On Krylov subspace approximations of functions of operators with continuous spectra for exterior wave problems

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Abstract

The Krylov subspace projection approach is a well-established tool for the reduced order modeling of dynamical systems in the time domain. In this talk, we address the main issues obstructing the application of this powerful approach to the time-domain solution of the exterior wave problems. Solutions of these problems can be obtained as $\cos(t\sqrt{-A})b$, where A is self-adjoint non-positive PDE operator with continuous spectrum, b is compactly supported initial condition. To avoid spurious reflections and resonances, the approximate solution should preserve the features of continuous spectral measure of the original problem. To resolve this problem, we introduce dumped non-Hermitian matrix $\tilde{A}_N \in \mathbb{C}^{N\times N}$, discretized initial condition b_N and so-called stability-corrected time-domain exponential (SCTDE) function $f(t,x)$, such that $f(t,\tilde{A}_N)b_N \approx \cos(t\sqrt{-A})b$ for large N in a targeted subdomain of the exterior domain, and the Laplace transform $f(\lambda,\tilde{A}_N) = \int_0^\infty e^{-t\sqrt{\lambda}} f(t,\tilde{A}_N)dt$ of $f(t,\tilde{A}_N)b$ is analytic function of λ in $\mathbb{C} \setminus [-\infty, 0]$ with the branch-cut on $[-\infty, 0]$, i.e., it behaves as resolvent of a self-adjoint non-positive operator with continuous spectrum. We present approximations via polynomial and extended Krylov subspaces and give a theoretical foundation of our method. To illustrate its performance, we show a number of large scale numerical examples of important applied problems.

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