TIME-BASED LOCALIZATION FOR ASYNCHRONOUS WIRELESS SENSOR NETWORKS

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ABSTRACT

In this paper, we propose time-based localization approaches for asynchronous wireless sensor networks (WSNs), where not only clock skews but also clock offsets are present at all nodes. We first propose a joint synchronization and localization approach using the two-way ranging (TWR) protocol. Furthermore, a novel ranging protocol, namely asymmetric trip ranging (ATR), is employed and a two-step joint synchronization and localization approach is developed. As a result, we achieve efficient closed-form least-squares (LS) estimators. We compare these two proposed approaches. Moreover, simulation results corroborate the efficiency of our time-based localization schemes.

***Index Terms*—** Localization, synchronization, clock skew, clock offset, least squares, two-way ranging

1. INTRODUCTION

With the burgeoning of wireless sensor networks (WSNs) in important applications, such as target tracking, environment monitoring, and geographical routing, it is clear that localization-awareness [1] is crucial for the successful deployment of WSNs. The unique properties of ultra-wideband impulse radio [1] facilitate localization based on time-of-arrival (TOA) or time-difference-of-arrival (TDOA) with high accuracy and potentially low cost. As the TOA or TDOA measurements are time-based, clock synchronization [2] is tightly coupled with localization.

Localization and synchronization are traditionally treated as standalone problems. Only few research works [3, 4, 5, 6, 7] have considered both recently. The two-way ranging (TWR) protocol proposed in the IEEE 802.15.4a standard [8] is employed in [3, 4] for asynchronous networks, where the relative clock skews are first calibrated, and then the node positions are estimated in a distributed way. The internal delay is further considered in [4]. The impact of the clock skew is approximated as random noise in [5], and intrasensor TDOAs are calculated to cancel the clock offset. A localization approach based on triple-differences, which are the differences of two differential TDOAs, is proposed in [6], where the corrupted one-way TOA measurements due to the relative clock offset and clock skew are corrected by several steps. Two-way message exchanges are used in [7] for a scenario, where all the anchors with known positions are synchronized and the sensor node runs freely. It jointly estimates the clock skew, the clock offset and the position of the sensor node. Furthermore, it considers the uncertainties of the anchors clocks and positions, and formulates a generalized total least-squares problem to cope with these uncertainties. However, it is a big challenge to first synchronize all the anchors.

In this paper, we tackle asynchronous WSNs. We first propose closed-form LS estimators for joint synchronization and localization using the TWR protocol, which does not require the consecutive [3] or periodic [4] transmission of ranging packets, and does not ignore the clock drift during time of flight. Furthermore, we devise a novel ranging protocol labeled asymmetric trip ranging (ATR), which is an extension of the one proposed in [9]. The ATR protocol makes all the other anchors listen to the ranging packets and record timestamps, when one anchor and the sensor node exchange their ranging packets. It can obtain more information than the TWR protocol, where all the other nodes are idle, when two nodes exchange their ranging packets. By exploring this extra information, we can not only estimate the sensor position, but also its processing time and the clock parameters of all the anchors by closed-form least-squares (LS) estimators. Moreover, we compare the approaches based on the ATR and the TWR protocols. In future work, we would like to develop distributed algorithms to cope with joint synchronization and localization problems in asynchronous WSNs.

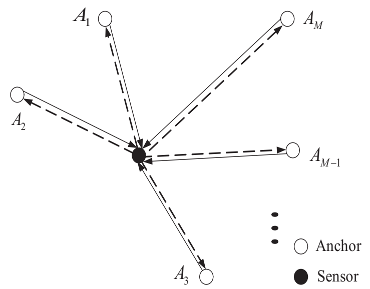
This research was supported in part by STW under the Green and Smart Process Technologies Program (Project 7976); X. Ma was supported in part by the Georgia Tech Ultra-wideband Center of Excellence (http://www.uwbtech.gatech.edu/).

2. JOINT SYNCHRONIZATION AND LOCALIZATION USING THE TWR PROTOCOL

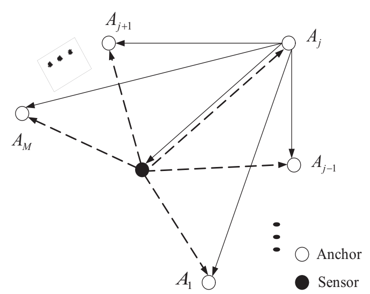
2.1. System Model

Since the TWR protocol is proposed in the standard [8], we first develop an approach based on this protocol to jointly synchronize and localize asynchronous WSNs thereby overcoming some of the drawbacks in [3, 4]. The consecutive [3] or periodic [4] transmission of ranging packets is required in order to make use of the prior knowledge of the packet length for relative clock skew estimation. Moreover, due to the fact that the relative clock skew is expressed in ppm (10^{-6}), [3, 4] ignores the clock drift during the time of flight, which is in the order of tens of nanoseconds for an indoor environment, and thus [3, 4] introduces an approximation. In our proposed scheme, we do not require a consecutive or periodic packet transmission, and do not ignore any clock drift. Furthermore, closed-form LS estimators are developed with computational efficiency.

We deal with asynchronous networks, where not only clock offsets but also clock skews are present at all the anchors and the sensor node. Without loss of generality, we consider M anchors and one sensor node. Our localization target is the sensor node. We assume that all the nodes are distributed in an l-dimensional space, e.g., $l = 2$ or $l = 3$. The coordinates of the anchor nodes are known and defined as $\mathbf{X}_a = [\mathbf{x}_1, \ldots, \mathbf{x}_M]_{l\times M}$, where the vector $\mathbf{x}_i = [x_{1i}, \ldots, x_{li}]^T$ indicates the coordinates of the ith anchor node. A vector $\mathbf{x}$ of length l denotes the unknown coordinates of the sensor node. Moreover, the model for the anchor clock [2] is given by $C_i(t) = \alpha_i t + \theta_i,\ i = 1, 2, \ldots, M$, where α_i and θ_i denote the unknown clock skew and clock offset of the ith anchor clock $C_i(t)$



(a) TWR protocol



(b) ATR protocol

Fig. 1. Examples of different protocols for asynchronous WSNs

relative to the absolute clock. The sensor clock is modeled in the same way using α_s and θ_s instead.

The ranging packet proposed in the standard is composed of a synchronization header preamble, a physical layer header (PHR) and a data field. A so-called ranging bit, which is a bit in the PHR, is set when the packet is used for ranging. Furthermore, the first pulse of the PHR is called the ranging marker (RMARKER). The moment when the RMARKER leaves or arrives at the antenna of a node is recorded as a timestamp to facilitate ranging. Each anchor carries out multiple iterations of the TWR protocol as shown in Fig. 1(a) to measure its distance to the sensor node. As a result, the ith anchor measures its round trip time at the jth iteration of the TWR protocol as V_{ij}, which is obtained by making the difference of its two timestamps recorded upon the departure and the arrival of the RMARKERs of the ranging request and the ranging response, respectively. The sensor correspondingly measures its processing time D_{ij} in the same way. The clock offset of the node is eliminated by making the difference of its timestamps. However, the clock skews of the nodes still remain. Thus the relation between V_{ij} and D_{ij} for the jth iteration can be modeled as

$$\frac{c}{2}\left(\frac{V_{ij}}{\alpha_i} - \frac{D_{ij}}{\alpha_s}\right) = d_i + \frac{\widetilde{n}_{ij}}{\alpha_i} - \frac{\widetilde{m}_{ij}}{\alpha_s},\ i = 1, 2, \ldots, M, \quad (1)$$

where $d_i = \|\mathbf{x}_i - \mathbf{x}\| = \sqrt{\|\mathbf{x}_i\|^2 - 2\mathbf{x}_i^T\mathbf{x} + \|\mathbf{x}\|^2}$ is unknown, and $\widetilde{n}_{ij}$ and $\widetilde{m}_{ij}$ are the distance error terms translated from the measurement errors in V_{ij} and D_{ij}, which can be modeled as zero mean Gaussian random variables. Once all the anchors execute the TWR protocol for one time, we obtain M equations, but $M + l + 1$ unknowns in total. Hence, the TWR protocol is executed N times by each anchor to obtain extra information. Defining $\mathbf{p}_i = \frac{c}{2}[V_{i1},\ V_{i2},\ \ldots,\ V_{iN}]^T$ and $\mathbf{q}_i = \frac{c}{2}[D_{i1},\ D_{i2},\ \ldots,\ D_{iN}]^T$ obtained by the ith anchor-sensor pair, we arrive at

$$\frac{1}{\alpha_i}\mathbf{p}_i - \frac{1}{\alpha_s}\mathbf{q}_i = d_i\mathbf{1}_N + \frac{1}{\alpha_i}\widetilde{\mathbf{n}}_i - \frac{1}{\alpha_s}\widetilde{\mathbf{m}}_i,\ i = 1, 2, \ldots, M. \quad (2)$$

2.2. Algorithm

For simplicity, we ignore the noise terms from now on. We are not interested in methods with a high computational complexity, such as the maximum likelihood (ML) method which also requires the unknown noise pdf. Because of the low cost and low power constraints for a WSN, we would like to explore low-complexity closed-form solutions. The equations (2) for different anchor-sensor pairs are coupled through α_s and $\mathbf{x}$. They are nonlinear with respect to (w.r.t.) $\mathbf{x}$ due to the nonlinear relation $\mathbf{d} \odot \mathbf{d} = \boldsymbol{\psi}_a - 2\mathbf{X}_a^T\mathbf{x} + \|\mathbf{x}\|^2\mathbf{1}_M$, where $\mathbf{d} = [d_1,\ d_2,\ \ldots,\ d_M]^T$, $\odot$ denotes element-wise product and $\boldsymbol{\psi}_a = [\|\mathbf{x}_1\|^2,\ \|\mathbf{x}_2\|^2,\ \ldots,\ \|\mathbf{x}_M\|^2]^T$, but each of them is linear w.r.t. $1/\alpha_i$, $1/\alpha_s$ and d_i. Therefore, we can first estimate the relative clock skew $\beta_i = \alpha_s/\alpha_i$ based on a set of equations modified from (2):

$$\beta_i\mathbf{p}_i - \mathbf{q}_i = \alpha_s d_i\mathbf{1}_N. \quad (3)$$

Note that α_s and d_i are coupled together. According to (3), $N \geq 2$ in order to estimate both β_i and $\alpha_s d_i$. Since we are only interested in β_i, an orthogonal projection matrix $\mathbf{P}_N = \mathbf{I}_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$ onto the complement of $\mathbf{1}_N$ is constructed. Applying $\mathbf{P}_N$ to both sides of (3), we can estimate β_i as

$$\hat{\beta}_i = \frac{\mathbf{p}_i^T\mathbf{P}_N\mathbf{q}_i}{\mathbf{p}_i^T\mathbf{P}_N\mathbf{p}_i}. \quad (4)$$

We remark that a different processing time is required in each iteration of the TWR protocol. Otherwise, $\mathbf{p}_i$ and $\mathbf{q}_i$ would be canceled out by the projection in the noiseless case. Sequentially, we plug $\hat{\beta}_i$ into (3) and average it to mitigate the noise, which leads to

$$\frac{1}{N\alpha_s}\mathbf{1}_N^T(\hat{\beta}_i\mathbf{p}_i - \mathbf{q}_i) = d_i,\ i = 1, 2, \ldots, M. \quad (5)$$

After element-wise multiplication of (5), moving knowns to one side and unknowns to the other side, we achieve

$$\boldsymbol{\psi}_a = \mathbf{K}\mathbf{z}, \quad (6)$$

where $\mathbf{z} = [\mathbf{x}^T,\ \|\mathbf{x}\|^2,\ \frac{1}{\alpha_s^2}]^T$, $\mathbf{K} = [2\mathbf{X}_a^T,\ -\mathbf{1}_M,\ \mathbf{f}]$ and the ith element of $\mathbf{f}$ is defined as $[\mathbf{f}]_i = \frac{1}{N^2}(\mathbf{1}_N^T(\hat{\beta}_i\mathbf{p}_i - \mathbf{q}_i)) \odot (\mathbf{1}_N^T(\hat{\beta}_i\mathbf{p}_i - \mathbf{q}_i))$. Consequently, the LS estimate of $\mathbf{z}$ is given by

$$\hat{\mathbf{z}} = (\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\boldsymbol{\psi}_a. \quad (7)$$

We remark that the rank of $\mathbf{K}$ should be $l + 2$, thus $M \geq l + 2$, e.g., $M \geq 4$ for $l = 2$.

3. JOINT SYNCHRONIZATION AND LOCALIZATION USING THE ATR PROTOCOL

3.1. System Model

We observe that all the other nodes are idle, when two nodes exchange their ranging packets using the TWR protocol. To make full use of the broadcast property of wireless signals, we propose the ATR protocol to make all the other anchors listen to the ranging packets and record timestamps, when one anchor and the sensor node exchange their ranging packets. It can obtain more information than the TWR protocol and reduce the communication load.

The same packet structure as proposed in the standard is employed here. As illustrated in Fig. 1(b), the ATR protocol starts with the jth anchor initiating a ranging request, and then the sensor node responses. The ith anchor records the timestamps T_{ij} and R_{ij} upon the arrival of the RMARKERs of the ranging request from the jth anchor and of the ranging response from the sensor, respectively. Note that T_{jj} is recorded when the jth anchor initiates the ranging request. Because we do not use any timestamps from the sensor node, the clock parameters of the sensor node do not have any influence on our scheme. This is also one advantage of the ATR protocol compared to the TWR protocol. By making differences of the timestamps from the same anchor, the clock offsets are canceled out. Thus the relations between T_{ij} and R_{ij} for the jth anchor as the initiator can be modeled as

$$\frac{c}{\alpha_i}(R_{ij} - T_{ij}) = d_i + d_j + \Delta_j - d_{ij} + \frac{n_{ij}}{\alpha_i} - \frac{m_{ij}}{\alpha_i}, \quad i = 1, 2, \ldots, M \tag{8}$$

where Δ_j is the unknown distance corresponding to the processing time of the sensor node formulating a response to the jth anchor, d_i is unknown, and $d_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|$ is known. Moreover, n_{ij} and m_{ij} are the distance error terms translated from the measurement errors in R_{ij} and T_{ij}, which can be modeled as zero mean Gaussian random variables. Defining $\mathbf{u}_j = c[R_{1j} - T_{1j},\ R_{2j} - T_{2j},\ \ldots,\ R_{Mj} - T_{Mj}]^T$, $\mathbf{g}_j = [d_{1j},\ d_{2j},\ \ldots,\ d_{Mj}]^T$, $\mathbf{n}_j = [n_{1j},\ n_{2j},\ \ldots,\ n_{Mj}]^T$, and $\mathbf{m}_j = [m_{1j},\ m_{2j},\ \ldots,\ m_{Mj}]^T$, we can now write (8) in vector form as

$$\text{diag}(\mathbf{u}_j)\boldsymbol{\gamma} = \mathbf{d} + (d_j + \Delta_j)\mathbf{1}_M - \mathbf{g}_j + \text{diag}(\boldsymbol{\gamma})(\mathbf{n}_j - \mathbf{m}_j), \tag{9}$$

where $\boldsymbol{\gamma} = [1/\alpha_1,\ 1/\alpha_2,\ \ldots,\ 1/\alpha_M]^T$.

The unknown $\mathbf{x}$, $\boldsymbol{\gamma}$ and Δ_j have to be estimated according to (9). There are $M + l + 1$ unknowns but only M equations in (9). Note that if an additional anchor plays the role of initiator, we obtain M new equations and one extra unknown distance corresponding to the processing time. Assuming that n anchors play the role of initiator, we have to fulfill the condition $nM \geq M + l + n$ in order to obtain enough equations to estimate all the parameters, where $M \geq n > 0$. It is possible that only a subset of anchors plays the role of initiator. However, since we are interested in the minimum number of anchors required for this approach, we take $n = M$, which means that all anchors participate. The minimum value of M is then given by $M_{\min} = \min\{M \in \{1, 2, \ldots\} | M^2 - 2M \geq l\}$, for instance, when $l = 2$, $M_{\min} = 3$. From now on, we ignore the error terms for simplicity, and assume $n = M$. We then obtain in total M^2 equations and can write them in vector form as follows

$$\mathbf{A}\boldsymbol{\gamma} = (\mathbf{B} + \mathbf{C})\mathbf{d} + \mathbf{C}\boldsymbol{\Delta} - \mathbf{g}, \tag{10}$$

where $\mathbf{A} = [\text{diag}(\mathbf{u}_1),\ \text{diag}(\mathbf{u}_2),\ \ldots,\ \text{diag}(\mathbf{u}_M)]^T$, $\mathbf{B} = \mathbf{1}_M \otimes \mathbf{I}_M$, $\mathbf{C} = \mathbf{I}_M \otimes \mathbf{1}_M$, $\boldsymbol{\Delta} = [\Delta_1,\ \Delta_2,\ \ldots,\ \Delta_M]^T$, $\mathbf{g} = [\mathbf{g}_1^T,\ \mathbf{g}_2^T,\ \ldots,\ \mathbf{g}_M^T]^T$, and $\otimes$ denotes Kronecker product. We remark that the sensor node is again required to use different processing times in response to different anchors. If $\Delta_i = \Delta_j$, we obtain the relation $(R_{ij} - T_{ij})/(R_{ji} - T_{ji}) = \alpha_i/\alpha_j$ without error terms, and it is only possible to estimate the relative clock skew. In that case, the equations (10) are not independent. Therefore, we assume that $\Delta_i \neq \Delta_j, i, j \in \{1, 2, \ldots, M\}$.

3.2. Algorithm

We would like to estimate $\mathbf{x}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\Delta}$, in total $2M + l$ unknown parameters based on (10), which is a complicated nonlinear equation w.r.t. $\mathbf{x}$. When we ignore the relations among the distances in $\mathbf{d}$ and regard them as independent unknowns, (10) is linear w.r.t. $\boldsymbol{\gamma}$, $\mathbf{d}$ and $\boldsymbol{\Delta}$. Therefore, we propose a two-step approach. We first jointly estimate $\boldsymbol{\gamma}$, $\mathbf{d}$ and $\boldsymbol{\Delta}$, obtaining a unique estimate for $\boldsymbol{\gamma}$ but ambiguous estimates of $\mathbf{d}$ and $\boldsymbol{\Delta}$. Secondly, we plug the estimate of $\boldsymbol{\gamma}$ into (10), linearize the equation w.r.t $\mathbf{x}$ via mathematical manipulations and then estimate $\mathbf{x}$. We remark that although our approach is accomplished in two steps, the first step is still a joint approach to estimate $\boldsymbol{\gamma}$, $\mathbf{d}$ and $\boldsymbol{\Delta}$, and in the second step the estimate of $\boldsymbol{\gamma}$ is used and the relation between $\mathbf{d}$ and $\mathbf{x}$ is explored. Furthermore, we use the same set of measurements to obtain all the estimates, and thus our method yields a joint synchronization and localization approach.

In the first step, there are $3M$ unknowns ($\boldsymbol{\gamma}$, $\mathbf{d}$ and $\boldsymbol{\Delta}$) and M^2 equations. Thus, $M_{\min} = 3$ is still valid. However, we note that $[\mathbf{B} + \mathbf{C},\ \mathbf{C}]$ is rank deficient because of the common basis of $\mathbf{B}$ and $\mathbf{C}$. Therefore, we can only jointly estimate $\mathbf{d}$ and $\boldsymbol{\Delta}$ with ambiguities. Since we are only interested in the result for $\boldsymbol{\gamma}$ based on (10), the subspace minimization method [10] is employed due to its computational efficiency, which is equivalent to the joint estimation of all the unknowns. Let us define $\mathbf{D} = [[\mathbf{B}]_{:,1:M-1},\ \mathbf{C}]$ of size $M^2 \times (2M - 1)$, and obtain an orthogonal projection matrix onto the orthogonal complement of $\mathbf{D}$ as $\mathbf{P}_d = \mathbf{I}_{M^2} - \mathbf{D}(\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T$, which fullfills the condition that $\mathbf{P}_d\mathbf{B} = \mathbf{P}_d\mathbf{C} = \mathbf{0}_{M^2 \times M}$. Premultiplying $\mathbf{P}_d$ to both sides of (10), we arrive at

$$\mathbf{P}_d\mathbf{A}\boldsymbol{\gamma} = -\mathbf{P}_d\mathbf{g}. \tag{11}$$

Consequently, the LS estimate of $\boldsymbol{\gamma}$ is given by

$$\hat{\boldsymbol{\gamma}} = -(\mathbf{A}^T\mathbf{P}_d\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}_d\mathbf{g}. \tag{12}$$

We remark that in order to obtain the LS estimate of $\boldsymbol{\gamma}$, the condition $M^2 + 1 - 2M \geq M$ has to be fullfilled taking the penalty of the projection into account, which means $M_{\min} = 3$ is still valid.

In the second step, plugging $\hat{\boldsymbol{\gamma}}$ into (10), we achieve

$$\mathbf{A}\hat{\boldsymbol{\gamma}} = (\mathbf{B} + \mathbf{C})\mathbf{d} + \mathbf{C}\boldsymbol{\Delta} - \mathbf{g}, \tag{13}$$

where we would like to get rid of the nuisance parameter $\boldsymbol{\Delta}$, and investigate the nonlinear relation between $\mathbf{d}$ and $\mathbf{x}$ in order to obtain a unique estimate of $\mathbf{x}$. Thus, the orthogonal projection matrix $\mathbf{P}_c$ onto the orthogonal complement of $\mathbf{C}$ is used, which is given by $\mathbf{P}_c = \mathbf{I}_{M^2} - \frac{1}{M}(\mathbf{I}_M \otimes (\mathbf{1}_M\mathbf{1}_M^T))$. Moreover, we find that $\mathbf{P}_c\mathbf{B}\mathbf{d} = \mathbf{B}\mathbf{d} - \bar{d}\mathbf{1}_{M^2}$, where $\bar{d} = \frac{1}{M}\sum_{i=1}^{M} d_i$. Consequently, premultiplying $\mathbf{P}_c$ to both sides of (13), we obtain

$$\mathbf{P}_c\mathbf{A}\hat{\boldsymbol{\gamma}} = \mathbf{B}\mathbf{d} - \bar{d}\mathbf{1}_{M^2} - \mathbf{P}_c\mathbf{g}. \tag{14}$$

Due to the special structure of $\mathbf{B}$, we have $\frac{1}{M}\mathbf{B}^T\mathbf{B} = \mathbf{I}_M$. Therefore, premultiplying $\frac{1}{M}\mathbf{B}^T$ to both sides of (14), moving $\mathbf{d}$ to one side, the other terms to the other side, and simplifying the equations, we arrive at

$$\mathbf{d} = \frac{1}{M}\mathbf{B}^T\mathbf{P}_c\mathbf{A}\hat{\boldsymbol{\gamma}} + \bar{d}\mathbf{1}_M + \frac{1}{M}\mathbf{B}^T\mathbf{P}_c\mathbf{g}. \tag{15}$$

After element-wise multiplication on both sides of the equation, moving unknown parameters to one side, known terms to the other side, we achieve

$$\boldsymbol{\phi} = \mathbf{H}\mathbf{y}, \tag{16}$$

where $\boldsymbol{\phi} = \boldsymbol{\psi}_a - \frac{1}{M^2}(\mathbf{B}^T\mathbf{P}_c(\mathbf{A}\hat{\boldsymbol{\gamma}} + \mathbf{g})) \odot (\mathbf{B}^T\mathbf{P}_c(\mathbf{A}\hat{\boldsymbol{\gamma}} + \mathbf{g}))$, $\mathbf{H} = [2\mathbf{X}_a^T,\ \mathbf{1}_M,\ \frac{2}{M}\mathbf{B}^T\mathbf{P}_c(\mathbf{A}\hat{\boldsymbol{\gamma}} + \mathbf{g})]$ and $\mathbf{y} = [\mathbf{x}^T,\ \bar{d}^2 - \|\mathbf{x}\|^2,\ \bar{d}]^T$. Thus, the LS estimate of $\mathbf{y}$ is given by

$$\hat{\mathbf{y}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\boldsymbol{\phi}. \tag{17}$$

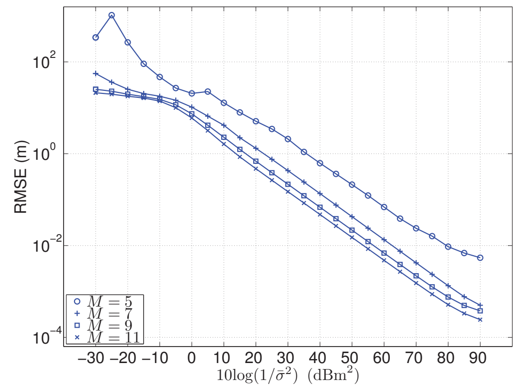
We remark that the rank of $\mathbf{H}$ should be $l + 2$, and thus $M \geq l + 2$, e.g., $M \geq 4$ for $l = 2$. One more anchor is required due to the linearization compared to $M_{\min} = 3$ mentioned before.

Let us now compare the communication load of this approach with the one of the approach in Section 2.2. In the worst case, the ATR protocol is executed by every anchor in the network (in total M anchors). Therefore, $2M$ ranging packets are transmitted, and $2M^2$ timestamps are recorded. On the other hand, if we run the minimum number of iterations for each anchor-sensor pair in the TWR protocol, which is $N_{\min} = 2$, $2N_{\min}M$ ranging packets are transmitted, and $4N_{\min}M$ timestamps are recorded. Obviously, using the approach based on the ATR protocol, we obtain more information and have a smaller communication load. Moreover, the estimate of $\boldsymbol{\gamma}$ is based on the whole set of measurements, but the estimate of β_i only depends on a subset of measurements. Furthermore, The computational complexities of the estimator (17) for $\mathbf{y}$ and (7) for $\mathbf{z}$ are similar, while the one of the estimator (4) for β_i is less than the one of (12) for $\boldsymbol{\gamma}$.

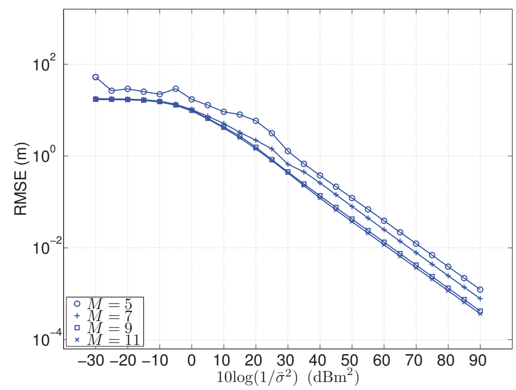
4. SIMULATION RESULTS

We now evaluate the performance of the proposed approaches by Monte Carlo simulations. We consider a simulation setup, where all the anchors and the sensor node are randomly distributed inside a $40\,\text{m} \times 40\,\text{m}$ rectangular to mimic an indoor geometry scale ($l = 2$). Furthermore, due to the broadcast property of the ATR protocol, we assume that the noise variances are related to the distances according to the path loss law. Hence, m_{ij} and $m_{ji}, i, j \in \{1, 2, \ldots, M\}$ have the same variance $\sigma_{ij}^2 = \sigma_{ji}^2$, while all $n_{ij}, j \in \{1, \ldots, M\}$ have the same variance σ_i^2. Sequentially, we define the average noise power as $\bar{\sigma}^2 = \frac{1}{M}\sum_{i=1}^{M} \sigma_i^2$, and choose σ_{ij}^2 and σ_i^2 to fulfill the condition that all σ_{ij}^2/d_{ij}^2 and σ_i^2/d_i^2 are equal. Note that since $d_{ii} = 0, i \in \{1, 2, \ldots, M\}$, we simply assume $\sigma_{ii}^2 = 0$ and $m_{ii} = 0$. For the TWR protocol, all $\widetilde{n}_{ij}$ and $\widetilde{m}_{ij}, j \in \{1, \ldots, N\}$ have the same variance σ_i^2, and the same $\bar{\sigma}^2$ is used. The processing time in the sensor node is randomly generated, uniformly distributed in the range of 2.5 ms to 7.5 ms. As a result, the corresponding distance Δ_i is in the range of 7.5×10^5 m ($3 \times 10^8 \times 2.5 \times 10^{-3}$) to 2.25×10^6 m. The clock skews of the anchors and the sensor are randomly generated and uniformly distributed in the range of $[1 - 100\,\text{ppm}, \quad 1 + 100\,\text{ppm}]$. The performance criterion is the root mean square error (RMSE) of $\hat{\mathbf{x}}$, which is $\sqrt{1/N_{exp} \sum_{j=1}^{N_{exp}} \|\hat{\mathbf{x}}^{(j)} - \mathbf{x}\|^2}$, where $\hat{\mathbf{x}}^{(j)}$ is the estimate obtained in the jth trial, and $N_{exp} = 2000$ is the number of Monte Carlo trials.

Fig. 2(a) and Fig. 2(b) show the RMSE of $\mathbf{x}$ vs. $1/\bar{\sigma}^2$ for both protocols, respectively. We have tested different numbers of anchors. As $M = 5$ is just one more than the minimum number of anchors required by the approaches, the curves with $M = 5$ (the lines with circle markers) are not as smooth as the ones with more anchors. More anchors improve the accuracy of the estimates. In Fig. 2(a), the performance gap between $M = 5$ and $M = 7$ (the line with $+$ markers) is larger than the one between $M = 7$ and $M = 9$ (the line with rectangular markers). Thus the improvement reduces as the number of anchors increases. In Fig. 2(b), we choose $N = 3$. We observe that the improvement first increases and then reduces along with the number of anchors. Although the approach based on the TWR protocol transmits $N - 1$ times more packets than the one based on the ATR protocol, we observe in general better performance achieved by the latter one.



(a) The ATR protocol



(b) The TWR protocol

Fig. 2. RMSE of $\mathbf{x}$ for different protocols

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