CENSORED TRUNCATED SEQUENTIAL SPECTRUM SENSING FOR COGNITIVE RADIO NETWORKS

Sina Maleki and Geert Leus

Faculty of Electrical Engineering, Delft University of Technology, Delft, The Netherlands
E-mail: {s.maleki, g.j.t.leus}@tudelft.nl

ABSTRACT

A truncated censored sequential spectrum sensing technique is considered as an energy saving approach for a cooperative spectrum sensing system. In order to design the underlying sensing parameters, the maximum energy consumption per sensor is minimized subject to a lower bounded global probability of detection and an upper bounded false alarm rate. We compare the performance of the proposed scheme with a fixed sample size censoring scheme. It is shown that the truncated censored sensing approach is highly energy efficient, particularly when the sensing cost is high.

***Index Terms*—** distributed spectrum sensing, sequential sensing, cognitive radio networks, censoring, energy efficiency.

1. INTRODUCTION

Cooperative spectrum sensing improves the detection reliability of a cognitive radio network [1], [2]. On the other hand, as the number of cognitive radios involved in spectrum detection increases, so does the network energy consumption. Further, recent standardization activities permit the operation of low-power sensors in TV bands [3]. This leads to a limited available energy per sensor. Therefore, energy efficient cooperative spectrum sensing techniques are necessary to give a reliable sensing performance while satisfying the energy constraint of the system.

The spectrum sensing module consumes energy in both the sensing and transmission stage. A combination of censoring and truncated sequential sensing is proposed to save energy. The sensors sequentially sense the spectrum before reaching a truncation point where they are forced to stop sensing. If the accumulated energy of the collected sample observations is in a certain region before the truncation point, a decision is sent to the fusion center (FC). Else a censoring policy is used by the sensor, and no bit will be sent. This way, a large amount of energy is saved for both sensing and transmission. Our goal is to minimize the maximum energy consumption per sensor subject to a specific detection performance constraint which is defined by a lower bound on the global probability of detection and an upper bound on the global probability of false alarm. In terms of cognitive radio system design, the probability of detection limits the harmful interference to the primary user and the false alarm rate controls the loss in spectrum utilization. The ideal case yields no interference and full spectrum utilization, but it is practically impossible to reach this point. Hence, current standards determine a bound on the detection performance to achieve an acceptable interference and utilization level [4]. To reduce the computational complexity of the system, a single-threshold truncated sequential test is proposed where each cognitive radio sends a decision to the FC upon the detection of the primary user and the related analytical expressions are derived. To make a fair comparison of the proposed technique with current energy efficient approaches, a fixed sample size censoring scheme is considered as a benchmark where each sensor employs a censoring policy after collecting a fixed number of samples. For this approach, it is proved that a single-threshold censoring policy is optimal in terms of energy consumption. Moreover, an explicit solution of the underlying problem is given.

The problem of energy efficient distributed sensing design for cognitive radio networks has been considered in [5]. In [5], a combined censoring and sleeping scheme is proposed with the goal of minimizing the network energy consumption subject to a specific detection performance constraint. It is shown that such a system can attain a high energy saving. Note that such a sleeping scheme can easily be incorporated in to this paper to gain an even higher energy saving. Censoring for cognitive radios is also considered in [6], [7]. In [6], a censoring rule similar to the one in this paper is considered in order to limit the bandwidth occupancy of the cognitive radio network. Our fixed sample size censoring scheme is different in two ways. First, in [6], the FC makes no decision in case it does not receive any decision from the cognitive radios which is ambiguous, since the FC has to make a final decision, while in our paper, the FC reports the absence of a primary user, if no local decision is received at the FC. Second, we give a clear optimization problem and an explicit expression of the solution while this is not presented in [6]. In [7], analytical expressions for the sensing parameters are given according to a Neyman-Pearson set-up for both soft and hard fusion schemes, but unlike [5] no constraint on the energy consumption is taken into account.

Sequential spectrum sensing is also considered for cognitive radio design. An infinite horizon sequential probability ration test (SPRT) is employed in [8]- [11] for different sensing techniques. It is shown that the sensing time dramatically reduces when employing sequential detection. The optimization of the cognitive network throughput under a constraint on the miss-detection probability is solved in [12], [13] in order to find the optimal stopping and access policies. This approach is infinite horizon which is a not a valid assumption considering the limited sensing time of cognitive radios. Further, a binary result has to be sent to the FC for each collected observation sample which entails a high transmission energy consumption. Nevertheless, the considered optimization problem is matched to the cognitive radio system requirements and an extension of [12] for the finite horizon case can also be considered. In [14], the sensing thresholds that minimize the average sample number (ASN) are derived subject to a constraint on the false alarm rate, miss-detection probability, outage probability, and interference level. This method is particularly designed for systems with real-time traffic. A truncated sequential sensing technique is employed in [15] to reduce the

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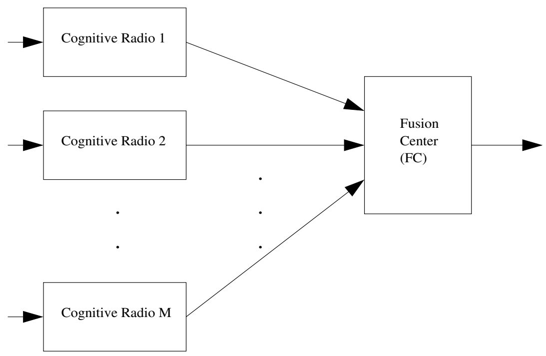

Fig. 1. Cooperative spectrum sensing configuration

sensing time of a cognitive radio system. The thresholds are determined such that a certain probability of false alarm and detection are obtained. In this paper, we are employing a similar technique, except that in [15], after the truncation point, a single threshold scheme is used to make a final decision, while in our paper, the sensor decision is censored if no decision is made before the truncation point. In addition, we assume a random signal for the primary user signal while in [15], the signal is assumed deterministic which leads to a different probability of detection and ASN. Further, [15] considers a single sensor detection scheme while we employ a distributed cooperative sensing system and finally, in our paper an explicit optimization problem is given to find the sensing parameters.

The remainder of the paper is organized as follows. In Section 2, the fixed size censoring scheme is described, including the optimization problem and the algorithm to solve it. The sequential censoring scheme is presented in Section 3. The analytical expressions for the underlying system parameters are given and the optimization problem is analyzed. We discuss some numerical results in Section 4 and conclusions are posed in Section 5.

2. FIXED SIZE CENSORING PROBLEM FORMULATION

To have a benchmark for the performance of the combined sequential and censoring scheme, a fixed size censoring scheme is presented in this section. We consider a parallel detection configuration as shown in Fig. 1 comprising a network of M cognitive radios. Each cognitive radio senses the spectrum and makes a local decision about the presence or absence of the primary user and informs the FC by employing a censoring policy. The final decision is then made at the FC by employing the OR rule. The OR rule is used because of its simplicity and low implementation cost. Denoting r_{ij} to be the i-th sample received at j-th cognitive radio, each radio solves a binary hypothesis testing problem as follows

$$\begin{aligned} \mathcal{H}_0 &: \quad r_{ij} = w_{ij},\ i = 1, ..., N,\ j = 1, ..., M \\ \mathcal{H}_1 &: \quad r_{ij} = h_j s_{ij} + w_{ij},\ i = 1, ..., N,\ j = 1, ..., M \quad (1) \end{aligned}$$

where w_{ij} is additive white Gaussian noise with zero mean and variance σ_w^2, s_{ij} is the transmitted primary user signal which is also assumed to be white Gaussian with zero mean and variance σ_s^2, and h_j is the channel gain between the primary user and the j-th cognitive radio which is assumed constant during each sensing period. Furthermore the $s_{ij}{}'$s and $w_{ij}{}'$s are assumed statistically independent.

An energy detector is employed by each cognitive sensor which calculates the accumulated energy over N observation samples. The received energy at the j-th radio is given by $\mathcal{E}_j = \sum_{i=1}^{N} \frac{|r_{ij}|^2}{\sigma_w^2}$. It is well known [16] that under such a model $\mathcal{E}_j$ follows a central chi-square distribution with $2M$ degrees of freedom under $\mathcal{H}_0$ and $\mathcal{H}_1$ and the related probability density functions are respectively given by

$$\begin{aligned} p(\mathcal{E}_j|\mathcal{H}_0) &= \frac{1}{2^N\Gamma(N)}\mathcal{E}_j^{N-1}e^{\mathcal{E}_j/2}I_{\{\mathcal{E}_j\geq 0\}}, & (2) \\ p(\mathcal{E}_j|\mathcal{H}_1) &= \frac{1}{2^N\Gamma(N)}\mathcal{E}_j^{N-1}e^{\mathcal{E}_j/2(1+\gamma_j)}I_{\{\mathcal{E}_j\geq 0\}}, & (3) \end{aligned}$$

where $I_{\{x\geq 0\}}$ is the indicator function, and $\gamma_j = |h_j|^2\sigma_s^2/\sigma_w^2$ is the SNR of the primary user received at the j-th cognitive radio.

A censoring policy is then employed at each radio where the local decisions are send to the FC only if they are deemed to be informative [5]. Censoring thresholds λ_1 and λ_2 are applied at each of the radios, where the range $\lambda_1 < \mathcal{E}_j < \lambda_2$ is called the censoring region. At the j-th radio, the local censoring decision rule is given by

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \mathcal{E}_j \geq \lambda_2, \\ \text{no decision} & \text{if } \lambda_1 < \mathcal{E}_j < \lambda_2, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \mathcal{E}_j \leq \lambda_1. \end{cases} \quad (4)$$

Based on such a decision policy, the local probabilities of false alarm and detection can be respectively written as

$$P_{fj} = Pr(\mathcal{E}_j \geq \lambda_2|\mathcal{H}_0) = \frac{\Gamma(N, \frac{\lambda_2}{2})}{\Gamma(N)}, \quad (5)$$

$$P_{dj} = Pr(\mathcal{E}_j \geq \lambda_2|\mathcal{H}_1) = \frac{\Gamma(N, \frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}, \quad (6)$$

where $\Gamma(a, x)$ is the incomplete gamma function given by $\Gamma(a, x) = \int_x^\infty t^{a-1}e^{-t}dt$, with $\Gamma(a, 0) = \Gamma(a)$.

Denoting C_{sj} and C_{ti} to be the energy consumed by the j-th radio in sensing per sample and transmission per bit, respectively, the average energy consumed for distributed sensing per user is given by,

$$C_j = NC_{sj} + (1 - \rho_j)C_{tj}, \quad (7)$$

where $\rho_j = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2)$ is denoted to be the average censoring rate. Defining $\pi_0 = Pr(\mathcal{H}_0)$, $\pi_1 = Pr(\mathcal{H}_1)$, $\delta_{0j} = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2|\mathcal{H}_0)$ and $\delta_{1j} = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2|\mathcal{H}_1)$, ρ_j is given by

$$\rho_j = \pi_0\delta_{0j} + \pi_1\delta_{1j}, \quad (8)$$

with

$$\delta_{0j} = \frac{\Gamma(N, \frac{\lambda_1}{2})}{\Gamma(N)} - \frac{\Gamma(N, \frac{\lambda_2}{2})}{\Gamma(N)}, \quad (9)$$

$$\delta_{1j} = \frac{\Gamma(N, \frac{\lambda_1}{2(1+\gamma_j)})}{\Gamma(N)} - \frac{\Gamma(N, \frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}. \quad (10)$$

Denoting $Q_{\rm F}^{\rm c}$ and $Q_{\rm D}^{\rm c}$ to be the respective global probability of false alarm and detection, the target detection performance is then quantified by $Q_{\rm F}^{\rm c} \leq \alpha$ and $Q_{\rm D}^{\rm c} \geq \beta$, where, α and β are pre-specified detection design parameters. Therefore, it is assured that the throughput of the cognitive radio is lower bounded and interference is constrained. Our goal is to determine the optimum censoring thresholds λ_1 and λ_2 such that the maximum energy consumption per sensor, i.e., $\max_j\ C_j$, is minimized subject to the constraints

$Q_{\mathrm{F}}^{c} \leq \alpha$ and $Q_{\mathrm{D}}^{c} \geq \beta$. Hence, our optimization problem can be formulated as

$$\min_{\lambda_1,\lambda_2} \max_{j} C_j \tag{11}$$
$$\text{s.t. } Q_{\mathrm{F}}^{c} \leq \alpha,\ Q_{\mathrm{D}}^{c} \geq \beta. \tag{12}$$

The FC employs an OR rule to make the final decision which is denoted by D_{FC}, i.e., $D_{FC} = 1$ if the FC receives at least one local decision declaring 1, else $D_{FC} = 0$. This way, the global probability of false alarm and detection can be derived as

$$Q_{\mathrm{F}}^{c} = Pr(D_{FC} = 1|\mathcal{H}_0) = 1 - \prod_{j=1}^{M}(1 - P_{fj}), \tag{13}$$

$$Q_{\mathrm{D}}^{c} = Pr(D_{FC} = 1|\mathcal{H}_1) = 1 - \prod_{j=1}^{M}(1 - P_{dj}). \tag{14}$$

Note that since all the cognitive radios employ the same upper threshold λ_2, we can state that $P_{fj} = P_f$ defined in (5). As a result (13) becomes $Q_{\mathrm{F}}^{c} = 1 - (1 - P_f)^M$.

Theorem 1. The optimal solution of (11) is obtained by $\lambda_1 = 0$.

Proof. Since $\frac{\partial C_j}{\partial \rho_j} = -C_{tj} \leq 0$ and $\frac{\partial \rho_j}{\partial \lambda_1} \leq 0$, we obtain $\frac{\partial C_j}{\partial \lambda_1} = \frac{\partial C_j}{\partial \rho_j}\frac{\partial \rho_j}{\partial \lambda_1} \geq 0$, and thus the optimal C_j is attained for the lowest λ_1 in the feasible set of the problem which is equal to 0.□

From Theorem 1, (9) and (10) can be simplified to $\delta_{0j} = 1 - P_f$ and $\delta_{1j} = 1 - P_{dj}$ and so (11) becomes,

$$\min_{\lambda_2} \max_{j} \big(NC_{sj} + (\pi_0 P_f + \pi_1 P_{dj})C_{tj}\big)$$
$$\text{s.t. } 1 - (1 - P_f)^M \leq \alpha,\ 1 - \prod_{j=1}^{M}(1 - P_{dj}) \geq \beta. \tag{15}$$

Since there is a one-to-one relationship between λ_2 and P_f, i.e., $\lambda_2 = 2\Gamma^{-1}[N, \Gamma(N)P_f]$, (15) can be formulated as [17, p.130]

$$\min_{P_f} \max_{j} \big(NC_{sj} + (\pi_0 P_f + \pi_1 P_{dj})C_{tj}\big)$$
$$\text{s.t. } 1 - (1 - P_f)^M \leq \alpha,\ 1 - \prod_{j=1}^{M}(1 - P_{dj}) \geq \beta. \tag{16}$$

Defining $P_f = F(\lambda_2) = \frac{\Gamma(N,\frac{\lambda_2}{2})}{\Gamma(N)}$ and $P_{dj} = G_j(\lambda_2) = \frac{\Gamma(N,\frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}$, we can write P_{dj} as $P_{dj} = G_j(F^{-1}(P_f))$. Calculating the derivative of C_j with respect to P_f, we find that $\frac{\partial C_j}{\partial P_f} \geq 0$. Therefore, we can simplify (16) as

$$\begin{aligned}&\min_{P_f} P_f\\ &\text{s.t. } 1 - (1 - P_f)^M \leq \alpha,\ 1 - \textstyle\prod_{j=1}^{M}(1 - P_{dj}) \geq \beta.\end{aligned} \tag{17}$$

which can be easily solved by a line search over P_f. However, defining $Q_{\mathrm{D}}^{c} = H(P_f) = 1 - \prod_{j=1}^{M}(1 - G_j(F^{-1}(P_f)))$, we can show that if the feasible set of (17) is not empty, then the optimal solution is given by $P_f = H^{-1}(\beta)$. When the received SNR of the primary user by the cognitive radios can be assumed to be the same, the optimal P_d is $P_d = 1 - (1 - \beta)^M$ and the optimal P_f is given by $P_f = F(G^{-1}(1 - (1 - \beta)^{1/M}))$. In the following section, a combination of the censoring and sequential approaches is presented which optimizes both the sensing and the transmission cost.

3. SEQUENTIAL CENSORING PROBLEM FORMULATION

3.1. System Model

In this section, each cognitive radio sequentially senses the spectrum and upon reaching a decision about the presence or absence of the parimary user, sends the result to the FC by employing a censoring policy. Here, a truncated censored sequential sensing scheme is employed where each cognitive radio carries on sensing until it reaches a decision while not passing a limit of N samples. Denoting Λ_{nj} to be the decision statistic at the j-th cognitive radio after n consecutive samples, the local decision rule to make a final decision is as follows,

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \Lambda_{nj} \geq b \text{ and } n \in [1, N], \\ \text{continue sensing} & \text{if } \Lambda_{nj} \in (a, b) \text{ and } n \in [1, N), \\ \text{no decision} & \text{if } a < \Lambda_{nj} < b \text{ and } n = N, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \Lambda_{nj} \leq a \text{ and } n \in [1, N], \end{cases} \tag{18}$$

where $a < 0$ and $b > 0$. To avoid the calculation of the LLR for each sample and because of the simple implementation of an energy detector, a sequential shifted chi-square test is employed, as in [15]. Therefore, the decision metric Λ_{nj} is defined as follows

$$\Lambda_{nj} = \sum_{i=1}^{n}(|r_{ij}|^2 - \Lambda), \tag{19}$$

where $\sigma_w^2 < \Lambda < \sigma_w^2(1 + \gamma_j)$ is a predetermined constant and $\gamma_j = |h_j|^2\sigma_s^2/\sigma_w^2$ is the SNR of the primary user received at the j-th cognitive radio. Dividing left and right hand sides of (19) by σ_w^2 we obtain

$$\bar{\Lambda}_{nj} = \Lambda_{nj}/\sigma_w^2 = \sum_{i=1}^{n}(|r_{ij}|^2 - \Lambda)/\sigma_w^2. \tag{20}$$

The probability density function of $x_{ij} = |r_{ij}|^2/\sigma_w^2$ under $\mathcal{H}_0$ and $\mathcal{H}_1$ is a chi-square distribution with $2n$ degrees of freedom. Thus, x_{ij} becomes exponentially distributed under both $\mathcal{H}_0$ and $\mathcal{H}_1$. Henceforth we obtain

$$Pr(x_{ij}|\mathcal{H}_0) = \frac{1}{2}e^{-x_{ij}/2}I_{\{x_{ij}\geq 0\}}, \tag{21}$$
$$Pr(x_{ij}|\mathcal{H}_1) = \frac{1}{2(1+\gamma_j)}e^{-x_{ij}/2(1+\gamma_j)}I_{\{x_{ij}\geq 0\}}, \tag{22}$$

Defining $\zeta_{nj} = \sum_{i=1}^{n}|r_{ij}|^2/\sigma_w^2 = \sum_{i=1}^{n}x_{ij}$, it is clear that, $\bar{\Lambda}_{nj} = \zeta_{nj} - n\bar{\Lambda}$, where $\bar{\Lambda} = \Lambda/\sigma_w^2$. Denoting $a_i = 0$, $i = 1, \ldots, p$, $a_i = \bar{a} + i\bar{\Lambda}$, $i = p+1, ..., N$ and $b_i = \bar{b} + i\bar{\Lambda}$, $i = 1, ..., N$, where $\bar{a} = a/\sigma_w^2$ and $\bar{b} = b/\sigma_w^2$, and where $p = \lfloor -a/\Lambda \rfloor$, (18) becomes

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \zeta_{nj} \geq b_n \text{ and } n \in [1, N], \\ \text{continue sensing} & \text{if } \zeta_{nj} \in (a_n, b_n) \text{ and } n \in [1, N), \\ \text{no decision} & \text{if } \zeta_{nj} \in (a_n, b_n) \text{ and } n = N, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \zeta_{nj} \leq a_n \text{ and } n \in [1, N]. \end{cases} \tag{23}$$

Defining $\zeta_{0j} = 0$, the local probability of false alarm and detection at the j-th cognitive radio, i.e., P_{fj} and P_{dj}, can be written as

$$P_{fj} = \sum_{n=1}^{N} Pr(\forall i \in [0, n-1] : \zeta_{ij} \in (a_i, b_i), \zeta_{nj} \geq b_n | \mathcal{H}_0), \tag{24}$$

$$P_{dj} = \sum_{n=1}^{N} Pr(\forall i \in [0, n-1] : \zeta_{ij} \in (a_i, b_i), \zeta_{nj} \geq b_n | \mathcal{H}_1). \tag{25}$$

Denoting ρ_j to be the average censoring rate at the j-th cognitive radio, and δ_{0j} and δ_{1j} to be the respective average censoring rate under $\mathcal{H}_0$ and $\mathcal{H}_1$, we have

$$\begin{aligned} \rho_j &= Pr(\zeta_{1j} \in (a_1, b_1), ..., \zeta_{Nj} \in (a_N, b_N)) \\ &= \pi_0 Pr(\zeta_{1j} \in (a_1, b_1), ..., \zeta_{Nj} \in (a_N, b_N) | H_0) \\ &+ \pi_1 Pr(\zeta_{1j} \in (a_1, b_1), ..., \zeta_{Nj} \in (a_N, b_N) | \mathcal{H}_1) \\ &= \pi_0 \delta_{0j} + \pi_1 \delta_{1j}, \end{aligned} \tag{26}$$

where,

$$\delta_{0j} = Pr(\zeta_{1j} \in (a_1, b_1), ..., \zeta_{Nj} \in (a_N, b_N) | H_0), \tag{27}$$

$$\delta_{1j} = Pr(\zeta_{1j} \in (a_1, b_1), ..., \zeta_{Nj} \in (a_N, b_N) | H_1), \tag{28}$$

The other parameter that is important in any sequential detection scheme is the average number of samples (ASN) required to reach a decision. Denoting N_j to be a random variable representing the number of samples required to announce presence or absence of the primary user, the ASN for the j-th cognitive radio, denoted as $\bar{N}_j$=$E(N_j)$, can be defined as

$$E(N_j) = \pi_0 E(N_j | \mathcal{H}_0) + \pi_1 E(N_j | \mathcal{H}_1), \tag{29}$$

where

$$\begin{aligned} E(N_j | \mathcal{H}_0) &= \sum_{n=1}^{N} n Pr(N_j = n | \mathcal{H}_0) \\ &= \sum_{n=1}^{N-1} n [Pr(\forall i \in [0, n-1] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_0) \\ &- Pr(\forall i \in [0, n] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_0)] \\ &+ N Pr(\forall i \in [0, N] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_0), \end{aligned} \tag{30}$$

and

$$\begin{aligned} E(N_j | \mathcal{H}_1) &= \sum_{n=1}^{N} n Pr(N_j = n | \mathcal{H}_1) \\ &= \sum_{n=1}^{N-1} n [Pr(\forall i \in [0, n-1] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_1) \\ &- Pr(\forall i \in [0, n] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_1)] \\ &+ N Pr(\forall i \in [0, N] : \zeta_{ij} \in (a_i, b_i) | \mathcal{H}_1). \end{aligned} \tag{31}$$

The total average energy consumption at the j-th cognitive radio for the censored sequential problem formulation becomes

$$C_j = \bar{N}_j C_{sj} + (1 - \rho_j) C_{tj}. \tag{32}$$

Denoting Q_{F}^{cs} and Q_{D}^{cs} to be the respective global probabilities of false alarm and detection, we define our problem as the minimization of the maximum energy consumption over all cognitive radios subject to a constraint on the global probabilities of false alarm and detection as follows

$$\begin{aligned} &\min_{\bar{a}, \bar{b}} \ \max_{j} \ C_j \\ &\text{s.t. } Q_{\mathrm{F}}^{cs} \leq \alpha, \ Q_{\mathrm{D}}^{cs} \geq \beta. \end{aligned} \tag{33}$$

As in (13), the global probability of false alarm and detection are

$$Q_{\mathrm{F}}^{cs} = Pr(D_{\mathrm{FC}} = 1 | \mathcal{H}_0) = 1 - \prod_{j=1}^{M} (1 - P_{fj}), \tag{34}$$

$$Q_{\mathrm{D}}^{cs} = Pr(D_{\mathrm{FC}} = 1 | \mathcal{H}_1) = 1 - \prod_{j=1}^{M} (1 - P_{dj}). \tag{35}$$

Note that since $P_{f1} = \cdots = P_{fM}$, in the rest of the paper, it is assumed that $P_{fj} = P_f$.

In the following subsection, analytical expressions for the probability of false alarm and detection as well as the censoring rate and ASN are extracted.

3.2. Problem Analysis

Looking at (24)-(29), we can see that the joint probability distribution function of $p(\zeta_{1j}, ..., \zeta_{nj})$ is the foundation of all the equations. Since, $x_{ij} = \zeta_{ij} - \zeta_{i-1j}$ for $i = 1, ..., N$, we have,

$$\begin{aligned} p(\zeta_{1j}, ..., \zeta_{nj}) &= p(\zeta_{2j}, ..., \zeta_{nj} | \zeta_{1j}) p(\zeta_{1j}) \\ &= p(\zeta_{3j}, ..., \zeta_{nj} | \zeta_{1j}, \zeta_{2j}) p(\zeta_{2j} | \zeta_{1j}) p(\zeta_{1j}) \\ &= . \\ &= . \\ &= p(\zeta_{nj} | \zeta_{1j}, ..., \zeta_{n-1j}) ... p(\zeta_{1j}) \\ &= p(x_{nj}) p(x_{n-1j}) ... p(x_{1j}). \end{aligned} \tag{36}$$

Therefore, the joint probability distribution function under $\mathcal{H}_0$ and $\mathcal{H}_1$ becomes

$$p(\zeta_{1j}, ..., \zeta_{nj} | \mathcal{H}_0) = \frac{1}{2^n} e^{-\zeta_{nj}/2} I_{\{\xi_{nj}\}}, \tag{37}$$

$$p(\zeta_{1j}, ..., \zeta_{nj} | \mathcal{H}_1) = \frac{1}{[2(1+\gamma_j)]^n} e^{-\zeta_{nj}/2(1+\gamma_j)} I_{\{\xi_{nj}\}}, \tag{38}$$

where, $\xi_{nj} = \{0 \leq \zeta_{1j} \leq \zeta_{2j} ... \leq \zeta_{nj}\}$ and $I_{\{\xi_{nj}\}}$ is again the indicator function.

The local probability of false alarm and the ASN under $\mathcal{H}_0$ in this work are similar to the one that is considered in [15] and [18]. The difference is that in [15], if the cognitive radio does not reach a decision after N samples, it employs a single threshold decision policy to give a final decision about the presence or absence of the cognitive radio. Henceforth, with a small modification we can use the results in [15] for our analysis. Further, since in our work the distribution of x_{ij} under $\mathcal{H}_1$ is exponential like the one under $\mathcal{H}_0$, unlike [15] where the primary user signal is assumed to be deterministic, we can also use the above approach to derive the analytical expressions for the local probability of detection, the ASN under H_1, and the censoring rate.

However, since the problem becomes computationally complex and a two-dimensional search is necessary, in order to reach a good solution in a reasonable time, we set $a < -N\Delta$. This way we obtain

$a_1 = \cdots = a_N = 0$ and we can relax one of the arguments of (33). Therefore, we only solve the following suboptimal problem

$$\min_{\bar{b}} \ \max_{j} \ C_j \quad \text{s.t. } Q_{\mathrm{F}}^{cs} \leq \alpha,\ Q_{\mathrm{D}}^{cs} \geq \beta. \tag{39}$$

Note that unlike Section 2, here the zero lower threshold is not necessarily optimal. The reason is that although the maximum censoring rate is achieved with the lowest $\bar{a}$, the minimum ASN is achieved with the highest $\bar{a}$, thus there is an inherent trade-off between a high censoring rate and a low ASN and a zero a_i is not necessarily the optimal solution.

Denoting E_n to be the event where $a_i < \zeta_{ij} < b_i,\ i = 1, ..., n-1$ and $\zeta_{nj} \geq b_n$, (24) becomes

$$P_f = \sum_{n=1}^{N} Pr(E_n|\mathcal{H}_0). \tag{40}$$

Hence, introducing $\Gamma_n = \{a_i < \zeta_{ij} < b_i,\ i = 1, ..., n-1\}$ and $p_n = \frac{1}{2^{n-1}} e^{-b_n/2}$, the local probability of false alarm P_f can be derived as

$$\begin{aligned} Pr(E_n|\mathcal{H}_0) &= \int_{\Gamma_n} ... \int \int_{b_n}^{\infty} \frac{1}{2^n} e^{-\zeta_{nj}/2} I_{\{\xi_{nj}\}} d\zeta_{1j} ... d\zeta_{nj} \\ &= p_n \int_{\Gamma_n} ... \int I_{\{\xi_{nj}\}} d\zeta_{1j} ... d\zeta_{n-1j} \\ &= p_n A(n) \end{aligned} \tag{41}$$

To find the analytical expression for P_f, we need to derive $A(n)$. Since $0 \leq \zeta_{1j} \leq \zeta_{2j} ... \leq \zeta_{n-1j}$ and $a_1 = \cdots = a_N \leq 0$, the lower bound for each integral is ζ_{i-1} and the upper bound is b_i where $i = 1, ..., n-1$. Thus we obtain,

$$A(n) = \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} ... \int_{\zeta_{n-2j}}^{b_{n-1}} d\zeta_{1j} d\zeta_{2j} ... d\zeta_{n-1j}, \tag{42}$$

which according to [18] is

$$A(n) = \frac{b_1 b_n^{n-2}}{(n-1)!},\ n = 1, ..., N. \tag{43}$$

hence, we have

$$P_f = \sum_{n=1}^{N} p_n A(n), \tag{44}$$

with $p_n = \frac{e^{-b_n/2}}{2^{n-1}}$. Similarly, for P_{dj}, we obtain

$$P_{dj} = \sum_{n=1}^{N} q_n A(n), \tag{45}$$

where $q_n = \frac{e^{-b_n/2(1+\gamma_j)}}{[2(1+\gamma_j)]^{n-1}}$. Further, defining $R_{nj} = \{\zeta_{ij} | \zeta_{ij} \in (0, b_i),\ i = 1, ..., n\}$, $Pr(R_{nj}|\mathcal{H}_0)$ and $Pr(R_{nj}|\mathcal{H}_1)$ are given by

$$Pr(R_{nj}|\mathcal{H}_0) = 1 - \sum_{i=1}^{n} p_i A(i), \tag{46}$$

$$Pr(R_{nj}|\mathcal{H}_1) = 1 - \sum_{i=1}^{n} q_i A(i), \tag{47}$$

and (30) and (31) become

$$\begin{aligned} E(N_j|\mathcal{H}_0) &= \sum_{n=1}^{N-1} n(Pr(R_{n-1j}|\mathcal{H}_0) \\ &- Pr(R_{nj}|\mathcal{H}_0)) + NPr(R_{N-1j}|\mathcal{H}_0) \\ &= 1 + \sum_{n=1}^{N-1} Pr(R_{nj}|\mathcal{H}_0), \end{aligned} \tag{48}$$

$$\begin{aligned} E(N_j|\mathcal{H}_1) &= \sum_{n=1}^{N} n(Pr(R_{n-1j}|\mathcal{H}_1) \\ &- Pr(R_{nj}|\mathcal{H}_1)) + NPr(R_{N-1j}|\mathcal{H}_1) \\ &= 1 + \sum_{n=1}^{N-1} Pr(R_{nj}|\mathcal{H}_1). \end{aligned} \tag{49}$$

Putting (46) and (47) in (48) and (49), we obtain,

$$E(N_j|\mathcal{H}_0) = 1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^{n} p_i A(i) \right\}, \tag{50}$$

$$E(N_j|\mathcal{H}_1) = 1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^{n} q_i A(i) \right\}, \tag{51}$$

and inserting (50) and (51) in (29), we obtain,

$$\begin{aligned} \bar{N}_j = \quad & \pi_0 \left(1 + \textstyle\sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^{n} p_i A(i) \right\} \right) \\ & + \pi_1 \left(1 + \textstyle\sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^{n} q_i A(i) \right\} \right). \end{aligned} \tag{52}$$

Finally, from (46) and (47), the censoring rate can be easily obtained as

$$\rho_j = \pi_0 \left(1 - \sum_{i=1}^{N} p_i A(i) \right) + \pi_1 \left(1 - \sum_{i=1}^{N} q_i A(i) \right). \tag{53}$$

Having the analytical expressions for (39), we can easily find the optimal maximum energy consumption per sensor by a line search over $\bar{b}$. Similar to the censoring problem formulation, here the sensing threshold is also bounded by $Q_{\mathrm{F}}^{cs^{-1}}(\alpha) \leq \bar{b} \leq Q_{\mathrm{D}}^{cs^{-1}}(\beta)$.

4. NUMERICAL RESULTS

A network of $M = 5$ cognitive radios is considered for the simulations. For the sake of simplicity it is assumed that all the sensors experiences the same SNR. The cost of sensing per sample $C_{sj} = 1$ and $C_{tj} = 10$. Further, the probability of false alarm constraint $\alpha = 0.1$ and $N = 10$. In Fig. 2 the maximum energy consumption per sensor is optimized for $\gamma = 0$dB, $0.1 \leq \beta < 1$, and $\pi_0 = 0.2,\ 0.8$, and it is compared with the reference energy consumption where only censoring is employed by the cognitive radios. As we can see, the proposed censored sequential scheme depresses the maximum energy consumption per sensor for both low and high π_0 as well as over the whole range of the detection probability constraint. Further, it is shown that the censored sequential scheme gives a higher energy efficiency than its censoring counterpart, particularly at high probability of detections. It is also shown that as the π_0

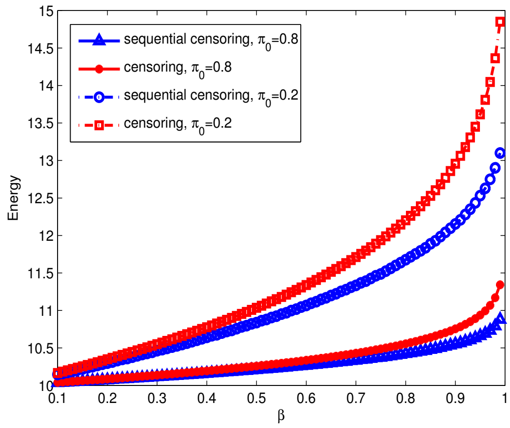

Fig. 2. Optimal maximum energy consumption per sensor versus β, $M = 5$, $N = 10$, SNR=0dB, $\alpha = 0.1$, $C_{sj} = 1$, and $C_{tj} = 10$.

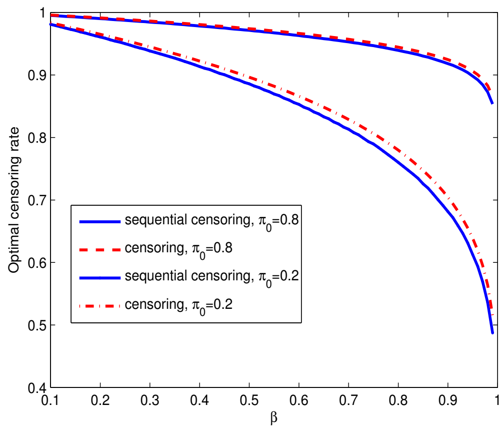

Fig. 3. Optimal censoring rate versus β, $M = 5$, $N = 10$, SNR=0dB, $\alpha = 0.1$, $C_{sj} = 1$, and $C_{tj} = 10$.

increases the maximum energy consumption per sensor decreases mainly due to a higher censoring rate.

Fig. 3 shows the optimal censoring rate versus β for the same scenario. Clearly, it is shown that the optimal censoring rate for higher π_0 is higher and further it is shown that the optimal censoring rate is slightly higher for censoring than for censored sequential sensing.

The optimal ASN versus β for the scenario in Fig. 2 is shown in Fig. 4. We can see that as π_0 increases the optimal ASN also increases which is expected due to the smaller probability of primary user appearance. Further, if the probability of detection increases the ASN is lower than the low detection rates, because the threshold $\bar{b}$ is lower for the higher detection rates and thus, cognitive radios sooner reach a decision.

Figures 5-7 consider a scenario where $M = 5$, $N = 30$, $C_{sj} = 1$, $C_{tj} = 10$, $\alpha = 0.1$, $\beta = 0.9$ and π_0 can take a value of 0.2 or 0.8. The performance of the system versus SNR is analyzed in this scenario. as for the two earlier scenarios, the censored sequential sensing gives a higher energy efficiency compared to censoring.

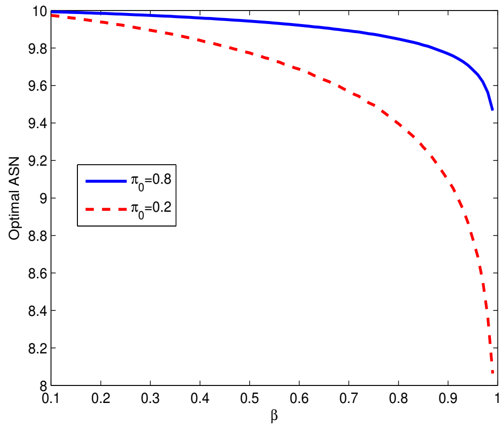

Fig. 4. Optimal ASN versus β for the censored sequential scheme, $M = 5$, $N = 10$, SNR=0dB, $\alpha = 0.1$, $C_{sj} = 1$, and $C_{tj} = 10$.

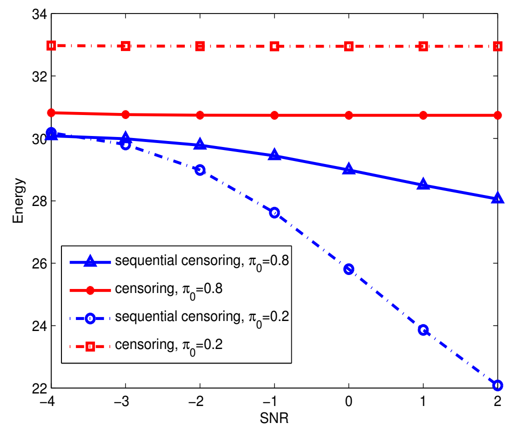

Fig. 5. Optimal maximum energy consumption per sensor versus SNR, $M = 5$, $N = 30$, $\alpha = 0.1$, $\beta = 0.9$, $C_{sj} = 1$, $C_{tj} = 10$.

While the optimal energy variation for the censoring scheme is almost the same for all the considered SNRs, the censored sequential scheme's energy consumption per sensor reduces significantly as the SNR increases. The reason is that as the SNR increases, the optimal ASN dramatically decreases.

Unlike the earlier scenarios, in Fig. 5 the optimal maximum energy per sensor for censored sequential sensing is lower over almost the whole SNR range (except for $\gamma = -4$ dB) for $\pi_0 = 0.2$. The reason is that in this scenario the number of samples is assumed to be $N = 30$ and thus, the ASN which manages the total sensing cost becomes more important compared to the censoring rate that controls the transmission energy at higher SNRs. Since the optimal ASN for $\pi_0 = 0.2$ is lower than the one for $\pi_0 = 0.8$, the energy consumption per sensor becomes lower, although the censoring rate for $\pi_0 = 0.8$ is higher. Furthermore, it is shown that as the number of samples increases, the censored sequential scheme gives a much lower energy consumption than its censoring counterpart.

Finally, Fig. 8 depicts the optimal maximum energy consumption per sensor versus the number of cognitive radios. The SNR is

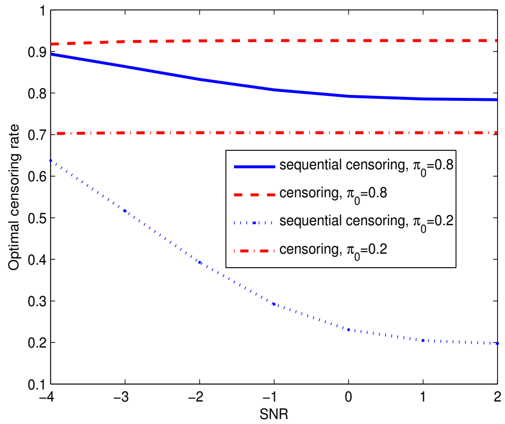

Fig. 6. Optimal censoring rate versus SNR, $M = 5$, $N = 30$, $\alpha = 0.1$, $\beta = 0.9$, $C_{sj} = 1$, $C_{tj} = 10$.

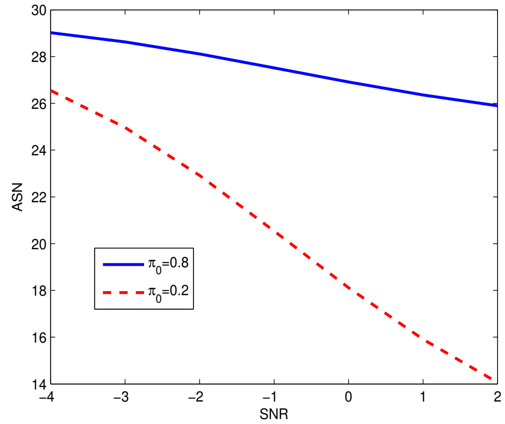

Fig. 7. Optimal ASN versus SNR for the censored sequential scheme, $M = 5$, $N = 30$, $\alpha = 0.1$, $\beta = 0.9$, $C_{sj} = 1$, $C_{tj} = 10$.

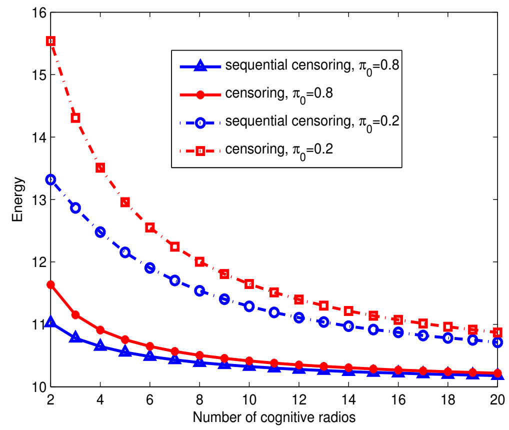

Fig. 8. Optimal maximum energy consumption per sensor versus number of cognitive radios, $N = 10$, $\alpha = 0.1$, $\beta = 0.9$, SNR=0dB, $C_{sj} = 1$, $C_{tj} = 10$.

assumed to be 0 dB, $N = 10$, $C_{sj} = 1$ and $C_{tj} = 10$. Furthermore, the probability of false alarm and detection constraints are assumed to be $\alpha = 0.1$ and $\beta = 0.9$ as determined by the IEEE 802.22 standard for cognitive radios [4]. It is shown that for both high and low values of π_0 censored sequential sensing outperforms censoring.

5. CONCLUSIONS

We presented two energy efficient techniques for a cognitive sensor network. First, a censoring scheme has been discussed where each sensor employs a censoring policy to reduce the energy consumption. Then a truncated censored sequential approach has been proposed based on the combination of censoring and sequential sensing policies. We defined our problem as the minimization of the maximum energy consumption per sensor subject to a global probability of false alarm and detection. The optimal lower threshold is shown to be zero for the censoring scheme and so the underlying optimization problem can be simplified to a line search problem. Further, an explicit expression is given to find the optimal solution. We have further derived the analytical expressions for the underlying parameters in the censored sequential scheme when the lower threshold is assumed to be zero.

Different scenarios regarding SNR, number of cognitive radios, and probability of detection constraints were simulated for low and high values of π_0. It was shown that under the practical assumption of low-power radios, sequential censoring outperforms censoring. Further, it was shown that the optimal ASN for low values of π_0 is lower than for high values and the same trend is also valid for the optimal censoring rate.

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