A RECEIVER ARCHITECTURE FOR MAXIMUM DIVERSITY TRANSMISSIONS OVER DOUBLY-SELECTIVE CHANNELS

Zijian Tang and Geert Leus

Delft University of Technology - Fac. EWI/Electrical Eng.
Mekelweg 4, 2628 CD Delft, The Netherlands,
{tang, leus}@cas.et.tudelft.nl

ABSTRACT

In this paper, equalization of time-varying channels is discussed, where the channel is modeled by a Complex Exponential Basis Expansion Model (CE-BEM). We consider a system where on the transmit side a precoder is employed to achieve maximum diversity. Along with an FFT decoder on the receive side, the resulting channel resembles an FIR filter on both matrix- and scalar-level. We propose therefore a Decision Feedback Equalizer (DFE), which bears a similar FIR structure as the channel. The equalizer taps can either be computed based on the channel knowledge, or adaptively estimated with the assistance of pilots. Simulations show that the proposed equalizer yields a satisfactory performance at a low hardware cost.

keywords: CE-BEM, doubly-selective channel, maximum diversity, adaptive filtering

1. INTRODUCTION

High data-rate mobile communications and thus advanced equalization techniques are attracting more and more attention. When the relative velocity between the transmitter and the receiver cannot be ignored, the resulting Doppler frequency gives rise to a time-varying (TV) channel, which makes many traditional equalization techniques less reliable. Jakes' model was proposed to model the time variation if the channel is composed of a large number of scatterers [1]. Despite its precision, this model is difficult to handle in many fields like channel estimation or equalization. Hence, a parametric model, though less precise, is preferred. In this paper, we adopt a Complex Exponential Basis Expansion Model (CE-BEM), initially proposed in [2] among others. This model is actually a superposition of $Q+1$ FIR filters, each of which is characterized by a set of $L+1$ taps. Different from a normal frequency-selective channel model, these FIR filters are weighted respectively by distinct complex exponential basis functions, i.e., we can express the νth channel tap at the nth time interval $h[n;\nu]$ as:

$$h[n;\nu] \approx \sum_{l=0}^{L} \delta[\nu - l] \sum_{q=0}^{Q} h_{q,l} e^{j2\pi qn/P}, \qquad (1)$$

where $\delta[\cdot]$ stands for a discrete Dirac function; P defines the size of the observation window, within which the coefficients of the CE-BEM $h_{q,l}$ remain constant. In comparison with other parametric channel models, such as the discrete prolate spheroidal expansion model in [3], the CE-BEM inflicts a larger modeling mismatch, especially on the edges of the observation window. On the other side, however, the average modeling mismatch can be made arbitrarily small if we increase the value of Q.

A major advantage of such a trigonometric expansion method might be its relative ease for analytical or numerical operations with respect to other algebraic or transcendental functions. For instance, the CE-BEM is uniquely characterized by its time-frequency duality: as we understand that the number of the channel taps $L+1$ results from time-domain sampling, we can likewise interpret the number of complex exponential basis functions $Q+1$ as a consequence of (virtual) frequency-domain sampling. The authors in [4] benefit from this time-frequency duality by proposing a precoding structure, which achieves a full multi-path/Doppler diversity amounting to a maximum of $(L+1)(Q+1)$ under the assumption of a CE-BEM channel. However, full diversity gain is generally exploited by a Maximum Likelihood (ML) equalizer, which becomes extremely expensive to implement even with a moderate number of data symbols. In this paper, we apply a suboptimal technique known as Decision Feedback Equalization (DFE) [5] to solve this problem. Unlike the work in [6] where a DFE is employed for uncoded transmission over a time-varying channel, the precoder under consideration in this paper forms a major hinder because it destroys the finite-alphabet property of the data symbols. In [7, Section IV.A], a special case of [4] is proposed together with a decoder that smartly makes use of the commutability between the channel and the (de-)precoder. The resulting effective channel, which associates the decoded samples directly with the transmitted data symbols, can then be characterized by a two-dimensional (2-D) FIR filter up to some frequency shifts. Since the effect of the precoder is seamlessly incorporated in this effective channel, the DFE becomes again functional.

In this paper, we present a parametric DFE, which is launched behind the decoder (see Fig. 1). It consists of a set of linear feed-forward filters and a feedback filter, both taking on a 2-D FIR structure analogous to the effective channel. We will show how to acquire the equalizer taps in two cases. In the first case, we need the knowledge of the CE-BEM channel defined in (1); in the second case we estimate the equalizer taps semi-blindly following an adaptive approach.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose and complex conjugate transpose (Hermitian), respectively. $\otimes$ is the Kronecker product. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all zero matrix as $\mathbf{0}_{M\times N}$. $\mathrm{mod}(p,q)$ denotes the remainder of p divided by q. Another important diag-

This research was supported in part by NWO-STW under project DTC.5893 (VICI-SPCOM) and project DTC. 6577 (VIDI-SPCOM)

onal matrix specific to this paper is $\mathbf{\Delta}_K^q$:

$$\mathbf{\Delta}_K^q = \text{diag}([1, e^{j\frac{2\pi q}{P}}, \cdots, e^{j\frac{2\pi q}{P}(K-1)}]) \quad (2)$$

where K denotes its size, q denotes the frequency bin it resides and P is the window length as we introduced in (1).

2. SYSTEM MODEL

Let us consider a SIMO system with one input and A outputs. The latter may be obtained by a combination of spatial and temporal oversampling. We follow the transceiver scheme depicted in Fig. 1, which starts with a data symbol sequence $\mathbf{s}$ of length $(N-Q)(M-L)$: $\mathbf{s} = [s[0], \cdots, s[(N-Q)(M-L)-1]]^T$. This data symbol sequence is first multiplied by an $NM \times (N-Q)(M-L)$ precoding matrix $\mathbf{\Theta}$ resulting in an NM-long data sequence $\mathbf{x} = [x[0], \cdots, x[NM-1]]^T$, and then cast to transmission. If we assume $P = NM$ and approximate the TV channel by the CE-BEM defined in (1), the nth received sample from the ath output can then be expressed as

$$y^{(a)}[n] = \sum_{l=0}^{L}\sum_{q=0}^{Q} h_{q,l}^{(a)} e^{j2\pi qn/P} x[n-l] + w^{(a)}[n], \quad (3)$$

where $w^{(a)}[n]$ stands for the additive noise. If we collect NM samples in one vector: $\mathbf{y}^{(a)} = [y^{(a)}[0], \cdots, y^{(a)}[NM-1]]^T$, the I/O relation defined in the above equation can be written in a matrix(vector) form:

$$\mathbf{y}^{(a)} = \mathbf{H}^{(a)}\mathbf{\Theta}\mathbf{s} + \mathbf{w}^{(a)}, \quad (4)$$

where $\mathbf{w}^{(a)}$ is similarly defined as $\mathbf{y}^{(a)}$; $\mathbf{H}^{(a)}$ denotes the ath channel matrix $\mathbf{H}^{(a)} := \sum_{q=0}^{Q} \mathbf{\Delta}_P^q \mathbf{H}_q^{(a)}$ where $\mathbf{\Delta}_P^q$ (see (2) for the notation) defines the qth exponential basis function, $\mathbf{H}_q^{(a)}$ denotes the Toeplitz matrix based on the qth FIR filter with its (k,m)th entry being $[\mathbf{H}_q^{(a)}]_{k,m} = h_{q,k-m}^{(a)}$.

Let us focus on the composition of the precoder $\mathbf{\Theta}$, which is defined in [4] [7, Section IV.A] as

$$\mathbf{\Theta} := (\mathbf{F}_N^H \mathbf{T}_1) \otimes \mathbf{T}_2, \quad (5)$$

with $\mathbf{F}_N$ being a unitary N-point FFT matrix with entries $[\mathbf{F}_N]_{p,q} := \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}(p-1)(q-1)}$, $\mathbf{T}_1 := [\mathbf{I}_{N-Q}, \mathbf{0}_{(N-Q)\times Q}]^T$ and $\mathbf{T}_2 := [\mathbf{I}_{M-L}, \mathbf{0}_{(M-L)\times L}]^T$. Hence, with the precoder $\mathbf{\Theta}$, the data sequence is in fact transformed into M coded OFDM symbols, each consisting of N tones. The zeros in $\mathbf{T}_1$ introduce a Q-long guard band in the frequency domain and the zeros in $\mathbf{T}_2$ introduce an L-long guard interval in the time domain. These guard bands (intervals) are sufficient, according to [4], to enable the maximum diversity of a CE-BEM channel.

Similar to the conventional OFDM system, we launch an $NM \times NM$ decoding matrix $\mathbf{\Psi}$ at the receive side, which is defined as

$$\mathbf{\Psi} := \mathbf{F}_N \otimes \mathbf{I}_M. \quad (6)$$

Integrating the effects of the precoder, the channel matrix and the decoder, we can express the ath input to the equalizer in Fig. 1 as

$$\begin{aligned}\bar{\mathbf{y}}^{(a)} &= \mathbf{\Psi}\mathbf{y}^{(a)} \\ &= \mathbf{\Psi}\mathbf{H}^{(a)}\mathbf{\Theta}\mathbf{s} + \bar{\mathbf{w}}^{(a)},\end{aligned} \quad (7)$$

where $\bar{\mathbf{y}}^{(a)}$ consists of in total NM decoded samples $\bar{\mathbf{y}}^{(a)} := [\bar{y}^{(a)}[0], \cdots, \bar{y}^{(a)}[NM-1]]^T$ and $\bar{\mathbf{w}}^{(a)}$ is similarly defined as $\bar{\mathbf{y}}^{(a)}$.

The precoding and decoding operations actually enable us to split the large-size $\mathbf{H}^{(a)}$ up into smaller matrices. This can be visualized by partitioning the data symbol sequence $\mathbf{s}$ into $N-Q$ blocks of length $M-L$ such that the ith data block is $\mathbf{s}(i) := [s[i(M-L)], ..., s[(i+1)(M-L)-1]]^T$ for $0 \le i \le N-Q-1$, and the decoded sequence $\bar{\mathbf{y}}^{(a)}$ into N blocks of length M such that the jth decoded block is $\bar{\mathbf{y}}^{(a)}(j) := [\bar{y}^{(a)}[jM], ..., \bar{y}^{(a)}[(j+1)M-1]]^T$, for $0 \le j \le N-1$. Assisted with these notations, we can rewrite (7), expressing the I/O relation on a block level as shown in [7, Section IV.A]:

$$\bar{\mathbf{y}}^{(a)}(i) = \sum_{q=0}^{Q} \bar{\mathbf{H}}_q^{(a)} \mathbf{s}(i-q) + \bar{\mathbf{w}}^{(a)}(i), \quad (8)$$

where $\bar{\mathbf{w}}^{(a)}(i)$ is similarly defined as $\bar{\mathbf{y}}^{(a)}(i)$ and $\bar{\mathbf{H}}_q^{(a)} := \mathbf{\Delta}_M^q \mathbf{H}_q^{(a)}$, with $\mathbf{H}_q^{(a)}$ being an $M \times (M-L)$ Toeplitz matrix with entries $[\mathbf{H}_q^{(a)}]_{k,m} := h_{q,k-m}^{(a)}$. Note that in (8) we assume $\mathbf{s}(i) = \mathbf{0}_{(M-L)\times 1}$ for $i < 0$ and $i \ge N-Q$, and $\bar{\mathbf{y}}^{(a)}(i) = \bar{\mathbf{w}}^{(a)}(i) = \mathbf{0}_{M\times 1}$ for $i < 0$ and $i \ge N$. It is worthy noting that the I/O relation rendered in (8) is analogous to a time invariant FIR on both matrix- and scalar-level.

3. PARAMETRIC EQUALIZER

In the following sections, we focus on the construction of a DFE for the equalizer part in Fig. 1. Its structure is depicted in Fig. 2: a set of linear feedforward filters, which process the decoded sequences $\bar{\mathbf{y}}^{(a)}$, are followed by a closed loop consisting of a decision device and a feedback filter. The decision device serves in many cases as a memoryless nonlinear quantizer $\mathcal{Q}(\cdot)$, which searches for the closest constellation point to its input signal $\hat{s}$: $\check{s} = \mathcal{Q}(\hat{s})$.

For the feedforward filters as well as the feedback filter, we recall that in case of conventional frequency-selective channels, they are often designed as a transversal filter corresponding to the FIR feature of the channel [5]. In the present situation, we observe in (8) that the effective channel is turned into a time-invariant FIR filter with $Q+1$ matrix taps; each of which is a frequency-shifted FIR filter with $L+1$ scalar taps. Let us call such a structure a 2-D FIR. Hence, analogous to the channel, we assign to the feedforward filters and the feedback filter also a 2-D FIR structure, i.e., an estimate of $\mathbf{s}(i)$ can be obtained as

$$\hat{\mathbf{s}}(i) = \sum_{a=1}^{A}\sum_{q_e=-Q_e}^{0} \bar{\mathbf{F}}_{q_e}^{(a)T} \bar{\mathbf{y}}^{(a)}(i - d_Q - q_e) - \sum_{q_b=0}^{Q_b} \bar{\mathbf{B}}_{q_b} \check{\mathbf{s}}(i - q_b). \quad (9)$$

with $\check{\mathbf{s}}(i)$ standing for the quantized estimate of $\mathbf{s}(i)$. In the above equation, the first term on the right-hand side represents the operation of the feedforward filters and the second term represents the operation of the feedback filter. We notice that each feedforward filter has $Q_e + 1$ matrix taps $\bar{\mathbf{F}}_{q_e}^{(a)}$ of size $M \times (M-L)$ for $q_e = 0, \cdots, -Q_e$. The parameter d_Q is included to denote the matrix-level delay for the feedforward filters, and thus determines which decoded blocks $\bar{\mathbf{y}}^{(a)}(i)$ should be fed to the equalizer. Apparently, d_Q must satisfy $-Q \le d_Q \le Q_e$ to ensure that $\mathbf{s}(i)$ is present in the equalizer input.

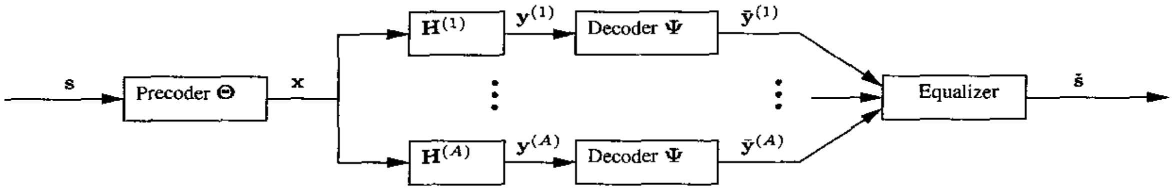

Fig. 1: *The transceiver diagram of a precoded SIMO system*

To stick to the 2-D FIR structure, we let each filter tap $\bar{\mathbf{F}}_{q_e}^{(a)}$ be the product of a diagonal exponential matrix $\boldsymbol{\Delta}_M^{q_e+d_Q}$ and an $M \times (M-L)$ Toeplitz matrix $\mathbf{F}_{q_e}^{(a)}$:

$$\bar{\mathbf{F}}_{q_e}^{(a)} := \boldsymbol{\Delta}_M^{q_e+d_Q}\mathbf{F}_{q_e}^{(a)}, \tag{10}$$

where $[\mathbf{F}_{q_e}^{(a)}]_{k,m} = [\mathbf{f}_{q_e}^{(a)}]_{k-m+d_L+1}$, with $\mathbf{f}_{q_e}^{(a)}$ denoting the $L_e + 1$ taps of the q_eth FIR filter for the ath channel output:

$$\mathbf{F}_{q_e}^{(a)} := \begin{bmatrix} [\mathbf{f}_{q_e}^{(a)}]_{d_L+1}\cdots & [\mathbf{f}_{q_e}^{(a)}]_1 & & \mathbf{0} \\ \vdots & \ddots & \ddots & \\ [\mathbf{f}_{q_e}^{(a)}]_{L_e+1} & & \ddots & [\mathbf{f}_{q_e}^{(a)}]_1 \\ & \ddots & \ddots & \vdots \\ \mathbf{0} & & [\mathbf{f}_{q_e}^{(a)}]_{L_e+1}\cdots & [\mathbf{f}_{q_e}^{(a)}]_{L+d_L+1} \end{bmatrix}. \tag{11}$$

Like Q_e, L_e defines the scalar-level order of the filter; The parameter d_L in the subscript is the scalar-level delay and should satisfy $-L \le d_L \le L_e$ to avoid all-zero columns in $\mathbf{F}_{q_e}^{(a)}$.

A similar composition can be observed in the feedback filter, i.e., it is assigned $Q_b + 1$ matrix taps $\bar{\mathbf{B}}_{q_b}$ for $q_b = 0, \cdots, Q_b$, each of which is again the product of a diagonal exponential matrix $\boldsymbol{\Delta}_{M-L}^{q_b}$ and an $(M-L) \times (M-L)$ Toeplitz matrix $\mathbf{B}_{q_b}$:

$$\bar{\mathbf{B}}_{q_b} := \boldsymbol{\Delta}_{M-L}^{q_b}\mathbf{B}_{q_b}, \tag{12}$$

where the (k,m)th entry of $\mathbf{B}_{q_b}$ is $[\mathbf{B}_{q_b}]_{k,m} = [\mathbf{b}_{q_b}]_{k-m+d_b+1}$, with $\mathbf{b}_{q_b}$ being an $(L_b+1) \times 1$ vector. Like d_L in (11), d_b is the delay parameter intrinsic to the feedback filter and should satisfy $0 \le d_b \le L_b$. Different from the feedforward filters, the feedback filter must take the channel causality into account, which implies that $[\mathbf{b}_0]_i = 0$ for $i = 1, \cdots, d_b + 1$.

The final estimate is the quantized result of (9):

$$\check{\mathbf{s}}(i) = \mathcal{Q}(\hat{\mathbf{s}}(i)). \tag{13}$$

From the above descriptions, we understand that the equalizer is characterized by $A(L_e+1)(Q_e+1) + Q_b(L_b+1) + L_b - d_b$ coefficients, which we put together in one vector: $\mathbf{v} := [\mathbf{f}^T, \mathbf{b}^T]^T$, with $\mathbf{f}$ representing the stack of the feedforward filter taps: $\mathbf{f} := [\mathbf{f}_0^{(1)T}, \cdots, \mathbf{f}_{-Q_e}^{(1)T}, \cdots, \mathbf{f}_{-Q_e}^{(A)T}]^T$, and $\mathbf{b}$ representing the stack of the feedback filter taps: $\mathbf{b} := [\mathbf{b}_{Q_b}^T, \cdots, \mathbf{b}_0^T]^T$. This notation enables us to rewrite (9) as an explicit function of $\mathbf{v}$:

$$\hat{\mathbf{s}}(i) = \tilde{\boldsymbol{\mathcal{U}}}(i)\mathbf{v}, \tag{14}$$

where $\tilde{\boldsymbol{\mathcal{U}}}(i)$ denotes the equalizer input, which consists of the feedforward filter input $\tilde{\boldsymbol{\mathcal{Y}}}(i)$ as well as the feedback filter input $\tilde{\boldsymbol{\mathcal{S}}}(i)$:

$$\tilde{\boldsymbol{\mathcal{U}}}(i) := [\tilde{\boldsymbol{\mathcal{Y}}}(i)^T, -(\tilde{\boldsymbol{\mathcal{S}}}(i))^T]. \tag{15}$$

It can be verified that $\tilde{\boldsymbol{\mathcal{Y}}}(i)$ has the expression

$$\tilde{\boldsymbol{\mathcal{Y}}}(i) = \begin{bmatrix} e^{-j\omega_{d_Q}}\boldsymbol{\Delta}_{L_e+1}^{d_Q}\boldsymbol{\mathcal{Y}}^{(1)}(i-d_Q)\boldsymbol{\Delta}_{M-L}^{d_Q} \\ \vdots \\ e^{-j\omega_{d_Q-Q_e}}\boldsymbol{\Delta}_{L_e+1}^{d_Q-Q_e}\boldsymbol{\mathcal{Y}}^{(1)}(i-d_Q+Q_e)\boldsymbol{\Delta}_{M-L}^{d_Q-Q_e} \\ \vdots \\ e^{-j\omega_{d_Q}}\boldsymbol{\Delta}_{L_e+1}^{d_Q}\boldsymbol{\mathcal{Y}}^{(A)}(i-d_Q)\boldsymbol{\Delta}_{M-L}^{d_Q} \\ \vdots \\ e^{-j\omega_{d_Q-Q_e}}\boldsymbol{\Delta}_{L_e+1}^{d_Q-Q_e}\boldsymbol{\mathcal{Y}}^{(A)}(i-d_Q+Q_e)\boldsymbol{\Delta}_{M-L}^{d_Q-Q_e} \end{bmatrix}, \tag{16}$$

where $\omega_q := 2\pi d_L q/P$ and $\boldsymbol{\mathcal{Y}}^{(a)}(i)$ denotes an $(L_e+1) \times (M-L)$ Hankel matrix with entries $[\boldsymbol{\mathcal{Y}}^{(a)}(i)]_{m,n} := [\bar{\mathbf{y}}^{(a)}(i)]_{m+n-d_L-1}$.

Similarly $\tilde{\boldsymbol{\mathcal{S}}}(i)$ can be expressed as

$$\tilde{\boldsymbol{\mathcal{S}}}(i) = \left[(\boldsymbol{\mathcal{S}}(i-Q_b)\boldsymbol{\Delta}_{M-L}^{Q_b})^T, \cdots, \boldsymbol{\mathcal{S}}(i)\right]^T, \tag{17}$$

with $\boldsymbol{\mathcal{S}}(i-q)$ being an $(L_b+1) \times (M-L)$ matrix, whose (m,n)th entry is $[\boldsymbol{\mathcal{S}}(i-q)]_{m,n} := [\check{\mathbf{s}}(i-q)]_{m+n-L_b+d_b-1}$.

Assuming that the channel state information (the coefficients of the CE-BEM channel in (1)) is known, there are several approaches to compute $\mathbf{v}$ in terms of different criteria. Examples of an MMSE solution and a zero-forcing solution can be found in [8], which we will not repeat here for the derivation is quite lengthy, but which we will apply in the simulation section.

4. SEMIBLIND EQUALIZATION

In the previous section, we have described how to build an equalizer with a limited number of coefficients. It has also been mentioned that to compute these coefficients, channel knowledge is indispensable. Often, this knowledge is not available or becomes soon obsolete for a fast varying channel. In such cases, direct channel estimation is less appealing, because to reduce the modeling error it is often desired to adopt a larger CE-BEM model (thus a bigger value of Q), which will inevitably increase the estimation difficulty. In this section, we propose a direct semi-blind equalization approach with the assistance of pilot symbols. The basic idea is to start with the pilots to acquire an estimate of the equalizer; in the subsequent iterations, we then refine the equalizer and the data symbol estimates interactively. In order to avoid the prohibitively large-size matrix inversion involved at each iteration, we employ the Normalized Least Mean Squares (NLMS) technique [9] to update the equalizer taps adaptively.

Prior to proceeding, let us first define $[\hat{\mathbf{s}}(i)]_j^{(k)}$ as the estimate of the jth data symbol of the ith block that is obtained at the kth iteration before quantization. The relationship between the indices

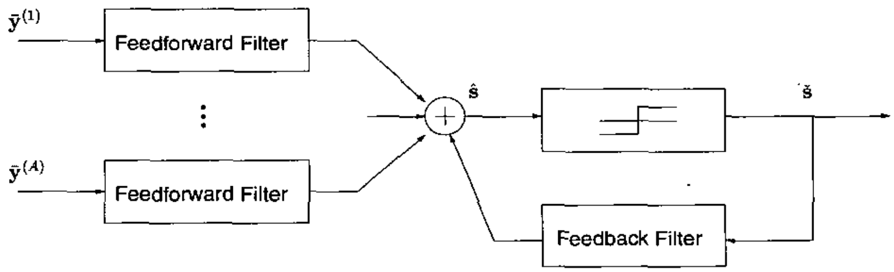

Fig. 2: *The block diagram of a decision feedback equalizer*

i, j and k will become more clear later on. Hence, if $\mathbf{v}^{(k)}$ is the vector of equalizer taps at the kth iteration, we have

$$[\hat{\mathbf{s}}(i)]_j^{(k)} = \tilde{\mathbf{u}}_{i,j}^{(k)T}\mathbf{v}^{(k)}, \quad (18)$$

with $\tilde{\mathbf{u}}_{i,j}^{(k)T}$ denoting the jth row of the equalizer input matrix $\tilde{\mathcal{U}}^{(k)}(i)$. The latter is similarly defined as in (15):

$$\tilde{\mathcal{U}}^{(k)}(i) := [\tilde{\mathcal{Y}}(i)^T, -(\bar{\mathcal{S}}^{(k)}(i))^T], \quad (19)$$

but different than (15), we use in the above expression a superscript k to underline the fact that the equalizer input (especially the feedback filter input) is constantly subject to updates from the previous iterations. Accordingly, (17) must be modified to:

$$\bar{\mathcal{S}}^{(k)}(i) = [(\mathcal{S}^{(k)}(i-Q_b)\mathbf{\Delta}_{M-L}^{Q_b})^T, \cdots, \mathcal{S}^{(k)}(i)]^T, \quad (20)$$

with $[\mathcal{S}^{(k)}(i-q)]_{m,n} := [\check{\mathbf{s}}(i-q)]_{m+n-L_b+d_b-1}^{(\tilde{k})}$. Here, $[\check{\mathbf{s}}(i-q)]_p^{(\tilde{k})}$ represents the quantized estimate of the pth symbol of the $(i-q)$th block that is obtained at the $\tilde{k}$th iteration: $[\check{\mathbf{s}}(i-q)]_p^{(\tilde{k})} = \mathcal{Q}([\hat{\mathbf{s}}(i-q)]_p^{(\tilde{k})})$. With a bit of confusion here, we use the index $\tilde{k}$ to indicate the iterations previous to k. Obviously, $\tilde{k}$ is linked with the present iteration index k as $\tilde{k} = k - j - q(M-L) + p$, as we recall that $M-L$ is the size of the symbol block.

We clarify now the relationship between the block index i, the symbol index j, and the iteration index k. (18) actually suggests that the iterations run on a symbol level, therefore, the indices j and k are incremented simultaneously by the same step. On the other hand, because of the rapidly varying nature of the channels, there are in general only a small amount of data samples available for adaptation. Hence, it might be helpful to run the iterative procedure over the whole symbol sequence for multiple loops. Thus we must associate the iteration index k with the block index i and the symbol index j as follows:

$$\mathrm{mod}(k, (M-L)(N-Q)) = i(M-L) + j, \quad (21)$$

for $i = 0, \cdots, N-Q-1$ and $j = 1, \cdots, M-L$.

Armed with the above notations and assuming the first N_t symbol blocks are pilots, we are able to launch the NLMS, which is aimed at minimizing the quantization error defined as $\mathcal{J} = |[\hat{\mathbf{s}}(i)]_j^{(k)} - [\check{\mathbf{s}}(i)]_j^{(k)}|^2$. Note that the NLMS operates in a decision-directed mode if the information symbol is not a pilot. It is easy to check that the derivative of $\mathcal{J}$ with respect to the equalizer taps $\mathbf{v}^{(k)}$ can be expressed as:

$$\frac{\partial \mathcal{J}}{\partial \mathbf{v}^{(k)}} = ([\check{\mathbf{s}}(i)]_j^{(k)} - \tilde{\mathbf{u}}_{i,j}^{(k)T}\mathbf{v}^{(k)})\tilde{\mathbf{u}}_{i,j}^{(k)*}, \quad (22)$$

which leads to the following updating formula:

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \frac{\mu}{\|\tilde{\mathbf{u}}_{i,j}^{(k)}\|^2}([\check{\mathbf{s}}(i)]_j^{(k)} - \tilde{\mathbf{u}}_{i,j}^{(k)T}\mathbf{v}^{(k)})\tilde{\mathbf{u}}_{i,j}^{(k)*}, \quad (23)$$

where $\|\cdot\|$ denotes the Frobenius norm and μ is a small positive value satisfying the convergence requirement.

We observe from the above description that although we interpret the receiver as a 2-D FIR filter, the structure of the channel is not explicitly assumed, which is indeed no longer crucial to this case. More strongly, we can apply this semi-blind algorithm without assuming any specific channel model.

5. SIMULATION RESULTS

We present simulations for a SIMO transmission using one transmit antenna and two receive antennas. Further, the signals from each receive antenna are oversampled by a factor of two. We thus obtain four outputs.

We apply the proposed equalizer to a time-varying channel, which has a normalized maximum Doppler spread $f_{max} \approx 1/400$. Jakes' model is utilized to generate the channel taps. In order to approximate the Jakes' channel tightly with a CE-BEM for an observation window of length $P = 400$, we set $Q = 2$ to satisfy the Nyquist criterion: $Q/(2P) \geq f_{max}$. Besides, we assume that the channel has a memory length of $L = 3$.

We generate a zero-mean white symbol sequence $s[n]$, which is comprised of $N - Q = 18$ blocks with each block containing $M - L = 17$ QPSK symbols. We assume the noise to be AWGN and uncorrelated with the information symbols. The obtained BER results are averaged over 1000 Monte Carlo runs, where in each run we generate a different channel, noise and data realization.

Test case 1. Equalization based on channel knowledge: We test the proposed equalizer with parameters $[L_e, Q_e, L_b, Q_b] = [9, 8, 12, 2]$, whose taps are obtained based on the CE-BEM coefficients (see e.g. [8]). We compare the performance in Fig. 3 with two Block Minimum Mean Square Error (BMMSE)-DFEs [5], whose taps are computed based on the knowledge of the true Jakes' channel and the CE-BEM coefficients, respectively. Although the last two equalizers yields a better performance, the complexity is high $\mathcal{O}(((N-Q)(M-L))^3) = \mathcal{O}(316^3)$. The proposed DFE, whose complexity is cubic in its size $\mathcal{O}(((L_e + L + 1)(Q_e + Q + 1))^3) = \mathcal{O}(143^3)$, offers an outstanding trade-off between complexity and performance.

Note that there is a performance gap between the BMMSE-DFE that are based on the true Jakes' channel and the equalizers that is based on the CE-BEM, which results from the channel modeling error. However, this gap can be further decreased by adopting a larger CE-BEM, e.g., if we take $Q = 4$.

Test case 2. Semi-blind equalization:, We implement for this test case a DFE with parameters $[L_e, Q_e, L_b, Q_b] = [3, 2, 2, 2]$. Besides, we assume $N_t = 1$ symbol block contains pilots, which, along with the zeros introduced in the precoder, results in a bandwidth efficiency of 71%. Fig. 4 shows that the semi-blind DFE suffers from a BER floor at high SNR. This problem is alleviated if the system can afford a bigger overhead, e.g., $N_t = 2$, equivalent to a bandwidth efficiency of 68%.

As we mentioned in Sec. IV that the iteration procedure will run over the data symbol sequence for several loops, Fig. 5 depicts the relationship between the SNR and the number of loops necessary to reach convergence. Since the complexity of the semi-blind DFE is linear in the number of loops, for an SNR higher than 12dB, the complexity is thus roughly $\mathcal{O}(316 \times 3)$, which is cheaper than the equalizer with channel knowledge.

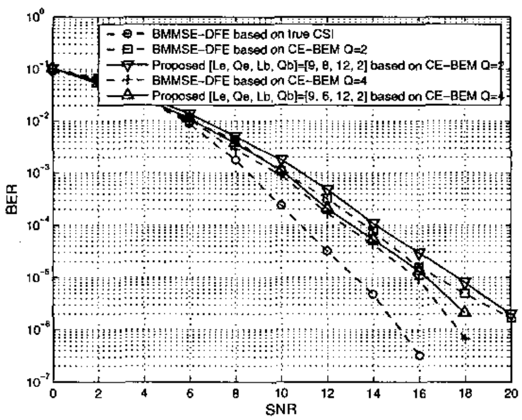

Fig. 3: *Equalization with CSI*

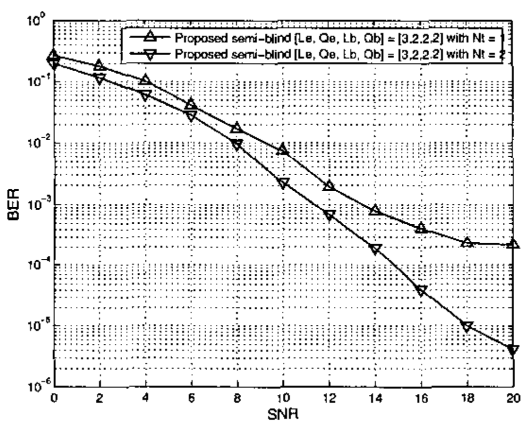

Fig. 4: *Semiblind equalization*

6. CONCLUSION

In this paper we have proposed a receiver architecture for maximum diversity transmissions over CE-BEM channels. We focused on a DFE that is similar in structure as the effective channel, and thus engages only a limited number of taps. The value of these taps are either computed based on channel knowledge or estimated using an adaptive filter. Simulation results reveal that the proposed DFE yields an outstanding trade-off between performance and complexity.

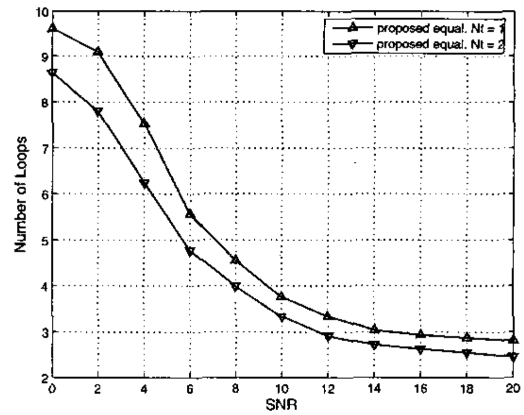

Fig. 5: *Average number of loops until convergence*

7. REFERENCES

[1] W. C. Jakes, *Microwave Mobile Channels.* New York: Wiley, 1974.

[2] M. K. Tsatsanis and G. B. Giannakis, "Modeling and equalization of rapidly fading channels," *International Journal of Adaptive Control and Signal Processing*, vol. 10, pp. 159–176, Mar. 1996.

[3] T. Zemen and C. F. Mechlenbräuker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *accepted by IEEE Transactions on Signal Processing.*

[4] X. Ma and G. B. Giannakis, "Maximum-diveristy transmissions over doubly selective wireless channels," *IEEE Transactions on Information Theory*, vol. 49, pp. 1832–1840, July 2003.

[5] N. Al-Dhahir and J. M. Cioffi, "MMSE decision-feedback equalizers: finite-length results," *IEEE Transactions on Information Theory*, vol. 41, pp. 961–975, Jul 1995.

[6] G. Leus, I. Barhumi, O. Rousseaux, and M. Moonen, "Time-varying FIR decision feedback equalization of doubly-selective channels," *IEEE Global Telecommunications Conference, GLOBECOM*, Dec. 2003.

[7] C. Tepedelenlioğlu and G. B. Giannakis, "Transmitter redundancy for blind estimation and equalization of time- and frequency-selective channel," *IEEE Transactions on Signal Processing*, vol. 48, pp. 2029–2043, July 2000.

[8] Z. Tang and G. Leus, "Low-complexity equalization of time-varying channels with precoding," *submitted to IEEE Transactions on Signal Processing.*

[9] S. Haykin, *Adaptive filter theory.* Prentice-Hall, 1996.