

# Recovering Second-Order Statistics from Compressive Measurements

Geert Leus\* and Zhi Tian†

\*Delft University of Technology, The Netherlands, g.j.t.leus@tudelft.nl

†Michigan Technological University, USA, ztian@mtu.edu

**Abstract**—This paper focuses on the reconstruction of second order statistics of signals under a compressive sensing framework, which can be useful in many detection problems. More specifically, the focus is on general cyclostationary signals that are compressed using random linear projections, and using those compressive measurements, the cyclic power spectrum is retrieved. Subsequently, this can for instance be used to detect the occupation of specific frequency bands, which has applications in cognitive radio. Surprisingly, if the span of the random linear projections is larger than the period of the cyclostationary signals, the cyclic power spectrum can be recovered without putting any sparsity constraints on it, which allows for simple least squares reconstruction methods. This result indicates that significant compression can be realized by directly reconstructing the second-order statistics rather than the random signals themselves.

## I. INTRODUCTION

In recent years, there have been considerable efforts devoted to the theory and algorithms for compressive sensing [1], [2]. Most efforts focus on deterministic and sparse signals in linear systems, such as sampling and reconstruction of the *frequency spectrum* of a signal waveform. In such systems, the compressive measurements are linearly related to the sparse unknowns to be recovered. This linear relationship is essential for effective sparse signal recovery, which is the cornerstone for those well-known sparse linear regression techniques such as Basis Pursuit (BP) based on  $\ell_1$ -norm minimization and Matching Pursuit (MP) based on signal-basis correlation [2], [3].

On the other hand, many signal processing problems deal with random processes. In this case, it is not meaningful to reconstruct the random signal itself; rather, the goal is to extract its (second-order or even higher-order) statistics such as the *power spectrum* and *cyclic power spectrum*. It has been well recognized that second-order statistics contain reliable information for random processes and are rich in useful features [4], [5], [6]. For wideband random processes, compressive measurements can be obtained via random linear projections at a sub-Nyquist rate to reduce the signal acquisition costs [7], [8]. Unfortunately, second-order statistics do not have a direct linear relationship with the compressive measurements, which renders existing linear regression algorithms inapplicable.

This paper develops a new compressive sensing framework for reconstruction of second-order statistics from compressive measurements. The focus is to reconstruct the two-dimensional cyclic power spectrum of a cyclostationary random process, while treating the problem of power spectrum recovery as a

special case. As a key step, this paper establishes a transformed linear system that connects the time-varying cross-correlations of compressive measurements to the desired second-order statistics. As long as the span of the random linear projections is larger than the period of the cyclostationary signals, the cyclic power spectrum can be recovered from the transformed linear system using simple least squares reconstruction methods, even when the random process is non-sparse. This surprising result is owing to the fact that we directly recover the second-order statistics which has less degrees of freedom than the random signal itself.

*Notation:* Upper (lower) boldface letters are used to denote matrices (column vectors);  $(\cdot)^T$  represents the transpose;  $(\cdot)^*$  the complex conjugate;  $(\cdot)^H$  the complex conjugate transpose or Hermitian;  $(\cdot)^\dagger$  the matrix pseudo-inverse;  $\otimes$  the Kronecker product;  $\text{vec}\{\cdot\}$  the column-wise matrix vectorization;  $\|\cdot\|_p$  the  $\ell_p$  norm;  $[f(k, l)]_{k, l}$  the matrix with the  $(k, l)$ th entry given by  $f(k, l)$  which can be any scalar function of  $k$  and  $l$  (the first entry is indicated by the index 0); and  $\mathbb{E}\{\cdot\}$  the expected value.

## II. RELATED WORK

Existing approaches mainly focus on the reconstruction of the power spectrum of stationary signals. For instance, the work in [9] estimates the power spectrum of the original stationary signal from the power spectra of the outputs of the linear projections constituting the compressive sampler, but it does not exploit the cross power spectra among those outputs. As a result, the reconstruction problem is underdetermined and requires the adoption of a sparsity constraint on the power spectrum in order to estimate it. On the other hand, [10] recovers the autocorrelation function of the signal for a limited span of lags based on the cross-correlation functions among the linear projection outputs at lag zero. The major breakthrough of this work is that an overdetermined problem can be obtained, even with significant compression, and as such, no sparsity constraints are required to solve this problem. For improved reconstruction accuracy, generalizations are considered in [11], [12], where all significant lags of the autocorrelation function of the signal are retrieved from all significant lags of the cross-correlation functions among the linear projection outputs, leading to satisfying power spectrum estimates. This again yields an overdetermined problem, provided that the number of linear projection outputs is larger than the square

root of the span of the linear projections (measured in Nyquist-rate samples).

The only work up to date recovering the cyclic power spectrum of a cyclostationary signal from compressive measurements can be found in [13], [14]. This paper considers a compressive sampler with linear projections that span one period of the cyclostationary signal. As in [10], only the cross-correlation functions of the linear projection outputs at lag zero are used, and thus the cyclic autocorrelation function of the signal can only be reconstructed for a limited span of lags, leading to an inaccurate cyclic power spectrum estimate. In this paper, we extend this approach, and estimate all significant lags of the cyclic autocorrelation function of the signal based on all significant lags of the cross-correlation functions of the linear projection outputs. However, even with this extension, an underdetermined system is obtained when the signal is compressed. The only way to realize an overdetermined system under compression is by increasing the span of the compressive sampler beyond one period of the cyclostationary signal. This is the topic of the current paper.

### III. PROBLEM STATEMENT

Consider a wide-sense cyclostationary signal  $x[t]$ , which means that the autocorrelation sequence  $r_x[t, \tau] = E\{x[t]x^*[t - \tau]\}$  is periodic in  $t$  with some period  $T$ . The corresponding cyclic autocorrelation sequence is given by

$$\tilde{r}_x[f, \tau] = \frac{1}{T} \sum_{t=0}^{T-1} r_x[t, \tau] e^{-j2\pi f(t-\tau/2)/T}, \quad (1)$$

and the cyclic power spectrum by

$$s_x[f, \phi] = \sum_{\tau=-\infty}^{\infty} \tilde{r}_x[f, \tau] e^{-j2\pi\phi\tau}, \quad (2)$$

where  $f \in \{0, 1, \dots, T-1\}$  is the cyclic frequency and  $\phi \in [0, 1)$  is the frequency. If  $x[t]$  is obtained by sampling at a rate of  $f_s$  Hz, then the variable  $f$  corresponds to an actual cyclic frequency of  $f f_s / T$  Hz and the variable  $\phi$  corresponds to an actual frequency of  $\phi f_s$  Hz.

Equivalently, we can also stack  $T$  successive samples  $x[t]$  in  $\mathbf{x}[n] = [x[nT], \dots, x[nT + T - 1]]^T$ , which is a stationary vector sequence with autocorrelation matrix sequence  $\mathbf{R}_x[\nu] = E\{\mathbf{x}[n]\mathbf{x}^H[n - \nu]\} = [r_x[t, \nu T + t - \tau]]_{t, \tau}$ . Hence,  $r_x[t, \tau]$  uniquely determines  $\mathbf{R}_x[\nu]$  and vice versa.

For wideband signals, the signal acquisition costs can be reduced by compressive sensing. More specifically, we consider a reduced-rate sampler of the form

$$\begin{bmatrix} y_1[k] \\ \vdots \\ y_M[k] \end{bmatrix} = \begin{bmatrix} \mathbf{c}_{1,1}^T & \dots & \mathbf{c}_{1,N}^T \\ \vdots & & \vdots \\ \mathbf{c}_{M,1}^T & \dots & \mathbf{c}_{M,N}^T \end{bmatrix} \begin{bmatrix} \mathbf{x}[kN] \\ \vdots \\ \mathbf{x}[kN + N - 1] \end{bmatrix}, \quad (3)$$

where the vectors  $\mathbf{c}_{i,j} \in \mathbb{C}^{T \times 1}$  are random sampling vectors with  $i = 1, \dots, M$  and  $j = 1, \dots, N$ . In other words, we take  $M$  random linear projections with a total span of  $NT$ , i.e.,  $N$  times the period of the cyclostationary signal. Compression is

achieved by taking  $M \leq NT$ . These compressive measurements allow us to compute  $r_{y_i, y_j}[\kappa] = E\{y_i[k]y_j^*[k - \kappa]\}$ , for  $i, j = 1, \dots, M$ . Thus, the problem we tackle in this paper is the reconstruction of  $r_x[t, \tau]$  ( $\mathbf{R}_x[\nu]$ ) or  $s_x[f, \phi]$  from  $r_{y_i, y_j}[\kappa]$ , for  $i, j = 1, \dots, M$ .

### IV. LINEAR RELATIONSHIPS

To simplify the reconstruction, we would benefit significantly from some explicit linear relationships. First, we derive a linear relationship between  $r_{y_i, y_j}[\kappa]$  and  $\mathbf{R}_x[\nu]$ . Then, we will use the expressions (1) and (2) to relate  $s_x[f, \phi]$  in a linear fashion to  $\mathbf{R}_x[\nu]$ . These simple linear equalities will form the basis of the reconstruction process, which will be discussed in Section V.

#### A. Relating $r_{y_i, y_j}[\kappa]$ to $\mathbf{R}_x[\nu]$

Let us start by observing that the sampling procedure in (3) can also be interpreted as a vector filtering operation on the vector sequence  $\mathbf{x}[n]$  followed by an  $N$ -fold decimation, or in other words  $y_i[k] = z_i[kN]$ , where

$$z_i[n] = \mathbf{c}_i^T[n] \star \mathbf{x}[k] = \sum_{m=1-N}^0 \mathbf{c}_i^T[m] \mathbf{x}[n - m],$$

with  $\mathbf{c}_i[n] = \mathbf{c}_{i, -n+1}$  for  $n = 1 - N, \dots, 0$ . Using this interpretation, it is easy to show that

$$\begin{aligned} r_{y_i, y_j}[\kappa] &= E\{y_i[k]y_j^*[k - \kappa]\} \\ &= E\{z_i[kN]z_j^*[(k - \kappa)N]\} = r_{z_i, z_j}[\kappa N], \end{aligned} \quad (4)$$

where

$$r_{z_i, z_j}[\nu] = \sum_{\mu=1-N}^0 \sum_{\eta=\mu}^{\mu-1+N} \mathbf{c}_i^T[\mu] \mathbf{R}_x[\nu - \eta] \mathbf{c}_j^*[\mu - \eta]. \quad (5)$$

Using the property  $\mathbf{a}^T \mathbf{X} \mathbf{b}^* = (\mathbf{b}^H \otimes \mathbf{a}^T) \text{vec}\{\mathbf{X}\}$  and defining  $\text{vec}\{\mathbf{R}_x[\nu]\} = \mathbf{r}_x[\nu]$ , we can also rewrite (5) as

$$\begin{aligned} r_{z_i, z_j}[\nu] &= \mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}^T[\nu] \star \mathbf{r}_x[\nu] \\ &= \sum_{\eta=-N+1}^{N-1} \mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}^T[\eta] \mathbf{r}_x[\nu - \eta], \end{aligned} \quad (6)$$

with

$$\mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}[\eta] = \sum_{\mu=\max\{1-N, 1-N+\eta\}}^{\min\{0, \eta\}} \mathbf{c}_j^*[\mu - \eta] \otimes \mathbf{c}_i[\eta].$$

From (4) and (6), we can thus write

$$\begin{aligned} r_{y_i, y_j}[\kappa] &= \sum_{\eta=-N+1}^{N-1} \mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}^T[\eta] \mathbf{r}_x[\kappa N - \eta] \\ &= \sum_{\lambda=0}^1 \tilde{\mathbf{r}}_{\mathbf{c}_i, \mathbf{c}_j}^T[\lambda] \tilde{\mathbf{r}}_x[\kappa - \lambda], \end{aligned} \quad (7)$$

where we have

$$\begin{aligned} \tilde{\mathbf{r}}_{\mathbf{c}_i, \mathbf{c}_j}[\kappa] &= [\mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}^T[\kappa N], \dots, \mathbf{r}_{\mathbf{c}_i, \mathbf{c}_j}^T[(\kappa - 1)N + 1]]^T, \\ \tilde{\mathbf{r}}_x[\kappa] &= [\mathbf{r}_x^T[\kappa N], \dots, \mathbf{r}_x^T[(\kappa + 1)N - 1]]^T. \end{aligned}$$

Stacking the  $M^2$  different cross-correlation functions  $r_{y_i, y_j}[\kappa]$ , i.e.,  $\mathbf{r}_y[\kappa] = [\dots, r_{y_i, y_j}^T[\kappa], \dots]^T$ , for  $i, j = 1, \dots, M$ , we finally obtain

$$\mathbf{r}_y[\kappa] = \sum_{\lambda=0}^1 \bar{\mathbf{R}}_c[\lambda] \bar{\mathbf{r}}_x[\kappa - \lambda], \quad (8)$$

where  $\bar{\mathbf{R}}_c[\kappa]$  is the  $M^2 \times NT^2$  matrix given by  $\bar{\mathbf{R}}_c[\kappa] = [\dots, \bar{\mathbf{r}}_{c_i, c_j}[\kappa], \dots]^T$ , for  $i, j = 1, \dots, M$ .

Assuming  $\mathbf{r}_y[\kappa]$  has a support limited to  $-L \leq \kappa \leq L$ , the support of  $\mathbf{r}_x[\nu]$  should be limited to  $-LN \leq \nu \leq LN$ , which means that the support of  $\bar{\mathbf{r}}_x[\kappa]$  should be limited to  $-L \leq \kappa \leq L$ . As a result, all the information can be gathered into the vectors

$$\mathbf{r}_y = [\mathbf{r}_y^T[0], \mathbf{r}_y^T[1], \dots, \mathbf{r}_y^T[L], \mathbf{r}_y^T[-L], \dots, \mathbf{r}_y^T[-1]]^T, \quad (9)$$

$$\bar{\mathbf{r}}_x = [\bar{\mathbf{r}}_x^T[0], \bar{\mathbf{r}}_x^T[1], \dots, \bar{\mathbf{r}}_x^T[L], \bar{\mathbf{r}}_x^T[-L], \dots, \bar{\mathbf{r}}_x^T[-1]]^T. \quad (10)$$

From (8), and the fact that the first  $T^2$  columns of  $\bar{\mathbf{R}}_c[1]$  as well as the last  $(N-1)T^2$  entries of  $\bar{\mathbf{r}}_x[L]$  are zero, the relation between  $\mathbf{r}_y$  and  $\bar{\mathbf{r}}_x$  can finally be expressed as

$$\mathbf{r}_y = \bar{\mathbf{R}}_c \bar{\mathbf{r}}_x, \quad (11)$$

where  $\bar{\mathbf{R}}_c$  is the  $(2L+1)M^2 \times (2L+1)NT^2$  matrix given by

$$\bar{\mathbf{R}}_c = \begin{bmatrix} \bar{\mathbf{R}}_c[0] & & & & \bar{\mathbf{R}}_c[1] \\ \bar{\mathbf{R}}_c[1] & \bar{\mathbf{R}}_c[0] & & & \\ & \bar{\mathbf{R}}_c[1] & \bar{\mathbf{R}}_c[0] & & \\ & & & \ddots & \\ & & & & \bar{\mathbf{R}}_c[1] & \bar{\mathbf{R}}_c[0] \end{bmatrix}.$$

### B. Relating $s_x[f, \phi]$ to $\mathbf{R}_x[\nu]$

Let us start by defining the  $T \times T$  matrix  $\mathbf{R}_\nu$  as

$$\mathbf{R}_\nu = \begin{bmatrix} r_x[0, \nu T] & \dots & r_x[0, (\nu+1)T-1] \\ \vdots & & \vdots \\ r_x[T-1, \nu T] & \dots & r_x[T-1, (\nu+1)T-1] \end{bmatrix},$$

and the  $T \times (2L+1)NT$  matrix  $\mathbf{R}$  as

$$\mathbf{R} = [\mathbf{R}_0, \mathbf{R}_1, \dots, \mathbf{R}_{(L+1)N-1}, \mathbf{R}_{-LN}, \dots, \mathbf{R}_{-1}].$$

We may assume that only the first column of the matrix  $\mathbf{R}_{(L+1)N-1}$  is non-zero, and thus we could virtually assume that the other entries of  $\mathbf{R}_{(L+1)N-1}$  are filled with the entries at the bottom corner of  $\mathbf{R}_x[(L+1)N-1]$  and the top corner of  $\mathbf{R}_x[-LN]$  (which are also all zero). In such a way, all the entries of  $\mathbf{R}$  can be linearly related to  $\bar{\mathbf{r}}_x$ , and we can write

$$\text{vec}\{\mathbf{R}\} = \mathbf{P} \bar{\mathbf{r}}_x,$$

where  $\mathbf{P}$  is a specific  $(2L+1)NT^2 \times (2L+1)NT^2$  permutation matrix.

Let us next also define the  $T \times T$  matrix  $\tilde{\mathbf{R}}_\nu$  as

$$\tilde{\mathbf{R}}_\nu = \begin{bmatrix} \tilde{r}_x[0, \nu T] & \dots & \tilde{r}_x[0, (\nu+1)T-1] \\ \vdots & & \vdots \\ \tilde{r}_x[T-1, \nu T] & \dots & \tilde{r}_x[T-1, (\nu+1)T-1] \end{bmatrix},$$

and the  $T \times (2L+1)NT$  matrix  $\tilde{\mathbf{R}}$  as

$$\tilde{\mathbf{R}} = [\tilde{\mathbf{R}}_0, \tilde{\mathbf{R}}_1, \dots, \tilde{\mathbf{R}}_{(L+1)N-1}, \tilde{\mathbf{R}}_{-LN}, \dots, \tilde{\mathbf{R}}_{-1}].$$

From (1), we can connect  $\tilde{\mathbf{R}}$  with  $\mathbf{R}$  through

$$\tilde{\mathbf{R}} = \sum_{\tau=-LNT}^{(L+1)NT-1} \mathbf{G}_\tau \mathbf{R} \mathbf{D}_\tau, \quad (12)$$

where  $\mathbf{G}_\tau$  is the  $T \times T$  matrix given by  $\mathbf{G}_\tau = 1/T [e^{-j2\pi f(t-\tau/2)/T}]_{f,t}$  and  $\mathbf{D}_\tau$  is the  $(2L+1)NT \times (2L+1)NT$  diagonal matrix with a one on the  $(\tau \bmod (2L+1)NT)$ th diagonal entry and zeros elsewhere.

Since  $\tilde{r}_x[f, \tau]$  is limited to  $-LNT \leq \tau \leq (L+1)NT-1$ , the cyclic power spectrum  $s_x[f, \phi]$  is uniquely determined by  $(2L+1)NT$  of its samples in  $\phi$ , i.e. by  $s_x[f, \frac{\varphi}{(2L+1)NT}]$  with  $0 \leq \varphi \leq (2L+1)NT-1$ . These samples can be stacked into

$$\mathbf{S} = \begin{bmatrix} s_x[0, 0] & \dots & s_x[0, \frac{(2L+1)NT-1}{(2L+1)NT}] \\ \vdots & & \vdots \\ s_x[T-1, 0] & \dots & s_x[T-1, \frac{(2L+1)NT-1}{(2L+1)NT}] \end{bmatrix},$$

which from (2) and (12) can be written as

$$\mathbf{S} = \tilde{\mathbf{R}} \mathbf{F} = \sum_{\tau=-LNT}^{(L+1)NT-1} \mathbf{G}_\tau \mathbf{R} \mathbf{F}_\tau, \quad (13)$$

where  $\mathbf{F}$  is the  $(2L+1)NT \times (2L+1)NT$  DFT matrix and  $\mathbf{F}_\tau = \mathbf{D}_\tau \mathbf{F}$  is the  $(2L+1)NT \times (2L+1)NT$  matrix that selects the  $(\tau \bmod (2L+1)NT)$ th row out of  $\mathbf{F}$ . Vectorizing this expression, we obtain

$$\begin{aligned} \mathbf{s}_x &= \text{vec}\{\mathbf{S}\} = \sum_{\tau=-LNT}^{(L+1)NT-1} (\mathbf{F}_\tau^T \otimes \mathbf{G}_\tau) \text{vec}\{\mathbf{R}\} \\ &= \sum_{\tau=-LNT}^{(L+1)NT-1} (\mathbf{F}_\tau^T \otimes \mathbf{G}_\tau) \mathbf{P} \bar{\mathbf{r}}_x = \mathbf{T} \bar{\mathbf{r}}_x, \end{aligned} \quad (14)$$

where  $\mathbf{T}$  is the  $(2L+1)NT^2 \times (2L+1)NT^2$  transformation matrix given by  $\mathbf{T} = \sum_{\tau=-LNT}^{(L+1)NT-1} (\mathbf{F}_\tau^T \otimes \mathbf{G}_\tau) \mathbf{P}$ . This matrix  $\mathbf{T}$  can be shown to be always full rank, and thus we can also write

$$\bar{\mathbf{r}}_x = \mathbf{T}^{-1} \mathbf{s}_x,$$

which combined with (11) also leads to

$$\mathbf{r}_y = \bar{\mathbf{R}}_c \mathbf{T}^{-1} \mathbf{s}_x. \quad (15)$$

### V. RECONSTRUCTION

When  $\bar{\mathbf{R}}_c$  has full column rank, we can directly solve (15) for  $\mathbf{s}_x$ , leading to

$$\hat{\mathbf{s}}_x = (\bar{\mathbf{R}}_c \mathbf{T}^{-1})^\dagger \mathbf{r}_y. \quad (16)$$

Alternatively, we can first solve (11) for  $\bar{\mathbf{r}}_x$  using least squares, leading to

$$\hat{\bar{\mathbf{r}}}_x = \bar{\mathbf{R}}_c^\dagger \mathbf{r}_y, \quad (17)$$

and from (14) this gives us

$$\hat{\mathbf{s}}_x = \mathbf{T} \hat{\bar{\mathbf{r}}}_x. \quad (18)$$

The computational load and memory requirements for solving (17) and (18) can be greatly reduced by noting that  $\bar{\mathbf{R}}_c$  is a block circulant matrix and thus can be converted into a block diagonal matrix via inverse DFT. Then,  $\{\hat{\bar{\mathbf{r}}}_x[l]\}_{l=-L}^L$  can be recovered in a parallel fashion in frequency domain [11], [12].

Note that these two approaches require  $M^2 \geq NT^2$ . When  $N = 1$ , then  $M^2 \geq NT^2$  only holds when  $M = T$ , which means we have achieved no compression. When  $N > 1$ , however, it is possible to obtain  $M^2 \geq NT^2$  for  $M < NT$ . Hence, when the span of the random linear projections  $NT$  is larger than the period of the cyclostationary signal  $T$ , it is possible to reconstruct the autocorrelation sequence of a signal from its compressive measurements using a simple least squares method, i.e., without taking any sparsity constraints into account and even when the signal is non-sparse.

On the other hand, when  $\bar{\mathbf{R}}_c$  does not have full rank, which for instance occurs when  $M^2 < NT^2$ , we can not reconstruct the autocorrelation sequence without taking some sparsity constraints into account, e.g., the sparsity of the cyclic spectrum. Hence, in that case, we could formulate an  $\ell_1$ -norm constrained least squares problem to solve (15) for  $\mathbf{s}_x$ :

$$\min_{\mathbf{s}_x} \|\mathbf{r}_y - \bar{\mathbf{R}}_c \mathbf{T}^{-1} \mathbf{s}_x\|_2^2 + \lambda \|\mathbf{s}_x\|_1, \quad (19)$$

where  $\lambda > 0$  is a weighting factor that balances between providing a small output error (through the first term) and a sparse cyclic spectrum (through the second term). This problem is well-known in the compressive sensing literature and can be solved by a plethora of algorithms [2], [3].

## VI. SPECIAL CASES

A first special case occurs when  $T = 1$ , which basically means that we are dealing with a stationary signal. In that case, many of the derivations simplify and then the proposed approach reduces to the approach discussed in [11] (see also [12] for earlier investigations for real-valued signals).

A second special case occurs when  $N = 1$ , which means that the span of the random linear projections is the same as the period of the cyclostationary signal. This case has also been discussed in [13], [14], although the focus therein was limited to reconstructing  $\mathbf{R}_x[0]$  from  $r_{y_i, y_j}[0]$  for  $i, j = 1, \dots, M$ , whereas this paper aims at reconstructing all relevant  $\mathbf{R}_x[\nu]$ 's from the related  $r_{y_i, y_j}[\kappa]$ 's for  $i, j = 1, \dots, M$ , allowing for a higher-resolution estimate of the cyclic spectrum. As discussed earlier, this case always requires some sparsity constraints.

## VII. SIMULATIONS

Consider a pulse amplitude modulated (PAM) waveform that is sampled  $T$  times per symbol period  $T_s$  to generate the cyclostationary signal  $x[t] = \sum_n a_n p((t/T - n)T_s)$ , where  $\{a_n\}$  are binary-valued symbols and  $p(\cdot)$  is the pulse shaper in the form of a  $T_s$ -long rectangle. A Gaussian random sampler is used to generate compressive samples  $y_i[k]$ , using parameters  $T = 16$ ,  $N = 4$ ,  $L = 20$ , and various values of  $M$ . The compression ratio is  $\eta = M/(NT)$ , where we choose  $M^2 \geq NT^2$  to avoid any sparsity constraints. The cyclic power spectrum  $\hat{\mathbf{s}}_x$  estimated from (16) is compared with the true one  $\mathbf{s}_x$  in (2). As Fig 1 illustrates, the normalized mean-square estimation error decreases as the compression ratio  $\eta$  becomes large, and stays low for a broad range of  $\eta < 1$ . Evidently, the proposed technique effectively reconstructs the second-order statistics of non-sparse signals, without any sparsity constraints.

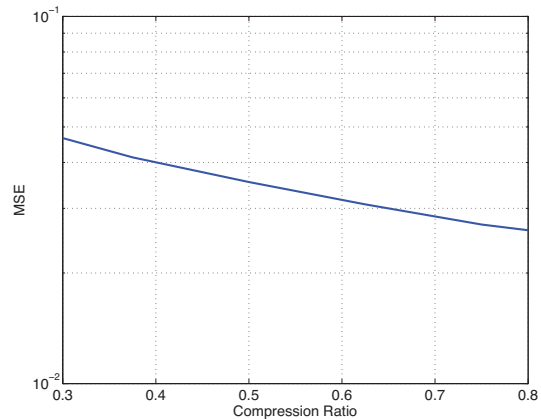


Fig. 1. Performance of compressive cyclic spectrum estimation: normalized mean square error (MSE) vs. compression ratio  $\eta = M/(NT)$ .

## VIII. SUMMARY

This paper develops a compressive sensing technique for extracting second-order (cyclic) statistics of random processes. The proposed technique utilizes all significant lags of the cross-correlation functions of the linear projection outputs, such that significant compression is effected even for non-sparse signals. Considering the rich features in second-order statistics, it will open up opportunities for a gamut of new techniques with enhanced capability in handling random processes in the wideband regime.

## REFERENCES

- [1] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. on Information Theory*, vol. 52(2), pp. 489-509, 2006.
- [2] D. L. Donoho, "Compressed sensing," *IEEE Trans. on Information Theory*, vol. 52 (4), pp. 1289-1306, 2006.
- [3] J. A. Tropp and A. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. on Information Theory*, vol. 53 (12), Dec. 2007.
- [4] W. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Processing Mag.*, vol. 8 (2), pp. 14-36, April 1991.
- [5] G. B. Giannakis, "Cyclostationary Signal Analysis," *Digital Signal Processing Handbook* CRC Press LLC, 1999.
- [6] K. Kim et al. "Cyclostationary Approaches to Signal Detection and Classification in Cognitive Radio," *IEEE Intl. Symp. on DySPAN*, pp. 212-215, 2007.
- [7] S. Kirolos et al., "Analog-to-Information Conversion via Random Demodulation," *IEEE Workshop on Design, Applications, Integration and Software*, pp. 71-74, 2006.
- [8] Z. Yu, S. Hoyos, and B. M. Sadler, "Mixed-signal parallel compressed sensing and reception for cognitive radio," *IEEE ICASSP Conf.*, pp. 3861-3864, 2008.
- [9] V. Havary-Nassab, S. Hassan, and S. Valaee, "Compressive Detection for Wideband Spectrum Sensing," *IEEE ICASSP Conf.*, March 2010.
- [10] G. Leus and D.D. Ariananda and G. Leus, "Wideband Power Spectrum Sensing using Sub-Nyquist Sampling," *IEEE SPAWC Conf.*, June 2011.
- [11] D.D. Ariananda, G. Leus, and Z. Tian, "Multi-coset Sampling for Power Spectrum Blind Sensing," *IEEE DSP Conf.* July 2011.
- [12] G. Leus and D.D. Ariananda, "Power Spectrum Blind Sampling," *IEEE Signal Processing Letters*, 2011.
- [13] Z. Tian, "Cyclic Feature based Wideband Spectrum Sensing using Compressive Sampling," *IEEE ICC Conf.*, June 2011.
- [14] Z. Tian, Y. Tafesse, and B. M. Sadler, "Compressive Cyclic Feature Spectrum Sensing for Wideband Cognitive Radios," *IEEE Journal of Selected Topics in Signal Processing*, submitted in 2011.