

# Simple Equalization of Time-Varying Channels for OFDM

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**Abstract**—We present a block minimum mean-squared error (MMSE) equalizer for orthogonal frequency-division multiplexing (OFDM) systems over time-varying multipath channels. The equalization algorithm exploits the band structure of the frequency-domain channel matrix by means of a band LDL<sup>H</sup> factorization. The complexity of the proposed algorithm is linear in the number of subcarriers and turns out to be smaller with respect to a serial MMSE equalizer characterized by a similar performance.

**Index Terms**—Equalization, intercarrier interference, OFDM, time-varying channels.

## I. INTRODUCTION

MULTICARRIER techniques such as OFDM gained a lot of attention for wireless mobile communications [1]. Indeed, thanks to the cyclic prefix (CP), OFDM systems easily equalize time-invariant multipath channels by 1-tap equalizers [1]. However, the request for communications with high mobility suggests that future OFDM designs should take into account also the Doppler spread associated with time-varying (TV) channels. This scenario complicates the equalization, because a TV channel generates intercarrier interference (ICI), thus destroying the orthogonality among OFDM data [2]-[5].

Recently, various techniques have been proposed to counteract such ICI effects in OFDM systems [6]-[12], and it has been shown that nonlinear equalizers based on ICI cancellation generally outperform linear approaches [7]-[10]. Anyway, linear schemes still preserve their importance. First, because linear equalizers are usually simpler, and therefore less complex. Second, because nonlinear schemes usually employ a linear equalizer to obtain the temporary decisions that they use to cancel out the ICI.

This letter presents a block MMSE equalizer that relies on the LDL<sup>H</sup> factorization [13] and, in order to reduce complexity, takes advantage of the band structure of the channel matrix in the frequency domain. The overall complexity is linear in the number of subcarriers, and quadratic in the bandwidth [13] of the channel matrix. The proposed scheme is a block equalizer, which jointly equalizes all the subcarriers, as in [7]

Manuscript received November 23, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. Alexander Haimovich. This work was partially supported by the Italian Ministry of University and Research under the project "MC-CDMA: an air interface for the 4th generation of wireless systems." G. Leus was also supported in part by NWO-STW under the VIDI program (DTC.6577).

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Digital Object Identifier 10.1109/LCOMM.2005.07031.

and [10]. Those schemes, such as [6][8][9][12], which separately equalize each subcarrier discarding the data received on the faraway subcarriers, are usually called serial equalizers. The comparison between the proposed block MMSE equalizer and a serial MMSE equalizer derived from [9] evidences a reduction in complexity, while preserving performance.

## II. OFDM SYSTEM MODEL

We consider an OFDM system with  $N$  subcarriers. Assuming time and frequency synchronization, and employing a CP length greater than the maximum delay spread of the channel, the OFDM input-output relation for the  $i$ -th OFDM symbol can be expressed by [6]-[10]

$$\mathbf{z}[i] = \underline{\Lambda}[i]\mathbf{a}[i] + \mathbf{n}[i] \quad (1)$$

where  $\mathbf{z}[i]$  is the  $N \times 1$  received vector,  $\mathbf{n}[i]$  is the  $N \times 1$  additive noise vector,  $\underline{\Lambda}[i]$  is the  $N \times N$  frequency-domain channel matrix, and  $\mathbf{a}[i]$  is the  $N \times 1$  OFDM symbol, which contains the frequency-domain data. Assuming that  $N_A$  subcarriers are active and  $N_V = N - N_A$  are used as frequency guard bands, we can write  $\mathbf{a}[i] = [\mathbf{0}_{1 \times N_V/2} \mathbf{a}[i]^T \mathbf{0}_{1 \times N_V/2}]^T$ , where  $\mathbf{a}[i]$  is the  $N_A \times 1$  data vector. Due to the TV nature of the channel  $\underline{\Lambda}[i]$  in (1) is not diagonal, and each diagonal is associated with a discrete Doppler frequency, which introduces ICI. Consequently, nontrivial equalization techniques are required.

## III. LOW-COMPLEXITY EQUALIZATION OF TV CHANNELS

We assume that the equalizer does not make use of the virtual subcarriers, which contain little signal power, and could also be affected by interference originated from adjacent transmissions. Moreover, we assume that  $\underline{\Lambda}[i]$  is known to the receiver. In practice,  $\underline{\Lambda}[i]$  could be estimated as in [8][10][11]. Since we neglect the data received on the  $N_V$  virtual subcarriers, by dropping the block index  $i$ , (1) becomes  $\mathbf{z} = \underline{\Lambda}\mathbf{a} + \mathbf{n}$ , where  $\mathbf{z}$  and  $\mathbf{n}$  are  $N_A \times 1$  vectors obtained by selecting the middle part of  $\mathbf{z}$  and  $\mathbf{n}$ , respectively, and  $\underline{\Lambda}$  is the  $N_A \times N_A$  matrix obtained by selecting the central block of  $\underline{\Lambda}$ . In order to recover  $\mathbf{a}$ , several options are possible [7]. We focus on linear block MMSE equalization, expressed by

$$\hat{\mathbf{a}}_{MMSE} = \underline{\Lambda}^H (\underline{\Lambda}\underline{\Lambda}^H + \gamma^{-1}\mathbf{I}_{N_A})^{-1}\mathbf{z}, \quad (2)$$

where  $\gamma$  is the signal-to-noise ratio (SNR). Although the MMSE outperforms other linear criteria [7], the matrix inversion in (2) requires  $O(N_A^3)$  flops [13], which represent a significant burden when  $N_A$  is high, such as for broadcasting applications. However, as already documented in [6] and

[9], TV multipath channels produce a nearly-banded channel matrix  $\mathbf{A}$ . This property can be exploited to further reduce complexity by means of an  $\text{LDL}^H$  factorization [13] of Hermitian band matrices.

#### A. Equalization by Band $\text{LDL}^H$ Factorization

Let us approximate the channel matrix  $\mathbf{A}$  with the band matrix  $\mathbf{B}$  obtained by selecting the main diagonal, the  $Q$  subdiagonals and the  $Q$  superdiagonals, of  $\mathbf{A}$ . Thus  $\mathbf{B} = \mathbf{A} \circ \mathbf{T}^{(Q)}$ , where  $\circ$  denotes element-wise product, and  $\mathbf{T}^{(Q)}$  is a matrix with lower and upper bandwidth  $Q$  [13] and all ones within its band. Accordingly, (2) can be approximated by

$$\hat{\mathbf{a}}_{LDL} = \mathbf{B}^H (\mathbf{B}\mathbf{B}^H + \gamma^{-1}\mathbf{I}_{N_A})^{-1} \mathbf{z}. \quad (3)$$

Since  $\mathbf{M} = \mathbf{B}\mathbf{B}^H + \gamma^{-1}\mathbf{I}_{N_A}$  is a Hermitian band matrix with lower and upper bandwidth  $2Q$ ,  $\mathbf{M}^{-1}$  can be obtained by using low-complexity decompositions such as the Cholesky or the  $\text{LDL}^H$  factorization, which are also characterized by a small sensitivity to rounding errors [13]. We consider the  $\text{LDL}^H$  factorization because it does not require square roots. The steps of the equalization algorithm are detailed below:

- 1) Choose  $Q$ , and construct the band matrix  $\mathbf{B} = \mathbf{A} \circ \mathbf{T}^{(Q)}$ ;
- 2) Construct the band matrix  $\mathbf{M} = \mathbf{B}\mathbf{B}^H + \gamma^{-1}\mathbf{I}_{N_A}$ ;
- 3) Perform the  $\text{LDL}^H$  factorization of  $\mathbf{M}$ , as expressed by  $\mathbf{M} = \mathbf{L}\mathbf{D}\mathbf{L}^H$ , where  $\mathbf{D}$  is diagonal, and the triangular factor  $\mathbf{L}$  has lower bandwidth  $2Q$ ;
- 4) Solve the system  $\mathbf{M}\mathbf{d} = \mathbf{z}$  by solving firstly the triangular system  $\mathbf{L}\mathbf{f} = \mathbf{z}$ , secondly the diagonal system  $\mathbf{D}\mathbf{g} = \mathbf{f}$ , and thirdly the triangular system  $\mathbf{L}^H\mathbf{d} = \mathbf{g}$ ;
- 5) Calculate  $\hat{\mathbf{a}}_{LDL} = \mathbf{B}^H\mathbf{d}$ .

The parameter  $Q$  can be chosen to trade off performance for complexity. Obviously, a larger  $Q$  implies a smaller approximation error and hence a performance improvement. On the other hand, the complexity increases due to the higher bandwidth of  $\mathbf{M}$ . As a rule of thumb, we can adopt  $Q \geq \lceil f_D/\Delta_f \rceil + 1$  [9], where  $f_D$  is the maximum Doppler frequency and  $\Delta_f$  is the subcarrier spacing.

#### B. Complexity Analysis

We evaluate the computational cost of the proposed algorithm in terms of complex additions (CA), complex multiplications (CM), and complex divisions (CD). Since  $\mathbf{B}$  has lower and upper bandwidth  $Q$ , the computation of  $[\mathbf{B}\mathbf{B}^H]_{m,p}$  in Step 2 requires  $2Q + 1 - |m - p|$  CM and  $2Q - |m - p|$  CA. Hence, taking into account that  $\mathbf{B}\mathbf{B}^H$  is Hermitian and neglecting some smaller terms in the complexity expression, Step 2 requires at most  $(2Q^2 + 3Q + 1)N_A$  CM and  $(2Q^2 + Q + 1)N_A$  CA. The band  $\text{LDL}^H$  factorization algorithm is reported in the following.

$\mathbf{L} = \mathbf{I}_{N_A}$ ;  $\mathbf{D} = \mathbf{M} \circ \mathbf{I}_{N_A}$ ;  $v = \mathbf{0}_{N_A \times 1}$ ;  
for  $j = 1 : N_A$   
 $m = \max\{1, j - 2Q\}$ ;  $M = \min\{j + 2Q, N_A\}$ ;  
for  $i = m : j - 1$   
 $[v]_i = [\mathbf{L}]_{j,i}^* [\mathbf{D}]_{i,i}$ ;  
end  
 $[v]_j = [\mathbf{M}]_{j,j} - [\mathbf{L}]_{j,m:j-1} [v]_{m:j-1}$ ;  $[\mathbf{D}]_{j,j} = [v]_j$ ;  
 $[\mathbf{L}]_{j+1:M,j} = \frac{[\mathbf{M}]_{j+1:M,j} - [\mathbf{L}]_{j+1:M,m:j-1} [v]_{m:j-1}}{[v]_j}$ ;  
end

The algorithm requires  $(2Q^2 + 3Q)N_A$  CM,  $(2Q^2 + Q)N_A$  CA, and  $2QN_A$  CD. This result is obtained by observing that the submatrix  $[\mathbf{L}]_{j+1:j+2Q,j-2Q:j-1}$  is strictly upper triangular. The two band triangular systems of Step 4 can be solved by band forward and backward substitution [13]. Since each algorithm requires  $2QN_A$  CM and CA, Step 4 requires  $4QN_A$  CM,  $4QN_A$  CA, and  $N_A$  CD. Moreover, each element of  $\hat{\mathbf{a}}_{LDL} = \mathbf{B}^H\mathbf{d}$  needs  $2Q + 1$  CM and  $2Q$  CA. Therefore, the whole algorithm requires roughly  $(4Q^2 + 12Q + 2)N_A$  CM,  $(4Q^2 + 8Q + 1)N_A$  CA, and  $(2Q + 1)N_A$  CD, leading to a total of  $(8Q^2 + 22Q + 4)N_A$  complex operations.

We now compare the complexity of the proposed block equalizer with the serial MMSE equalizer expressed by

$$[\hat{\mathbf{a}}_{serial}]_n = \bar{\mathbf{b}}_n^H (\bar{\mathbf{B}}_n \bar{\mathbf{B}}_n^H + \gamma^{-1}\mathbf{I}_{2Q+1})^{-1} \bar{\mathbf{z}}_n, \quad (4)$$

where  $n$  is the subcarrier index,  $\bar{\mathbf{z}}_n = [\mathbf{z}]_{n-Q:n+Q}$ ,  $\bar{\mathbf{B}}_n = [\mathbf{B}]_{n-Q:n+Q, n-2Q:n+2Q}$ , and  $\bar{\mathbf{b}}_n = [\bar{\mathbf{B}}_n]_{:,2Q+1}$ . This equalizer is similar to the one adopted to initialize the iterative symbol MMSE estimator of [9]. In the complexity analysis of the serial MMSE, we have assumed that the inversion of the Hermitian matrix in (4) is performed by means of the  $\text{LDL}^H$  factorization. This requires roughly half the number of complex operations of standard inversion methods like the Gaussian elimination [13]. We have also assumed that the products involving different  $\bar{\mathbf{B}}_n$ 's are performed by reusing intermediate computations. The results of this complexity analysis show that the serial MMSE equalizer (4) [9] requires  $(4/3 Q^3 + 10Q^2 + 26/3 Q + 2)N_A$  CM,  $(4/3 Q^3 + 8Q^2 + 17/3 Q + 1)N_A$  CA, and  $(2Q^2 + 3Q + 1)N_A$  CD, leading to a total of  $(8/3 Q^3 + 20Q^2 + 52/3 Q + 4)N_A$  complex operations.

For the serial MMSE, instead of using the  $\text{LDL}^H$  factorization, the matrix inversion for index  $n$  can also be done by reusing the inverse computed for index  $n - 1$ , as in the recursive inversion algorithm of [8]. In this case, without assuming a banded channel matrix, the serial MMSE equalizer of size  $2Q + 1$  has complexity  $O(QN_A^2)$  [8]. For banded channel matrices, by neglecting the complexity of the first inversion, the serial MMSE equalizer (4) [9] requires about  $(14Q^2 + 15Q + 3)N_A$  CM,  $(14Q^2 + 7Q + 1)N_A$  CA, and  $(2Q + 1)N_A$  CD, leading to a total of  $(28Q^2 + 24Q + 5)N_A$  complex operations. Therefore, the recursive inversion approach is cheaper than  $\text{LDL}^H$  factorization only for  $Q \geq 4$ . Hence, with respect to the proposed approach, the complexity of the serial MMSE equalizer (4) [9] is 1.75 and 2.50 times higher for  $Q = 2$  and  $Q = 4$ , respectively. Asymptotically (i.e.,  $N_A, Q \rightarrow \infty$ ), the complexity of the serial equalizer is 3.5 times higher than the proposed block approach.

#### C. Performance Comparison

We compare the uncoded BER performance of different equalizers by means of simulation results. We consider an OFDM system with  $N = 128$ ,  $N_A = 96$ ,  $L = 8$ , and QPSK modulation. We assume Rayleigh fading channels, exponential power delay profile, and Jakes' Doppler spectrum with maximum Doppler frequency  $f_D = 0.15\Delta_f$ .

In Fig. 1, we compare serial and block approaches (assuming a banded channel matrix or not) with  $Q = 2$ . For a fair comparison with [8] and [9], we incorporated the presence

of the guard frequency bands in the expressions of the serial MMSE equalizers. Among the banded approaches, the serial MMSE equalizer [9] has the best performance. Anyway, the proposed block MMSE equalizer has almost the same performance with a complexity reduction of 43% for  $Q = 2$ . The small difference between the performance of the two banded equalizers is mainly due to the use of the signal received on the guard bands, and can be reduced further by applying the block MMSE on an enlarged block of size  $N_A + 2Q$  that includes part of the guard bands. Worse performances are obtained using the serial zero-forcing (ZF) equalizer [6], the conventional 1-tap equalizer for time-invariant channels, and an approximated block MMSE equalizer (Eq. 28 of [10]). On the other hand, the non-banded approaches [8][7] outperform the banded ones. However, the complexity is quadratic [8] or even cubic [7] instead of linear in  $N_A$ . In addition, the non-banded equalizers rely on the assumption of a good estimation of the small Doppler components (i.e., those that fall outside the channel matrix band), which seems to be unrealistic for practical SNR values. It is also worth noting that the BER floor for the (non-banded) serial MMSE is lower than for the proposed equalizer. This points out that the modelling error due to the band matrix approximation  $\mathbf{A} \approx \mathbf{B}$  is higher than the error caused by using a serial instead of a block equalizer.

#### D. Further Developments

The proposed block MMSE algorithm opens the way to the investigation of other topics that we cannot treat in detail due to the lack of space. First, the proposed equalizer can be easily combined with time-domain windowing to improve the BER performance, as done in [9] for the serial MMSE equalizer. In this case, since the noise is no longer white, the matrix  $\mathbf{I}_{N_A}$  in (3) should be replaced by the noise covariance matrix, which in general is not banded. Hence, the window design should be carried out within the class of cosine-based windows, which leads to perfectly banded noise covariance matrices. Interestingly, this class includes some common windows such as Hamming, Hann, and Blackman. Moreover, low-pass windows lead to noise covariance matrices with an approximately banded structure, and therefore the proposed approach can still be applied by approximating the noise covariance matrix as banded. In most cases, the increase in computational complexity is negligible: for the Hamming window, only  $3N_A$  extra operations are needed.

The proposed equalizer can also form the basis for more advanced equalizers based on decision-driven ICI cancellation. Some non-exhaustive examples in the context of serial equalization include the iterative estimator of [9] and the decision-feedback ICI canceler of [8]. In addition, the proposed method can be extended to multicarrier CDMA systems [14]. Finally, studying the effects of channel estimation on banded and non-banded equalizers is a topic of interest.

#### IV. CONCLUSION

We have proposed a block MMSE equalizer for OFDM systems over TV multipath channels. By making use of the band LDL<sup>H</sup> factorization, we have shown that the complexity of the proposed algorithm is  $O(Q^2 N_A)$ , and significantly smaller

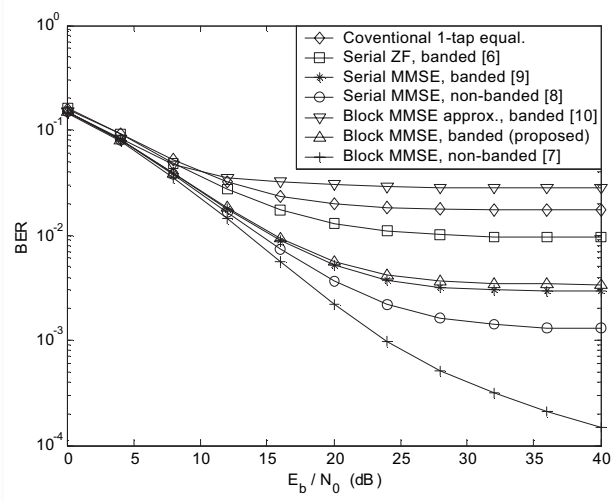


Fig. 1. BER comparison ( $Q = 2$  for serial and banded approaches).

than for other MMSE approaches, while preserving good performance. Future research topics include the incorporation of windowing and the development of nonlinear equalizers.

#### ACKNOWLEDGMENT

The authors thank Prof. P. Schniter and the anonymous reviewers for their useful suggestions and comments.

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