conclusion

In this paper we proposed a method that can be used for estimation of the strongest and first arriving paths in a multipath propagation scenario. We assumed that both the channel and the associated pulse shapes are unknown. In simulations we observed that matched filter approach gives the best estimation accuracy and detection performace due to the fact that we have more information available. On the other when the pulse shape is not known we can still achieve reasonable estimation and detection performace by adjusting parameters like pulse repetition amount and SNR. In conclusion the proposed method can be used for positioning

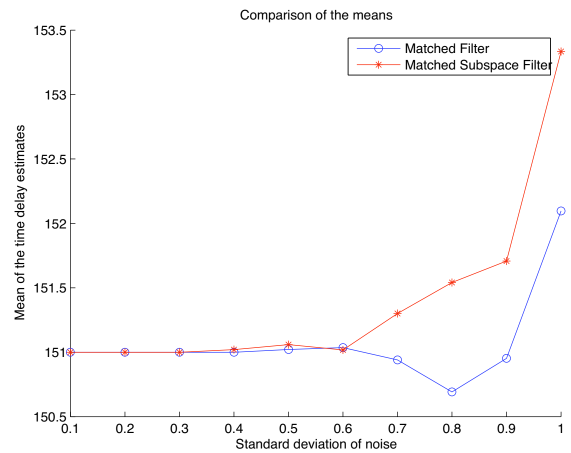

Fig. 1. Mean of the time delay estimates

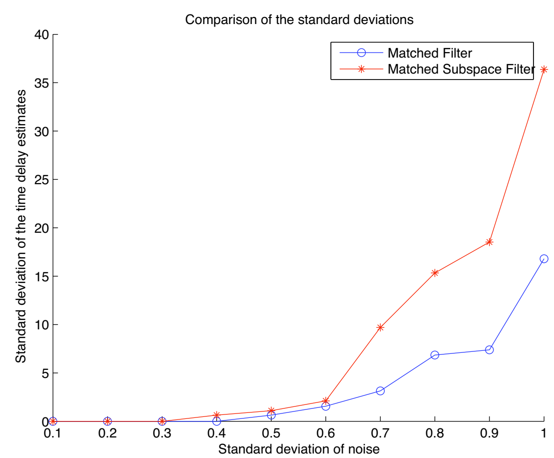

Fig. 2. Standard deviation of the time delay estimates

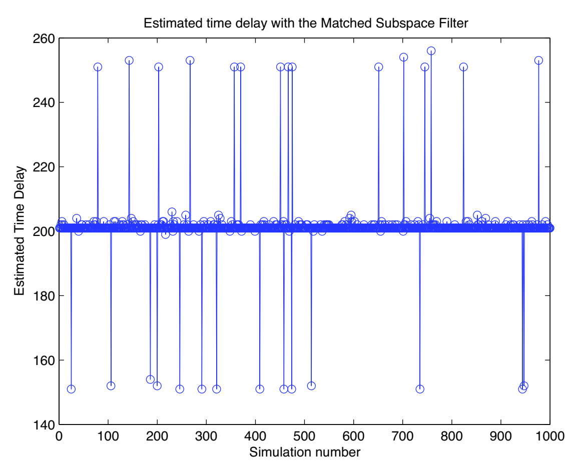

Fig. 3. Time delay estimates with matched subspace filter

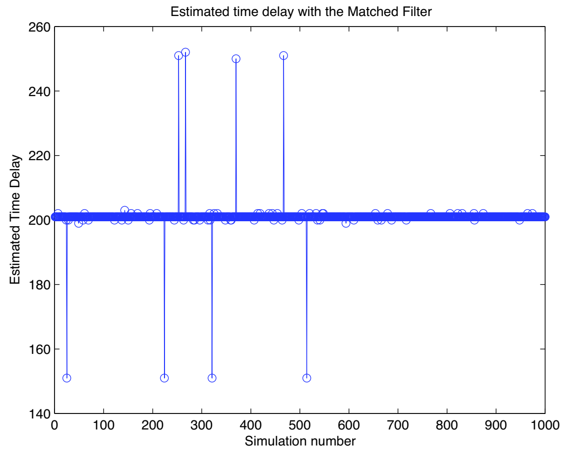

Fig. 4. Time delay estimates with matched filter

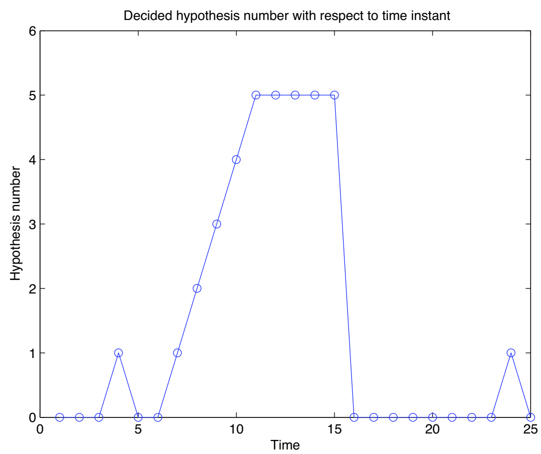

Fig. 5. Selected hypothesis for a long signal. True time delay is 11.

purposes in scenarios where the pulse shape is not known beyond some *regularity conditions* as discussed earlier.

It is also interesting to note the interplay between estimation theory and detection theory in the problem of positioning. Usually estimation and detection are regarded as separate problems yet they can be used interchangeably. In order to find the correct range information from time delays, decision theory and estimation theory must be used cooperatively.

Throughout the paper, we assumed that noise is Gaussian. This is a reasonable assumption for most of the cases. However, the case of non-Gaussian noise and subspace interference is also relevant. For non-Gaussian noise the algorithms in [16] can be applied to the problem. In the case of narrowband interference we may adopt the results in [13]. One difficulty in dealing with subspace interference is the fact that we are applying a selection matrix $\mathbf{A}_\tau$ to the observed data. It means that we are applying this selection matrix to the interference as well. Because of this we need to develop novel methods to handle subspace interference.

In the simulations we observed two types of errors. The first type is small deviations from the true time delay estimate. Such an error is expected from any estimation procedure. The second type of errors are large errors. These errors stem mainly from the fact that we used periodic signals. Such signals have large sidelobes in the matched filter output. This brings the possibility of make a large error mainly by choosing the peak in the correlation wrongly. We observed that both matched filter and matched subspace filter are influenced from such large errors especially at low SNR. Such errors can easily be reduced by using appropriate codes that have low sidelobes. A good example is Barker codes.

Finally we must note that we worked on periodic signals for simplicity. However our results are applicable to more general modulations like pulse amplitude modulation (PAM) and pulse position modulation (PPM). The only change will occur in the selection matrix $\mathbf{A}_\tau$. All the remaining results will be the same. This stems from the fact that both PAM and PPM has a structure that can be expressed with subspaces.

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