# Joint Sensor Placement and Power Rating Selection in Energy Harvesting Wireless Sensor Networks

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***Abstract*—In this paper, the focus is on optimal sensor placement and power rating selection for parameter estimation in wireless sensor networks (WSNs). We take into account the amount of energy harvested by the sensing nodes, communication link quality, and the observation accuracy at the sensor level. In particular, the aim is to reconstruct the estimation parameter with minimum error at a fusion center under a system budget constraint. To achieve this goal, a subset of sensing locations is selected from a large pool of candidate sensing locations. Furthermore, the type of sensor to be placed at those locations is selected from a given set of sensor types (e.g., sensors with different power ratings). We further investigate whether it is better to install a large number of cheap sensors, a few expensive sensors or a combination of different sensor types at the optimal locations.**

***Index Terms*—Wireless sensor networks, sensor selection, convex optimization, energy harvesting, estimation.**

## I. INTRODUCTION

Advanced sensor networks are needed in order to meet the increasing needs of internet of things applications, such as automated surveillance, environmental monitoring, smart cities, and so on. To guarantee a durable autonomous sensor network, sensing nodes should be capable of processing and communicating data with restricted energy harvesting (EH) and consumption budgets.

Sensors are usually restricted to be placed in certain locations to protect them from defects or simply due to physical space constraints. Therefore, optimal sensor placement, i.e., to select the best subset of sensing locations out of a large set of available locations, keeping in mind the network infrastructure and the inference task, forms an important sensor network design task.

Sensor placement is a combinatorial problem, which can be solved optimally through an exhaustive search by evaluating a performance measure (e.g., inference accuracy, budget constraints) for all possible combinations of sensing locations. However, when the number of candidate locations is large, this process will be computationally intractable. Instead, a suboptimal solution can be obtained by greedily selecting sensors one by one. Such a greedy algorithm is near optimal, if the performance measure can be expressed as a submodular set function of the sensing locations [1], [2]. Alternatively, the sensor placement problem can be solved suboptimally using convex optimization [3], which utilizes the convexity of the cost and constraint functions to solve the optimization problem [4], [5].

The sensor selection problem can be performed either online or offline. In online sensor selection [6], real time measurements are usually collected and these are used to make decisions on whether to activate or deactivate sensors. In offline sensor selection [3], [5], the objective is to place a set of sensors such that a desired performance is met based on some prior statistics, which do not depend on real measurements. We focus on the offline sensor selection in this paper.

The goal of this paper is to combine optimal sensor placement with a novel and important dimension that adds the flexibility to the network designer, namely, to choose the type of sensor from a pool of available sensor types. For example, sensor types may have different modulation schemes, power budgets/battery capacities, quantization levels and costs. This gives network designers the flexibility to place more expensive sensors (with more power budget, quantization levels, data rate, etc.) in strategic locations while cheaper sensors (with cheaper batteries, fewer bits, and so on) in less important locations. In [3], [5] the sensing locations are selected based only on the measurement accuracy at the sensor level. We present in this work a solution for the sensor placement problem considering practical aspects such as the quality of the communication channel (between the sensors and the fusion center) and the available harvested energy at the candidate sensor location in addition to the measurement accuracy at the sensing nodes. Furthermore, at each selected location, we also select the type of sensor that needs to be placed based on its battery capacity. This design problem is solved for the parameter estimation task with estimation accuracy being the inference performance metric. A similar derivation for the error of the minimum mean squared error (MMSE) estimator is formulated in [7]. Nevertheless, only one type of sensor is considered in [7], which restricts the system flexibility. Moreover, the problem statement in this work is novel and accounts for a trade off between the system performance and

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the total budget. We further consider the changes of the amount of harvested energy over different snapshots of the day.

## II. System model

Consider a set of $N$ geographically distributed candidate sensing locations. It is assumed that a sensor can be placed at any of these sensing locations, and the sensors record data related to a physical phenomenon. Assuming a linear measurement model, we denote the observation at the $n$-th sensor by

$$x_n = \mathbf{h}_n^T \boldsymbol{\theta} + v_n, \tag{1}$$

where $\mathbf{h}_n \in \mathbb{R}^m$ is the regressor, $\boldsymbol{\theta} \in \mathbb{R}^m$ is the unknown parameter, which is assumed to be a Gaussian random vector, i.e., $\boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}})$, and $v_n$ is the sensor noise. We further assume that the sensor noise is Gaussian distributed with zero mean and variance $\sigma_v^2$, and that it is uncorrelated with the unknown parameter vector $\boldsymbol{\theta}$. At each sensor location, one of $K$ sensor types can be installed. Each sensor type has the same observation accuracy, however, it has a different EH capability and battery capacity. Thus, its price and physical size are also different.

All selected sensors send their observations to the fusion center through orthogonal additive white Gaussian noise (AWGN) channels. Figure 1 shows the system setup. Sensors are assumed to harvest enough energy to charge a small battery, which will be used for collecting measurements and transmitting them to the fusion center. The transmission power at the $n$-th sensing location is a function of the amount of power available at that location at the $t$-th time interval, $t \in \{1, \ldots, T\}$, as well as the EH efficiency and battery capacity of the $k$-th sensor type. To be more specific, the transmission power will be $p_{n,k}^{(t)} = f(\rho_n^{(t)}, \eta_k)$, where $\rho_n^{(t)}$ is the average power available at the $n$-th location and $t$-th time instance, and $\eta_k$ is the EH efficiency of the $k$-th sensor type. For instance, the transmission power can be formulated as, $p_{n,k}^{(t)} = \min(\rho_n^{(t)}\eta_k, b\eta_k)$, where $b$ is a positive constant representing an upper limit for EH. Figure 2 shows an illustration of the average EH intensity over the sensor locations at some time of the day, $\rho_n^{(t)}$ (more details are given in Section 5).

At the fusion center, the received signal from the $k$-th sensor type at the $n$-th location can be expressed as,

$$y_{n,k}^{(t)} \quad = \quad w_{n,k}\left(\frac{\sqrt{p_{n,k}^{(t)}}g_n x_n}{\sigma_{x(n)}} + \phi_n\right), \tag{2}$$

where $g_n$ is the wireless fading channel gain between the $n$-th sensor location and the fusion center, and $\phi_n$ is zero-mean Gaussian receiver noise with variance $\sigma_\phi^2$. The $\phi_n$ values are assumed to be uncorrelated with $v_n$ and $\boldsymbol{\theta}$. For simplicity, we assume $g_n$ is deterministic. Further, $w_{n,k}$ is a selection variable, where $w_{n,k} = 1$ indicates the placement of the $k$-th sensor type at the $n$-th sensing location and $w_{n,k} = 0$ indicates otherwise. To force the average transmitted power to $p_{n,k}^{(t)}$, the

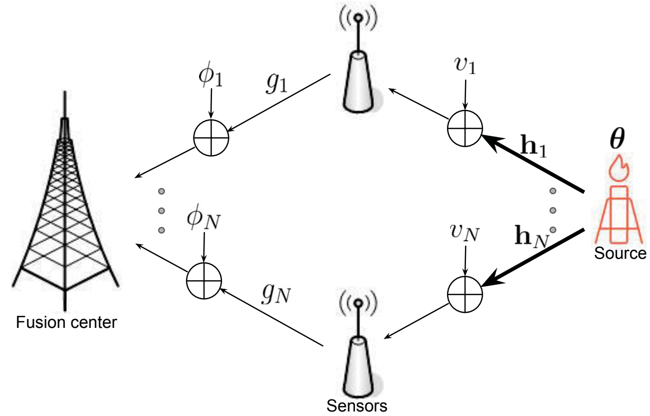


Fig. 1. System setup.

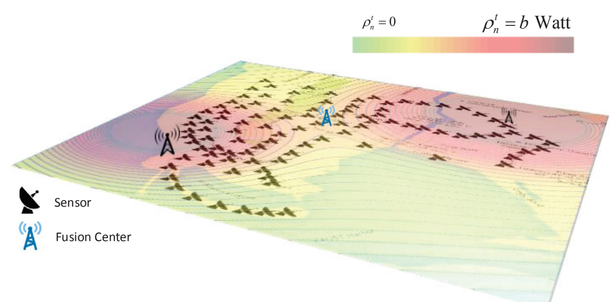


Fig. 2. Sensor placement and power map.

transmission signal is scaled by $\sigma_{x(n)}$, where $\sigma_{x(n)}^2$ denotes the average power of the measurement $x_n$ and is given by

$$\sigma_{x(n)}^2 = E\{|x_n|^2\} = E\{|\mathbf{h}_n^T\boldsymbol{\theta} + v_n|^2\} = \mathbf{h}_n^T \boldsymbol{\Sigma}_{\boldsymbol{\theta}} \mathbf{h}_n + \sigma_v^2. \tag{3}$$

It is assumed that $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, the AWGN channel gains and the receiver noise statistics are known at the fusion center. Also, the regressors $\mathbf{h}_n$ are assumed to be known[1]. We finally assume that the fusion center has the statistics of the average EH over time at each sensor location and type.

Based on the observations received at the fusion center given by (2), the unknown parameters can be reconstructed using the MMSE estimator. Denoting the MMSE estimate of $\boldsymbol{\theta}$ as $\hat{\boldsymbol{\theta}}$, the MMSE error covariance matrix at each time snapshot, $\boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{y}}^{(t)} = E\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\}$ is given by, [8]

$$\boldsymbol{\Sigma}_{\boldsymbol{\theta}|\mathbf{y}}^{(t)} = \left(\boldsymbol{\Sigma}_\theta^{-1} + \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{w_{n,k}}{\sigma_{q(n,k,t)}^2}\{\mathbf{h}_n\mathbf{h}_n^T\}\right)^{-1}. \tag{4}$$

[1]Some known methods (e.g., RSS, TOA, etc) can be used to localize the source provided some extra information is sent to the fusion center. These methods are beyond the scope of this work.

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Observe how the selection elements $\{w_{n,k}\}$ in the numerator affect the MMSE error. $\sigma^2_{q(n,k,t)}$ is the aggregate noise variance of the observation and receiver noises. From (1) and (2), the aggregate noise can be modeled as $q^{(t)}_{n,k} = v_n + \frac{\sigma_{x_n}\phi_n}{\sqrt{p^{(t)}_{n,k}}g_n}$. Therefore, $q^{(t)}_{n,k}$ is a zero mean Gaussian noise with variance formulated as,

$$\sigma^2_{q(n,k,t)} = \sigma^2_v + \frac{(\mathbf{h}_n^T \mathbf{\Sigma}_\theta \mathbf{h}_n + \sigma^2_v)\sigma^2_\phi}{g_n^2 p^{(t)}_{n,k}}. \tag{5}$$

## III. Problem statement

Given $N$ candidate sensing locations and $K$ sensor types with different EH capabilities, battery capacities and prices, we would like to jointly choose the optimal sensor locations and types that minimize the total cost of the sensor network subject to a prescribed reconstruction error at any time. Or equivalently, we want to minimize the maximum error at any time subject to a threshold on the cost. The reconstruction error is caused by the noisy measurements and the noisy communication channels between the sensors and the fusion center.

The reconstruction error is a function of the error covariance matrix. To guarantee a small reconstruction error, one might, for example, minimize the sum of the eigenvalues of the error covariance matrix (which is called the A-optimality criterion), denoted by $E^{(t)}(\mathbf{W}) = tr\{\mathbf{\Sigma}^{(t)}_{\boldsymbol{\theta}|\mathbf{y}}\} =$

$$E^{(t)}(\mathbf{W}) = tr\left(\mathbf{\Sigma}_{\boldsymbol{\theta}}{}^{-1} + \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{w_{n,k}}{\sigma^2_{q(n,k,t)}}\{\mathbf{h}_n\mathbf{h}_n^T\}\right)^{-1}, \tag{6}$$

where $\mathbf{W}$ is the selection matrix as defined later and $tr(.)$ is the trace of the matrix. Recall that the element $w_{n,k}$ is equal to one if the sensor at location $n$ and type $k$ is selected, otherwise, $w_{n,k} = 0$. We assume that no more than one sensor can be selected at any location. Therefore, the $\ell_0$ norm of the vector of all sensor types at the $n$-th location, $\mathbf{w}_n^T = [w_{n,1}, ..., w_{n,K}]$, is either equal to 1 or 0 based on whether a sensor at the $n$-th location is selected or not. To simplify this constraint, an auxiliary sensor type that represents no sensor selection, $k = 0$, is introduced such that its cost and EH efficiency are $c_0 = \eta_0 = 0$. This means that we will redefine $\mathbf{w}_n$ as $\mathbf{w}_n^T = [w_{n,0}, \dots, w_{n,K}]$. Hence, $||\mathbf{w}_n^T||_0 = 1$ always holds, where $w_{n,0} = 1$, if no sensor is selected at the $n$-location.

The selection matrix $\mathbf{W}$ is then defined as the matrix with $n$-th row given by $\mathbf{w}_n^T = [w_{n,0}, ..., w_{n,K}]$, $\forall n \in \{1, \ ..., N\}$. Finally, we define a cost vector $\mathbf{c} = [c_0 \ c_1 \ ... \ c_K]^T$ and an EH efficiency vector $\boldsymbol{\eta} = [\eta_0 \ \eta_1 \ ... \ \eta_K]^T$ such that $c_k$ and $\eta_k$, $\forall k \in \{0, ..., K\}$ correspond to the given price and EH efficiency of sensor type $k$, respectively.

## IV. Joint sensor placement and type selection

We start the analysis considering only one snapshot of the average EH at each candidate sensing location, i.e., $t = 1$. The optimization problem can then be formulated as,

$$\begin{aligned} \arg\min_{\{w_{n,k}\}} \quad & \sum_{n=1}^{N} \mathbf{w}_n^T \mathbf{c} && (7)\\ \text{subject to} \quad & E^{(t)}(\mathbf{W}) \le \lambda && (7a)\\ & w_{n,k} \in \{0,1\}, \quad \forall n,k && (7b)\\ & ||\mathbf{w}_n||_0 = 1, \quad \forall n, && (7c) \end{aligned}$$

where $\lambda$ is the maximum allowed error at any time. The optimization problem (7) is not convex because of the non-convex Boolean constraints in (7b) and the $\ell_0$ norm constraints in (7c). To obtain a convex problem which can be solved using well established tools, the constraints (7b) are relaxed to $w_{n,k} \in [0,1]$, $\forall n,k$ and the constraints (7c) are relaxed to $\mathbf{w}_n^T\mathbf{1} = 1$, $\forall n$. The convex problem is then written as,

$$\begin{aligned} \arg\min_{\{w_{n,k}\}} \quad & \sum_{n=1}^{N} \mathbf{w}_n^T \mathbf{c} && (8)\\ \text{subject to} \quad & E^{(t)}(\mathbf{W}) \le \lambda && (8a)\\ & w_{n,k} \in [0,1], \quad \forall n,k && (8b)\\ & \mathbf{w}_n^T\mathbf{1} = 1, \quad \forall n. && (8c) \end{aligned}$$

Equivalently, the reconstruction error can be minimized subject to a prescribed system cost, $\xi$, which may be beneficial for the applications in which the goal is to minimize the error, i.e.,

$$\begin{aligned} \arg\min_{\{w_{n,k}\}} \quad & E^{(t)}(\mathbf{W}) && (9)\\ \text{subject to} \quad & \sum_{n=1}^{N} \mathbf{w}_n^T \mathbf{c} \le \xi && (9a)\\ & w_{n,k} \in [0,1], \quad \forall n,k && (9b)\\ & \mathbf{w}_n^T\mathbf{1} = 1, \quad \forall n. && (9c) \end{aligned}$$

In general, the solution of (8) and (9) can be $0 \le w_{n,k} \le 1$, $\forall n,k$. Hence, some rounding algorithm should be used to obtain a Boolean solution [5], [3].

*Multiple snapshots:* The amount of harvested energy is generally not fixed over time, therefore, it is useful to choose the optimal sensors based on the average amount of harvested energy over many time intervals, i.e., $t \in \{1, 2, \dots, T\}$. In this case, the sensors are selected such that the maximum error for any snapshot is minimized. The optimization problem is written as,

$$\begin{aligned} \arg\min_{\{w_{n,k}\}} \quad & \max_t \quad E^{(t)}(\mathbf{W}) && (10)\\ \text{subject to} \quad & \sum_{n=1}^{N} \mathbf{w}_n^T \mathbf{c} \le \xi && (10a)\\ & w_{n,k} \in [0,1], \quad \forall n,k && (10b)\\ & \mathbf{w}_n^T\mathbf{1} = 1, \quad \forall n. && (10c) \end{aligned}$$

Note that (10) is a convex function because the maximum of a non-decreasing convex function is also convex [4].

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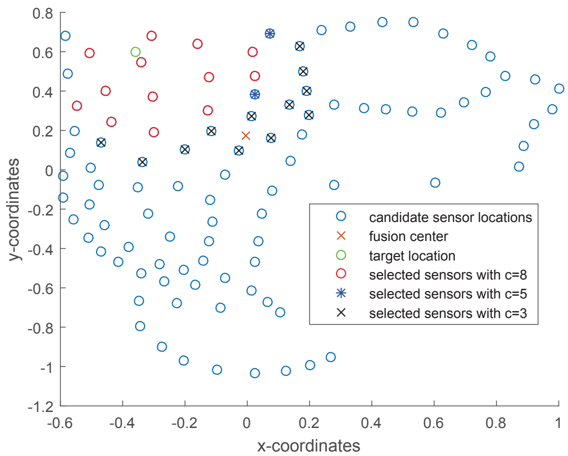


Fig. 3. Selected sensors for fixed source location

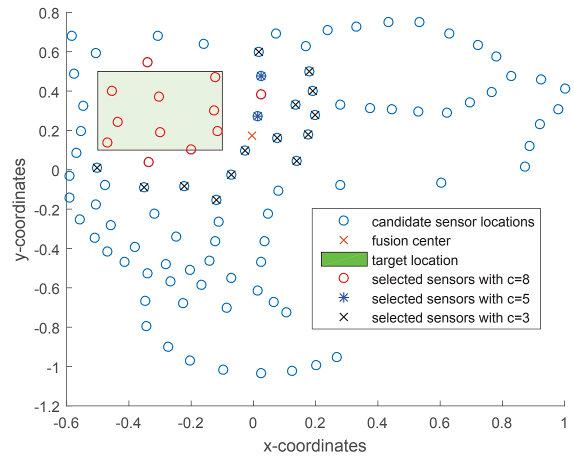


Fig. 5. selected sensors for a source location uniformly distributed over the green area

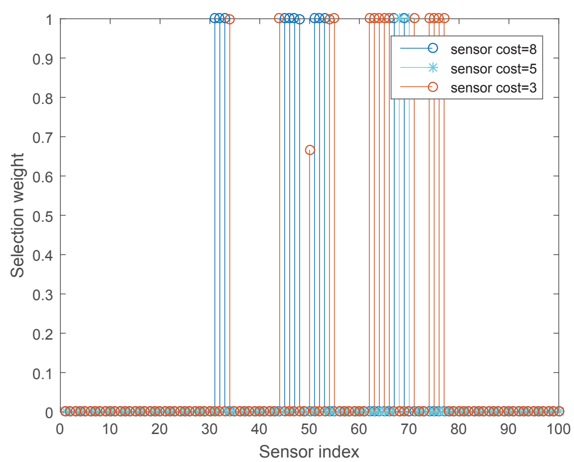


Fig. 4. Sensor selection weights

## V. Numerical Experiments

Figure 2 shows part of the KAUST campus map with 100 candidate sensor locations. This work serves an envisioned project to measure the gasoline leakage in the atmosphere. Note that the provided data in this section are not real data and only serve the purpose of validating the obtained selection solutions presented in Section IV. Although the gasoline observations might be non-linear, a linearization process is necessary in order to make the MSE independent of the unknowns and hence make offline selection possible [5], [6].

The selected sensors communicate directly with the fusion center (also shown in Figure 2) via wireless channels. The number of unknowns is $M = 2$ with a covariance matrix $\boldsymbol{\Sigma}_\theta$ modeled as $[\boldsymbol{\Sigma}_\theta]_{1,1} = [\boldsymbol{\Sigma}_\theta]_{2,2} = 1$ and $[\boldsymbol{\Sigma}_\theta]_{2,1} = [\boldsymbol{\Sigma}_\theta]_{1,2} = 0.5$, i.e., 2 unknowns to be estimated with 0.5 correlation. The unknown parameters could for instance represent the intensity of two gases at a specific location. Two cases are considered: (1) fixed source location and (2) random source location. The gains $\mathbf{h}_n$ are modeled as decaying signals with the distance from the gas source, i.e., $\mathbf{h}_n = [h_{n,1}\ h_{n,2}\ ...\ h_{n,M}]^T$ with

$$h_{n,m} = \alpha_m exp(\frac{d_n}{\beta_m}), \tag{11}$$

where $d_n$ is the distance between the gas source and the $n$-th sensor and $\alpha_m$ and $\beta_m$ are the sources diffusion parameters. We set $\alpha_m = [10\ 20]^T$ and $\beta_m = [1/3\ 1/6]^T$. All measurements are affected by independent zero-mean Gaussian noise with equal variance, $\sigma_v^2 = 1$.

The maximum amount of average EH at any location in the map at one snapshot is shown in Figure 2. Such maps for the average EH can be obtained every half an hour over a whole day period, i.e., $t \in \{1, \ldots, 48\}$. The transmission power is modeled as, $p_{n,k}^{(t)} = \min(\rho_n^{(t)}\eta_k, 10\eta_k)$.

The wireless channel gain is modeled as $g_n = r_n^{-\gamma}$, where $r_n$ is the distance between the $n$-th sensor and the fusion center and $\gamma = 2$ is the path loss exponent, i.e., we assume radio propagation in free space. The receiver AWGN is modeled such that $\phi_n \sim \mathcal{N}(0, 1)$. Perfect channel state information (CSI) is assumed to be available at the fusion center. We assume $K = 3$ sensor types with the cost and EH efficiency vectors $\mathbf{c} = [0\ 3\ 5\ 8]^T$ and $\boldsymbol{\eta} = [0\ .3\ .5\ .8]^T$. As mentioned before, two scenarios are studied:

1. Fixed source location: In this case, the measurements gains $\mathbf{h}_n$ are fixed. Hence, to minimize the reconstruction error subject to a prescribed system cost ($\xi = 150$), the optimization problem in (10) is solved. The selected set of sensors is shown in Figure 3.

2. Random source location: In this scenario, the average error is minimized considering a uniformly distributed source location over a known area. We replace the objective function in (10) with $\sum_i [\max_t E^{(t)}(\mathbf{W})]_i$, where $i$ is the source location index, and solve for a total system price of $\xi = 150$. Figures 4 and 5 show the selected set of sensors for a source location uniformly distributed over the green area in Figure

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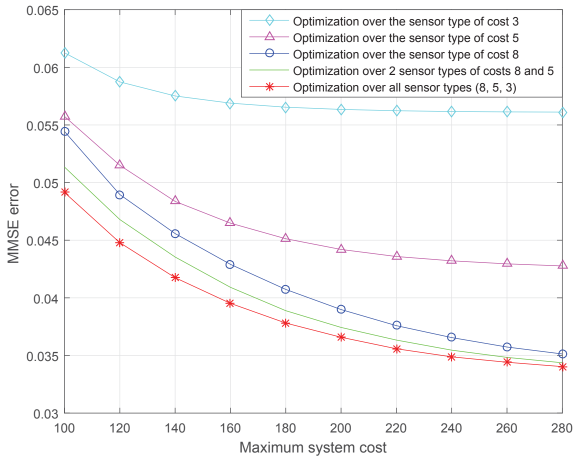


Fig. 6. Reconstruction error vs. system cost with the option to select from a pool of one, two or three sensor kinds

5. An application example for this case is a sensor network that measures the gas leakage from gas pipes in an industrial plant. The sensor selection is optimized based on the a priori expectation of a leakage from the pipes.

Interestingly, the achieved solutions for the described systems above and for other tested systems with adjusted parameters are either 0 or 1, as shown in Figure 4. Hence, the achieved solutions are already Boolean and there is no need for rounding. This being said, the solution is not Boolean in general. Therefore, a rounding algorithm must be utilized to decide the selection of sensors with non binary solution.

Finally, to highlight the importance of having more than one sensor type, we show in Figure 6 the obtained error by solving (10) versus the prescribed system costs, with the option of selecting from a pool of one, two or three sensor types.

## VI. Conclusions

A sensor selection optimization problem was derived and solved in order to jointly select optimal sensor locations and types from a pool of candidate locations and available sensor types. The sensors are selected such that the reconstruction error at the fusion center is minimized taking into account the observation accuracy, the amount of harvested energy and the communication link costs.

We show that indeed having the option to choose different sensor types with different power ratings at optimal locations might reduce the reconstruction error.

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