Frequency Agile Waveform Adaptation for Cognitive Radios

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***Abstract*— In the context of cognitive radio (CR) networks, this paper develops a frequency-agile waveform adaptation technique that dynamically adjusts the spectral shape, power and frequency of transmission waveforms for efficient network spectrum utilization. A general framework is presented based on generalized transmitter and receiver basis functions, which allow to jointly carry out dynamic resource allocation (DRA) and waveform adaptation, two procedures that are traditionally carried out separately. New objective function and cognitive spectral mask constraints are formulated for DRA optimization tailored to CR applications. The joint DRA and waveform adaptation approach permits distributed games in multiple access networks, in which participating CR users optimize their respective local utility functions by taking actions from the action space defined by allowable basis function parameters. It results in not only enhanced radio spectrum resource efficiency due to joint optimization, but also affordable complexity scalable in the network size, by virtue of the distributed game approach adopted.**

I. Introduction

The emerging paradigm of open spectrum access shows promise to alleviate today's spectrum scarcity problem [1]. Key to this new paradigm are frequency-agile cognitive radios (CRs) that can rapidly tune to desired spectrum bands at affordable cost. Broadly, CRs dynamically decide the allocation of available spectrum resources to improve the overall efficiency, also known as dynamic resource allocation (DRA) [2]. When the spectrum resources are cast as the spectral shapes of transmit waveforms, DRA subsumes waveform adaptation.

This paper develops a frequency-agile waveform adaptation technique that dynamically adjust spectral shape, power and frequency of radio transmission waveforms for DRA optimization. Such a DRA can be carried out in a distributed fashion using distributed games [3], [4]. In that case, every radio will iteratively sense the available resources and adjust its own usage of these resources accordingly. Such resources could for instance be represented by transmitter and receiver basis functions, which can be judiciously chosen to enable various agile platforms, such as frequency, time, or code division multiplexing (FDM, TDM, CDM). For convenience, an OFDM platform with carriers as basis functions is generally considered, leading to what is known as dynamic spectrum allocation (DSA) [5]. However, many platforms are TDM- or CDM-based and make use of different types of basis functions such as pulses, codes, or wavelets. In this paper, we will therefore extend the DSA approach to a more general DRA approach that allows us to handle and/or combine all kinds of basis functions.

Based on the basis expansion framework, this paper formulates a DRA optimization problem using CR-unique objective function and spectral mask constraints. The objective function aims at optimally utilizing the network radio spectrum resources, while the mask constraints are cognitive of both government spectrum regulations and the need to protect license-holding primary users. By virtue of the basis expansion framework adopted, the DRA outcomes naturally lead to adaptation of the transmit waveform spectral shapes. It is thus possible to combine DRA with waveform adaptation [6], [7], two operations that are generally carried out separately.

II. System Model

In DRA, the basic goal is to jointly optimize the usage of available radio resources [2]. In this paper, the resources are represented by means of a set of transmitter and receiver basis functions $\{\psi_k(t)\}_{k=0}^{K-1}$ and $\{\phi_k(t)\}_{k=0}^{K-1}$. Hence, each cognitive radio q, $q = 0, 1, \ldots, Q-1$, communicates data using the basis functions $\{\psi_k(t)\}_k$ and preprocesses the data at the receiver using the basis functions $\{\phi_k(t)\}_k$. In this context, waveform adaptation boils down to judicious usage of these basis functions to form transmit waveforms of desired spectral shapes and power levels, in response to channel and network dynamics.

Suppose CR q modulates the data symbol $u_{q,k}$ on $\psi_k(t)$. Assuming the channel impulse response of CR q is given by $g_q(t)$, the signal received at CR q can then be written as

$$x_q(t) = g_q(t) \star \sum_k u_{q,k}\psi_k(t) + v_q(t),$$

where $v_q(t)$ represents the interfering signal, which includes the interference from other CRs, primary users, and noise. The resulting sample at CR q received on $\phi_l(t)$ can then be described as

$$\begin{aligned} x_{q,k} &= \int x_q(t) \star \phi_l^*(-t)dt \\ &= \sum_k u_{q,k} \int g_q(t) \star \psi_k(t) \star \phi_l^*(-t)dt \\ &\quad + \int v_q(t) \star \phi_l^*(-t)dt \\ &= \sum_k u_{q,k} h_{q,k,l} + v_{q,k}, \end{aligned} \tag{1}$$

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where $\{h_{q,k,l}\}_{k,l}$ represents the set of composite channel coefficients for CR q. For the sake of simplicity, we only consider the transmission of a single block of K data symbols. Other blocks are transmitted in a similar fashion. Interblock interference (IBI) can for instance be avoided by the use of a cyclic prefix, as illustrated in the example below.

The above set-up incorporates FDM, TDM, as well as CDM scenarios. In FDM, we could take

$$\psi_k(t) = \frac{1}{\sqrt{K+N}} \sum_{n=0}^{K+N-1} e^{j2\pi k(n-N)/K} p(t-nT) \quad (2a)$$

$$\phi_l(t) = \frac{1}{\sqrt{K}} \sum_{n=N}^{K+N-1} e^{j2\pi l(n-N)/K} p(t-nT) \quad (2b)$$

whereas in TDM, we could take

$$\psi_k(t) = \frac{\sqrt{K}}{\sqrt{K+n_0}} \sum_{n=0}^{K+N-1} \delta_{<n-N-k>_K} p(t-nT) \quad (3a)$$

$$\phi_l(t) = \sum_{n=N}^{K+N-1} \delta_{n-N-k} p(t-nT). \quad (3b)$$

Both cases actually describe a baseband digital implementation, where $< n >_K$ denotes the remainder after dividing n by K, and $p(t)$ is the normalized pulse used at the DAC and ADC. It is assumed that $p(t)$ has a span $[0, T)$ and a bandwidth $[-B/2, B/2)$ ($B \approx 1/T$). Here, the considered range is $t \in [0, (K+N)T)$, where NT is an upper bound on the length of any channel $g_q(t)$. We have assumed that the transmit functions $\psi_k(t)$ include a cyclic prefix of length NT, and that the receive functions $\phi_k(t)$ remove this cyclic prefix. That way IBI is avoided. Note that the above described FDM scheme actually corresponds to OFDM (orthogonal frequency division multiplexing), whereas the TDM scheme actually corresponds to SCCP (single carrier with a cyclic prefix). Passband and analog versions of FDM and TDM can be described in a similar fashion. CDM covers intermediate schemes, where the functions $\{\psi_k(t)\}_k$ and $\{\phi_k(t)\}_k$ could for example be related to spreading codes or wavelets.

To carry out joint DRA and waveform adaptation, we have to perform channel and data estimation, i.e., the estimation of $\{h_{q,k,l}\}_{k,l}$ and $\{u_{q,k}\}_k$. These two processes will take place in different phases, the training phase and the data transmission phase. Then, in every data transmission phase, we are set to cancel the estimated $\{\sum_k u_{q,k} h_{q,k,l}\}_l$ from the received $\{x_{q,l}\}_l$, allowing us to sense $\{v_{q,l}\}_l$ using for instance a sparse sampling mechanism [8]. After sensing this interference, we can compute an estimate of the interference covariance matrix $\mathbf{R}_q = E(\mathbf{v}_q \mathbf{v}_q^H)$, with $\mathbf{v}_q = [v_{q,0}, \ldots, v_{q,K-1}]^T$, and we have everything we need to run the joint DRA and waveform adaptation. This step boils down to deciding the vector $\mathbf{p}_q = [p_{q,0}, \ldots, p_{q,K-1}]^T$, with $p_{q,k} = \sqrt{E(|u_{q,k}|^2)}$, based on some objective function and constraints. The resulting $\mathbf{p}_q$ is fed back to the transmitter of CR q and a new data transmission phase can begin using the updated vector $\mathbf{p}_q$. In ensuing sections, we assume that the channel and interference parameters $\{h_{q,k,l}\}_{k,l}$ and $\mathbf{R}_q$ have already been estimated.

III. Dynamic Resource Allocation

To design our DRA strategy, we have to select objective functions and constraints to effect agile DRA. The design objective we choose here are the capacities of the different CRs, given the waveform approach:

$$C(\mathbf{p}_q) = \frac{1}{K} \log_2 |\mathbf{I}_K + \text{diag}(\mathbf{p}_q)\mathbf{H}_q^H \mathbf{R}_q^{-1} \mathbf{H}_q \text{diag}(\mathbf{p}_q)|, \quad (4)$$

where $\mathbf{H}_q$ is a $K \times K$ matrix with $[\mathbf{H}_q]_{k,l} = h_{q,k,l}$.

In a centralized DRA, the objective is to determine the collective of all allocation vectors $\{\mathbf{p}_q\}_q$ that maximizes the sum-rate of all users, that is,

$$\max_{\{\mathbf{p}_q \succeq \mathbf{0}\}_q} \sum_{q=0}^{Q-1} C(\mathbf{p}_q). \quad (5)$$

However, this formulation leads to a centralized non-convex optimization problem, which is NP-hard with complexity scaling exponentially with the number of users. Furthermore, it requires knowledge of all the available resource information $\{\mathbf{R}_q, \mathbf{H}_q\}_q$, which can be infeasible to obtain even for a central spectrum controller such as a base station. For any-to-any connections, it is more appropriate to perform distributed DRA by adopting individual (per-user) objective functions defined in (4).

The constraints we formulate are given by the so-called cognitive spectral masks that are imposed to control interference and they come in the following flavors:

- policy-based long-term spectral mask $S_{FCC}(f)$, imposed by government regulations;
- cognition-based notch mask $S_{notch}(f)$ used to protect active primary users licensed for some spectrum band(s).

The intersection of these masks yields the composite cognitive mask $S_c(f) = S_{FCC}(f) \cap S_{notch}(f)$. Defining the transmitted power spectral density of CR q as

$$S_q(f; \mathbf{p}_q) = \sum_k p_{q,k}^2 |\psi_k(f)|^2, \quad (6)$$

we can thus impose the constraint

$$S_q(f; \mathbf{p}_q) \leq S_c(f), \ \forall f.$$

Furthermore, we also consider an average transmit-power constraint per CR, given by

$$\int S_q(f; \mathbf{p}_q) df \leq P_q.$$

As we will show in Section IV, both these conditions are convex. Given all the above assumptions, distributed DRA boils down to solving

$$\max_{\mathbf{p}_q \succeq \mathbf{0}} \quad C(\mathbf{p}_q) \quad (7a)$$

$$s.t. \quad \int S_q(f; \mathbf{p}_q) df \leq P_q, \quad (7b)$$

$$S(f; \mathbf{p}_q) \leq S_c(f), \ \forall f. \quad (7c)$$

On a per-user basis, (7) presents a decentralized DRA formulation for deciding the optimal allocation vector $\mathbf{p}_q$, without requiring knowledge of other users' allocation vectors $\{\mathbf{p}_r\}_{r\neq q}$. Nevertheless, the interference covariance matrix $\mathbf{R}_q$ needs to be sensed according to (4), while the sensing task needs to be carried out when other users are transmitting with allocation vectors $\{\mathbf{p}_r\}_{r\neq q}$. On the network level, the intricacy among sensing, transmission and distributed DRA suggests a repeated game approach, in which users repeat the optimization process in (7) multiple iterations, until converging to steady-state DRA decisions $\mathbf{p}_q^*$, if existent. It has been shown that iterative games, even when performed in an asynchronous fashion among users, can considerably enhance the network-wide spectrum efficiency measured by (5) [3], [4].

The signal basis expansion framework for DRA provides a feasible means to jointly optimize the allocation decisions $\mathbf{p}_q$ via (7) and thereby implicitly adapt the transmitted waveforms. In contrast, existing work on cognitive radios separately treats DRA and waveform adaptation: the DRA literature focuses on direct optimization of the power spectrum $S_q(f)$ based on proper spectrum efficiency criteria [3], [4], while the waveform design literature investigates analog or digital pulse shaping techniques to comply with the allocated power spectrum $S_q(f)$ [6], [7]. The separate approach to DRA and waveform design has several limitations:

- When waveform adaptation is based on finalized DRA decisions and thus completely decoupled from the DRA process, sensing the aggregate interference is impossible prior to transmission. One way to solve this problem is to formulate centralized DRA, but this will cause a large communications overhead from the CRs to the spectrum controller.
- In the absence of waveform adaptation, distributed DRA is still possible. However, this mandates each DRA decision be made from the knowledge of the individual channels received from all interfering users, in combination with either a one-shot game or an iterative game. The one-shot games do not require knowledge of other users' DRA decisions, but exhibit a considerable performance gap from the socially optimal sum-capacity in (5). Iterative games, on the other hand, require users to broadcast their DRA decisions during iterations, resulting in a heavy communications overhead over a dedicated control channel.
- It can be difficult or costly to generate a transmitted waveform that perfectly match the allocated $S_q(f)$ of any flexible shape [7]. Without respecting the implementation limitations on waveform agility, the DRA decisions made on $S_q(f)$ are no longer optimal when implemented in practical radios.

Our DRA approach overcomes the above limitations. It offers a truly distributed framework in which the allocation vectors are optimized under a practical transmitter implementation structure. Waveform adaptation naturally arises as a result of DRA optimization.

IV. Implementation Issues

In this section, we study the formulations of the joint DRA and waveform adaptation problem in order to facilitate computationally-efficient optimization. Given the general basis function sets $\{\psi_k(t), \phi_k(t)\}_k$ and the set size K, we seek to formulate the optimization problem in (7) in convex form.

First, we change the maximization problem in (7a) to a minimization problem by multiplying the objective function by -1. By doing so, the modified objective function becomes convex, because $-C(\mathbf{p}_q)$ is convex in $\mathbf{p}_q$.

Next, we move on to the power and mask constraints. We rewrite the power constraint in (7b) in matrix-vector form as

$$\int S_q(f;\mathbf{p}_q)df = \mathbf{p}_q^T\mathbf{S}\mathbf{p}_q \leq P_q, \tag{8}$$

where $\mathbf{S}$ is a $K\times K$ diagonal matrix with its k-th diagonal element given by $[\mathbf{S}]_{k,k} = \int |\psi_k(f)|^2 df$. Therefore, the power constraint (8) is a convex function for $\mathbf{p}_q$.

To reformulate the cognitive mask constraint in (7c), we collect the frequency responses of all basis functions into a $K\times 1$ vector

$$\boldsymbol{\psi}(f) = \text{diag}\{\psi_1(f), \ldots, \psi_K(f)\}.$$

We define $\Psi(f) = [\text{Re}(\boldsymbol{\psi}(f), -\text{Im}(\boldsymbol{\psi}(f))]^T$, where $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote the real and imaginary parts, respectively. Recognizing $\sqrt{S_q(f;\mathbf{p}_q)} = ||\Psi(f)\mathbf{p}_q||_2$, we can rewrite (7c) as

$$||\Psi(f)\mathbf{p}_q||_2 \leq \sqrt{S_c(f)}, \quad \forall f. \tag{9}$$

The feasible set in (9) is defined by the intersection of an infinite number of second-order cone constraints on linear transformations of $\mathbf{p}_q$, one for each f. Hence, the mask constraint is a convex function for $\mathbf{p}_q$. To render the number of constraints in (9) finite, one way is to approximate it by sampling uniformly in frequency at N points $F_N := \{f_1, \ldots, f_N\}$ and replace it by

$$||\Psi(f_n)\mathbf{p}_q||_2 \leq \sqrt{S_c(f_n)} - \epsilon, \quad \forall f_n \in F_N. \tag{10}$$

Here $\epsilon \geq 0$ can be chosen such that satisfaction of (10) guarantees satisfaction of (9) even for $f \notin F_N$. The feasible set of the discretized problem may slightly differs from that of the original one. Nevertheless, the feasible set remains convex.

Putting together, we now have a DRA formulation with a convex objective subject to a set of convex-cone constraints:

$$\begin{aligned} \min_{\mathbf{p}_q} \quad & -C(\mathbf{p}_q) && \text{(11a)}\\ s.t. \quad & \mathbf{p}_q^T\mathbf{S}\mathbf{p}_q \leq P_q && \text{(11b)}\\ & ||\Psi(f_n)\mathbf{p}_q||_2 \leq \sqrt{S_c(f_n)} - \epsilon, \quad \forall f_n && \text{(11c)}\\ & -\mathbf{p}_q \leq 0. && \text{(11d)} \end{aligned}$$

Being convex, the formulation in (11) permits efficient numerical algorithms to reach its global optimum.

The general framework in (11) applies to a number of multiple access systems by properly choosing the basis functions $\{\psi_k(t)\}_k$ and $\{\phi_k(t)\}_k$. There are a number of options for the basis functions, such as carriers, pulses, codes, wavelets, and other forms. A special case is OFDM based [5], where carrier functions in (2) are used as the basis functions. Carrier functions not only are convenient to implement in practice, but also possess some nice mathematical properties that simplify the DRA optimization design. More specifically, if carriers functions are used, $\mathbf{H}_q$ is perfectly diagonal and the capacity formula (4) can be rewritten as

$$C(\mathbf{p}_q) = \frac{1}{K} \sum_{k=0}^{K-1} \log_2 \left(1 + \frac{p_{q,k}^2 |h_{q,k,k}|^2}{r_{q,k}} \right) \qquad (12)$$

where $r_{q,k} = E(|v_{q,k}|^2)$. Furthermore, the matrix $\mathbf{S}$ in the power constraint expression (11b) becomes an identity matrix. Notice that the objective function in (12) is the sum over K individual functions each solely defined by the weight $p_{q,k}$. The corresponding DRA problem admits a closed-form solution to $\mathbf{p}_q$ that is given by the well-known water-filling scheme [3].

V. Simulations

Consider a Q-user peer-to-peer CR network. Each user corresponds to one pair of unicast transmitter and receiver, giving rise to Q^2 channel links. We suppose that each link experiences frequency-selective fading modeled by an N_t-tap tapped delay line, where each tap coefficient is complex Gaussian with zero-mean and unit variance. The link power gain is denoted by a scalar $c_{rq} > 0$, $\forall r, q \in [1, Q]$, which captures both the path loss and the fading power. The noise variance is assumed to be 1 in all cases.

Fig. 1 demonstrates the flexibility of the distributed formulation in (7) for joint DRA and waveform adaptation. Three users are considered, who operate in two different scenarios described by following parameters: (a) $P_2 = P_3 = 10$, $P_1 = 5$, $c_{rq} = 5$, $\forall r \neq q$, and $N_t = 8$; (b) $P_q = 20$, $\forall q$, $c_{rq} = 0.2$, and $N_t = 4$. In both cases, we assume $c_{qq} = 1$ w.l.o.g.

We adopt $K = 32$ digital carriers as the basis functions. Figure 1 depicts the transmitted power spectra of the resulting multicarrier power allocation. In (a), both users experience strong interference, and the DRA game results in an FDM-type solution where the power spectra of different users are non-overlapping in frequency. In contrast, users in (b) have high transmit power and low interference, and the optimal DRA suggests spread spectrum frequency reuse. Users overlap in frequency to occupy nearly the entire bandwidth, and adapt mainly to their own channels. Such results confirm the theoretical prediction in [4], [5], and demonstrate the flexibility of our basis expansion framework in instantiating various optimal multiple access schemes under available resources.

VI. Summary

A general framework for joint DRA and waveform adaptation is presented based on the signal basis expansion approach. Generalized transmitter and receiver basis functions are employed to formulate CR-oriented DRA design objectives and cognitive spectral mask constraints. Such a framework promises several benefits to CR networks, including asynchronous and distributed DRA among CR users, protection of primary users, and adaptation capability in the presence of changing network traffic. This basis function structure for waveform adaptation is also amenable to carrying out additional flexible designs including quantized feedback and low-resolution DSP implementations, which we will explore in future work.

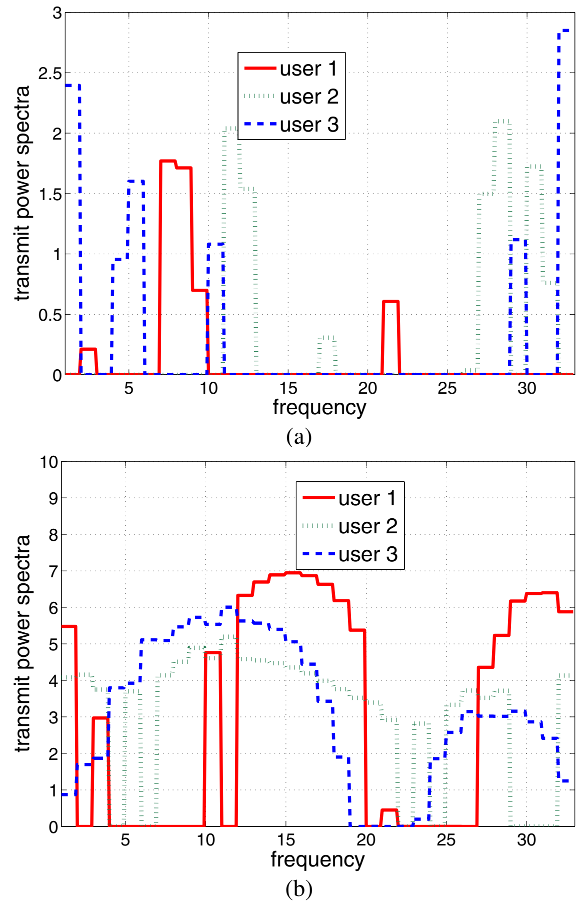

Fig. 1. Optimal transmitted power profiles of two users: (a) strong interference case, (b) weak interference case.

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