ON THE ESTIMATION OF RAPIDLY TIME-VARYING CHANNELS

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ABSTRACT

Relying on the basis expansion model (BEM) for rapidly time-varying channels, we propose a novel training-based BEM channel estimation approach. While the existing approach applies a rectangular window to the received sequence and employs a BEM with a period equal to the window length, the novel approach applies a general window to the received sequence and employs a BEM with a period equal to a multiple of the window length. Simulation results show that these extensions can significantly improve the channel estimation performance.

1. INTRODUCTION

In high-mobility wireless applications, rapidly time-varying channels may be encountered. Such channels have a coherence time that is of the same order as the symbol period, and thus can not be viewed as being invariant over a frame. Many equalization methods have already been developed for such channels. Most of them rely on an approximate yet accurate channel model, e.g., the basis expansion model (BEM) [7, 5, 2, 3]. We can distinguish between direct equalization methods and equalization methods that require knowledge of the BEM coefficients. The latter rely on a BEM channel estimation procedure, which can be blind [8, 5, 2, 4] or training-based [6].

In this paper, we only focus on training-based BEM channel estimation. The existing approach [6] applies a rectangular window to the received sequence and employs a BEM with a period equal to the window length, which corresponds to critically sampling the Doppler spectrum of the windowed channel. Since only a limited Doppler range of the windowed channel is considered for the channel estimation procedure, it might be important to reduce the sidelobes and/or to take more samples within that range. Therefore, we extend the existing approach by applying a general window to the received sequence and employing a BEM with a period equal to a multiple of the window length, which corresponds to oversampling the Doppler spectrum of the windowed channel.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts ^{T}, ^{H}, and † represent transpose, Hermitian, and pseudo-inverse, respectively. Continuous-time (discrete-time) variables are denoted as $x(\cdot)$ $(x[\cdot])$. We define the convolution operation as $\star$. Finally, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on the diagonal.

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2. DATA MODEL

We consider a baseband description of a wireless system with a single transmit and receive antenna. If we want to transmit a symbol sequence $x[n]$, it is first filtered by the transmit filter $g_{\mathrm{tr}}(t)$, distorted by the physical channel $g_{\mathrm{ch}}(t;\tau)$, corrupted by the additive noise $v(t)$, and finally filtered by the receive filter $g_{\mathrm{rec}}(t)$. The received signal $y(t)$ can then be written as

$$y(t) = \sum_{n=-\infty}^{\infty} g(t;t-nT)x[n] + w(t),$$

where T is the symbol period, $w(t) := g_{\mathrm{rec}}(t) \star v(t)$, and

$$g(t;\tau) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{\mathrm{rec}}(s)g_{\mathrm{tr}}(\tau-\theta-s)g_{\mathrm{ch}}(t-s;\theta)dsd\theta. \tag{1}$$

Sampling each receive antenna at symbol rate, the received sequence $y[n] := y(nT)$ can be written as

$$y[n] := \sum_{\nu=-\infty}^{\infty} g[n;\nu]x[n-\nu] + w[n], \tag{2}$$

where $w[n] := w(nT)$ and $g[n;\nu] := g(nT;\nu T)$.

Most wireless links experience multipath propagation, where clusters of reflected or scattered rays arrive at the receiver. All the rays within the same cluster experience the same (resolvable) delay, but each of them is characterized by its own complex gain and frequency offset. Hence, we can express the physical channel $g_{\mathrm{ch}}(t;\tau)$ as [1]

$$g_{\mathrm{ch}}(t;\tau) = \sum_c \delta(\tau-\tau_c)\sum_r G_{c,r}e^{j2\pi f_{c,r}t}, \tag{3}$$

where τ_c is the (resolvable) delay of the cth cluster, and $G_{c,r}$ and $f_{c,r}$ are respectively the complex gain and frequency offset of the rth ray of the cth cluster.

Assuming the time-variation of the physical channel $g_{\mathrm{ch}}(t;\tau)$ over the span of the receive filter $g_{\mathrm{rec}}(t)$ is negligible, we can replace $g_{\mathrm{ch}}(t-s;\theta)$ by $g_{\mathrm{ch}}(t;\theta)$ in (1), leading to

$$\begin{aligned} g(t;\tau) &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} g_{\mathrm{rec}}(s)g_{\mathrm{tr}}(\tau-\theta-s)ds\right)g_{\mathrm{ch}}(t;\theta)d\theta \\ &= \int_{-\infty}^{\infty}\psi(\tau-\theta)g_{\mathrm{ch}}(t;\theta)d\theta \\ &= \sum_c \psi(\tau-\tau_c)\sum_r G_{c,r}e^{j2\pi f_{c,r}t}, \end{aligned}$$

where $\psi(t) := g_{\mathrm{rec}}(t) \star g_{\mathrm{tr}}(t)$. This means that $g[n;\nu]$ can be expressed as

$$
\begin{aligned}
g[n;\nu] &= g(nT;\nu T) \\
&= \sum_c \psi(\nu T - \tau_c) \sum_r G_{c,r} e^{j2\pi f_{c,r} nT}.
\end{aligned} \tag{4}
$$

For the sake of simplicity, we assume in the following that we have a single cluster with (resolvable) delay $\tau_0 = 0$ consisting of rays with complex gain $G_{0,r} = G_r$ and frequency offset $f_{0,r} = f_r$, and that $\psi(t)$ is a Nyquist filter. Then, (4) simplifies to

$$
g[n;\nu] = g[n] = \sum_r G_r e^{j2\pi f_r nT},
$$

and the received sequence can be written as

$$
y[n] = g[n]x[n] + w[n].
$$

3. EXISTING CHANNEL ESTIMATION

In the existing channel estimation approach [6], we apply a rectangular window $d_r[n]$ to $y[n]$, where

$$
d_r[n] = \begin{cases} 1 & \text{if } n = 0,1,\ldots,N-1 \\ 0 & \text{elsewhere} \end{cases},
$$

and we assume that the periodic extension with period N of the windowed channel $g_{w_r}[n] = d_r[n]g[n]$ can be modeled by a BEM with period N:

$$
g_{w_r}[n \bmod N] \approx h[n] = \frac{1}{N} \sum_{q=-Q}^{Q} h_q e^{j2\pi qn/N},
$$

where Q should be chosen such that $Q/(NT)$ is larger than the Doppler spread of the channel $Q/(NT) \geq f_{\max} = \max_r |f_r|$. Defining $\mathbf{g}_{w_r} = [g_{w_r}[0], \ldots, g_{w_r}[N-1]]^T$ and $\mathbf{h} = [h_{-Q}, \ldots, h_Q]^T$, the least squares fit for $\mathbf{h}$ is obtained by solving

$$
\min_{\mathbf{h}} \|\mathbf{g}_{w_r} - \frac{1}{N}\mathbf{F}\mathbf{h}\|^2,
$$

where

$$
\mathbf{F} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j2\pi Q/N} & \cdots & e^{j2\pi Q/N} \\ \vdots & & \vdots \\ e^{-j2\pi Q(N-1)/N} & \cdots & e^{j2\pi Q(N-1)/N} \end{bmatrix}.
$$

The solution of this problem is given by

$$
\hat{\mathbf{h}} = N\mathbf{F}^{\dagger}\mathbf{g}_{w_r} = \mathbf{F}^H \mathbf{g}_{w_r}. \tag{5}
$$

Note that this solution corresponds to taking the $2Q+1$ samples around zero from the critically sampled Doppler spectrum of the windowed channel $g_{w_r}[n]$.

In practice, only a few pilot symbols $x[n_p]$, $p = 0,1,\ldots,P-1$, are known within the window $n = 0,1,\ldots,N-1$, and thus only a few entries of $\mathbf{g}_{w_r}$ can be estimated using the windowed output $y_{w_r}[n] = d_r[n]y[n]$. More specifically, defining $\tilde{\mathbf{g}}_{w_r} = [g_{w_r}[n_0], \ldots, g_{w_r}[n_{P-1}]]^T$, an estimate of $\tilde{\mathbf{g}}_{w_r}$ can be obtained as $\hat{\tilde{\mathbf{g}}}_{w_r} = [y_{w_r}[n_0]/x[n_0], \ldots, y_{w_r}[n_{P-1}]/x[n_{P-1}]]^T$. The BEM coefficients $\mathbf{h}$ are then computed by solving

$$
\min_{\mathbf{h}} \|\hat{\tilde{\mathbf{g}}}_{w_r} - \frac{1}{N}\tilde{\mathbf{F}}\mathbf{h}\|^2,
$$

where

$$
\tilde{\mathbf{F}} = \begin{bmatrix} e^{-j2\pi Q n_0/N} & \cdots & e^{j2\pi Q n_0/N} \\ \vdots & & \vdots \\ e^{-j2\pi Q n_{P-1}/N} & \cdots & e^{j2\pi Q n_{P-1}/N} \end{bmatrix},
$$

resulting in the following estimate:

$$
\hat{\mathbf{h}} = N\tilde{\mathbf{F}}^{\dagger}\hat{\tilde{\mathbf{g}}}_{w_r}. \tag{6}
$$

It has been shown in [6] that for this channel estimate an equispaced positioning for the pilot symbols is optimal.

4. PROPOSED CHANNEL ESTIMATION

Since only a limited Doppler range of the windowed channel is considered for the channel estimation procedure, it might be beneficial to reduce the sidelobes and/or to take more samples within that range. That is why we will apply a general window to the received sequence and employ a BEM with a period equal to a multiple of the window length, which corresponds to oversampling the Doppler spectrum of the windowed channel. This is discussed in more detail next.

We apply a general window $d[n]$ to $y[n]$, where

$$
d[n] = \begin{cases} s[n] & \text{if } n = 0,1,\ldots,L-1 \\ 1 & \text{if } n = L, L+1, \ldots, N-L-1 \\ s[N-1-n] & \text{if } n = N-L, N-L+1, \ldots, N-1 \\ 0 & \text{elsewhere} \end{cases},
$$

and we assume that the periodic extension with period KN of the windowed channel $g_w[n] = d[n]g[n]$ can be modeled by a BEM with period KN:

$$
g_w[n \bmod KN] \approx h^{(K)}[n] = \frac{1}{KN} \sum_{q=-Q}^{Q} h_q^{(K)} e^{j2\pi qn/(KN)},
$$

where Q should be chosen such that $Q/(KNT)$ is larger than the Doppler spread of the channel $Q/(KNT) \geq f_{\max} = \max_r |f_r|$. Defining $\mathbf{g}_w^{(K)} = [g_w[0], \ldots, g_w[KN-1]]^T$ and $\mathbf{h}^{(K)} = [h_{-Q}^{(K)}, \ldots, h_Q^{(K)}]^T$, the least squares fit for $\mathbf{h}^{(K)}$ is obtained by solving

$$
\min_{\mathbf{h}^{(K)}} \|\mathbf{g}_w^{(K)} - \frac{1}{KN}\mathbf{F}^{(K)}\mathbf{h}^{(K)}\|^2, \tag{7}
$$

where

$$
\mathbf{F}^{(K)} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j2\pi Q/(KN)} & \cdots & e^{j2\pi Q/(KN)} \\ \vdots & & \vdots \\ e^{-j2\pi Q(KN-1)/(KN)} & \cdots & e^{j2\pi Q(KN-1)/(KN)} \end{bmatrix}.
$$

The solution of this problem is given by

$$
\hat{\mathbf{h}}^{(K)} = KN\mathbf{F}^{(K)\dagger}\mathbf{g}_w^{(K)} = \mathbf{F}^{(K)H}\mathbf{g}_w^{(K)}. \tag{8}
$$

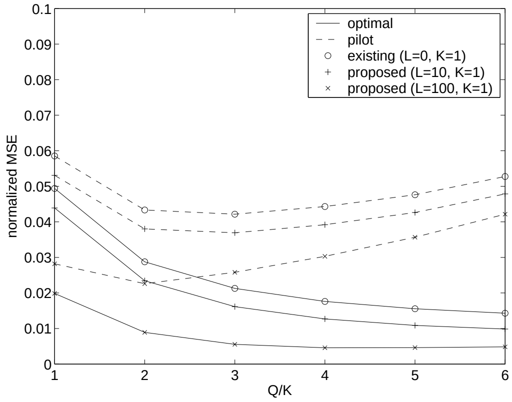

Figure 1: Effect of increasing the smoothness of the window when critical sampling is employed.

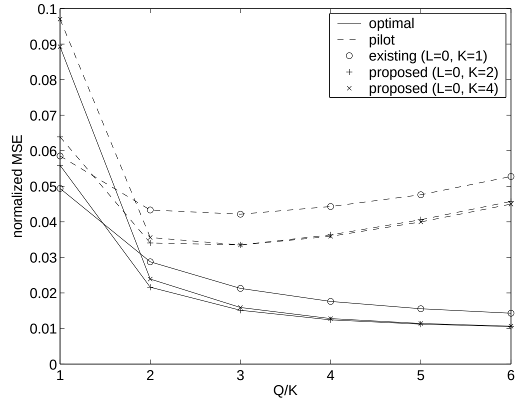

Figure 2: Effect of increasing the oversampling factor when a rectangular window is employed.

Note that this solution corresponds to taking the $2Q+1$ samples around zero from the K times oversampled Doppler spectrum of the windowed channel $g_w[n]$.

At this point, it is convenient to rewrite (7) in a different fashion. Defining $\mathbf{g}_w = [g_w[0], \ldots, g_w[N-1]]^T$ and $\mathbf{h}_k^{(K)} = [h^{(K)}_{-a_kK+k}, h^{(K)}_{(-a_k+1)K+k}, \ldots, h^{(K)}_{(b_k-1)K+k}, h^{(K)}_{b_kK+k}]$, with $a_k = (Q+k) \bmod K$ and $b_k = (Q-k) \bmod K$, for $k = 0, 1, \ldots, K-1$, it is easy to show that the solution for $\mathbf{h}_k^{(K)}$ of (7) is obtained by solving

$$\min_{\mathbf{h}_k^{(K)}} \|\mathbf{D}_k^{(K)}\mathbf{g}_w - \frac{1}{N}\mathbf{F}_k^{(K)}\mathbf{h}_k^{(K)}\|^2,$$

where $\mathbf{D}_k^{(K)} = \operatorname{diag}\{[1, e^{-j2\pi k/(KN)}, \ldots, e^{-j2\pi k(N-1)/(KN)}]^T\}$ and

$$\mathbf{F}_k^{(K)} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j2\pi a_k/N} & \cdots & e^{j2\pi b_k/N} \\ \vdots & & \vdots \\ e^{-j2\pi a_k(N-1)/N} & \cdots & e^{j2\pi b_k(N-1)/N} \end{bmatrix}.$$

The solution of this problem is given by

$$\hat{\mathbf{h}}_k^{(K)} = N\mathbf{F}_k^{(K)\dagger}\mathbf{D}_k^{(K)}\mathbf{g}_w = \mathbf{F}_k^{(K)H}\mathbf{D}_k^{(K)}\mathbf{g}_w. \tag{9}$$

Note that this solution corresponds to taking the a_k+b_k+1 samples around zero from the critically sampled Doppler spectrum of the $k/(KN)$ frequency shifted version of the windowed channel $g_w[n]$. It is clear that all these samples together (for $k = 0, 1, \ldots, K-1$) indeed consist of the $2Q+1$ samples around zero from the K times oversampled Doppler spectrum of the windowed channel $g_w[n]$, which have been derived in (8).

As before, in practice, only a few pilot symbols $x[n_p]$, $p = 0, 1, \ldots, P-1$, are known within the window $n = 0, 1, \ldots, N-1$, and thus only a few entries of $\mathbf{g}_w$ can be estimated using the windowed output $y_w[n] = d[n]y[n]$. More specifically, defining $\tilde{\mathbf{g}}_w = [g_w[n_0], \ldots, g_w[n_{P-1}]]^T$, an estimate of $\tilde{\mathbf{g}}_w$ can be obtained as $\hat{\tilde{\mathbf{g}}}_w = [y_w[n_0]/x[n_0], \ldots, y_w[n_{P-1}]/x[n_{P-1}]]^T$. The BEM coefficients $\mathbf{h}_k^{(K)}$ are then computed by solving

$$\min_{\mathbf{h}_k^{(K)}} \|\tilde{\mathbf{D}}_k^{(K)}\hat{\tilde{\mathbf{g}}}_w - 1/N\tilde{\mathbf{F}}_k^{(K)}\mathbf{h}_k^{(K)}\|^2,$$

where $\tilde{\mathbf{D}}_k^{(K)} = \operatorname{diag}\{[e^{-j2\pi kn_0/(KN)}, \ldots, e^{-j2\pi kn_{P-1}/(KN)}]^T\}$ and

$$\tilde{\mathbf{F}}_k^{(K)} = \begin{bmatrix} e^{-j2\pi a_k n_0/N} & \cdots & e^{j2\pi b_k n_0/N} \\ \vdots & & \vdots \\ e^{-j2\pi a_k n_{P-1}/N} & \cdots & e^{j2\pi b_k n_{P-1}/N} \end{bmatrix},$$

resulting in the following estimate:

$$\hat{\mathbf{h}}_k^{(K)} = N\tilde{\mathbf{F}}_k^{(K)\dagger}\tilde{\mathbf{D}}_k^{(K)}\hat{\tilde{\mathbf{g}}}_w. \tag{10}$$

5. DISCUSSION

First of all, it is clear that the proposed approach generalizes the existing one. More specifically, if we take $L = 0$ (the general window becomes a rectangular window) and $K = 1$ (oversampling becomes critical sampling), the proposed approach is equivalent to the existing one.

In the proposed approach, the windowed channel $g_w[n]$ only corresponds to the true channel $g[n]$ in the range $n = L, L+1, \ldots, N-L-1$. As a result, we will only use the estimated BEM coefficients in that range. In other words, the performance criterion that we are interested in is

$$\frac{1}{N-2L}\sum_{n=L}^{N-L-1}\left|g[n] - \frac{1}{KN}\sum_{q=-Q}^{Q} h_q^{(K)} e^{j2\pi qn/(KN)}\right|^2. \tag{11}$$

If this is the criterion of interest, one could wonder why we solve (7) instead of minimizing the above criterion. The reason for this is that the resulting BEM coefficients are rather unpredictable and turn out to have a huge dynamic range. The BEM coefficients obtained in the previous section, on the other hand, are related to the Doppler spectrum of the windowed channel $g_w[n]$, and as a result, have a limited dynamic range.

For the proposed approach, many windows are possible. However, as long as $s[n]$ realizes a smooth transition from 0

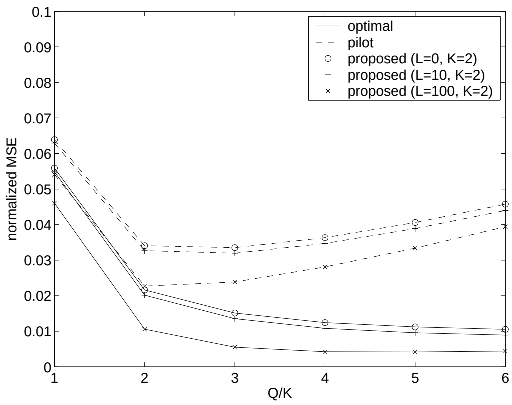

Figure 3: Effect of increasing the smoothness of the window when oversampling is employed.

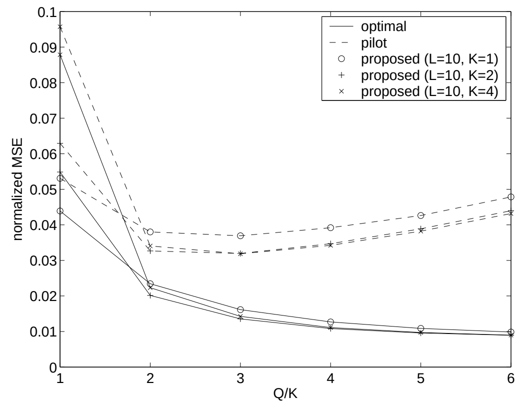

Figure 4: Effect of increasing the oversampling factor when a general window is employed.

to 1 for $n = 0, 1, \ldots, L-1$, there will not be a large difference in channel estimation performance. Therefore, we simply consider a cosine window: $s[n] = 1/2(1 - \cos(\pi(1/2 + n)/L)$.

6. SIMULATION RESULTS

In this section, we compare the proposed approach to the existing approach. We consider a channel consisting of 100 rays with complex gain $G_r = 1/\sqrt{100}$ and frequency offset $f_r = f_{\max}\cos(\theta_r)$, where θ_r is uniformly distributed in $[0, 2\pi)$. We further assume $N = 1000$ and $f_{\max}T = 1/1000$. In all simulations, we compare the existing approach with the proposed approach and consider the optimal solutions of (5) and (9) (indicated by 'optimal') as well as the pilot-based solutions of (6) and (10) (indicated by 'pilot'). The pilot-based solutions will rely on 100 pilot symbols that are distributed over the $N = 1000$ transmitted symbols in an equispaced fashion. In other words, we transmit a pilot symbol every other 9 data symbols. As a performance measure, we consider the normalized MSE of (11) versus Q/K at an SNR of 5 dB, where Q/K is an indication of the Doppler range considered in the channel estimation procedure.

First, we compare the existing approach ($L = 0$, $K = 1$) with the proposed approach taking $L = 10, 100$ and $K = 1$, i.e., we only change the smoothness of the window. From Figure 1, we observe that increasing the smoothness of the window significantly improves the performance for all values of Q/K.

Second, we compare the existing approach ($L = 0, K = 1$) with the proposed approach taking $L = 0$ and $K = 2, 4$, i.e., we only change the oversampling factor. From Figure 2, we observe that increasing the oversampling factor only improves the performance above an increasing value of Q/K and deteriorates the performance below this value. In addition, we see that the improvement saturates. Hence, it is not always beneficial to use a large oversampling factor. Note that when we would minimize criterion (11) directly, oversampling always leads to an improved performance, but as already indicated, we then obtain BEM coefficients that are rather unpredictable and turn out to have a huge dynamic range.

Finally, as can be observed in Figures 3 and 4, the effect of changing the smoothness of the window when oversampling is used is the same as when critical sampling is used, and the effect of changing the oversampling factor when a general window is used is the same as when a rectangular window is used.

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