A FULL-DIVERSITY DISTRIBUTED SPACE-TIME CODING SYSTEM WITH REGENERATIVE RELAYS

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ABSTRACT

We propose a distributed space-time coding (DSTC) systems based on the Alamouti design. We discuss the limitations in the relay channel of the "out of the box" Alamouti scheme and the additional complexity required to overcome its loss of diversity. Using a bit error rate based antenna selection approach, we design DSTC systems with one regenerative relay that improve on the classical Alamouti scheme when utilized in a two-hop channel. We prove that the proposed one relay DSTC system collects the full diversity of the distributed MISO channel. We also introduce a less complex DSTC system in which the relaying energies depend on the error probabilities at the relays. Numerical results show that the proposed systems perform close to the error probability lower bound obtained by considering error-free reception at the relays.

1. INTRODUCTION AND SYSTEM MODEL

In this paper we contribute to the area of regenerative relay systems by proposing and analyzing distributed space-time coding (DSTC) schemes with one and two regenerative relays. Unlike non-regenerative relays, regenerative relays do not naturally induce diversity in the system. It has been observed that due to errors at the relays the systems with distributed antennas using no-coding or standard (space-time) coding lose diversity when compared to a one-hop MIMO system with the same number of antennas [1,2]. For example, the cooperative schemes in [2–4] use the standard Alamouti space-time code in [5] and can at most achieve the diversity of the channel between source and relay. Moreover, the work of [2,4] does not suggest any alternative scheme that recovers the diversity.

The DSTC systems proposed in this paper are designed to induce and collect diversity in a distributed MISO channel by allowing feedback from the destination and error probability feedforward from the relays. We propose two schemes: a quasi-optimum scheme that requires feedback from the destination to the relays and an ad-hoc scheme, which dispenses with the feedback by taking advantage of the relative distance between source, relays, and the destination. The proposed systems can be utilized in cellular systems as well as in multihop networks without centralized control.

We consider a multiuser interference free wireless communication system that uses wireless relay stations. The relays have no data symbols of their own to transmit; their goal is to improve the quality of the link between the source and the destination. In order to eliminate the interference from the relay's own transmitter we impose a half-duplex constraint, i.e., we consider two different frequency bands A and B for transmitting and receiving signals at the relays. More precisely, the relays monitor only band A on which they receive the information signal from the active source, and transmit in band B to the destination. All the radios in the system use *one antenna* per transceiver.

Through a relay discovery process and protocol, which is not the focus of this paper, it is assumed that the source has access to one fixed relay station R_1. The source uses energy E per symbol to communicate with the destination. During the generic time slot i the source broadcasts in band A to the relay and the destination the data block $\sqrt{\varepsilon_A}\boldsymbol{s}[i] = \sqrt{\varepsilon_A}[s[2i], s[2i+1]]^{\mathrm{T}}$, where $\varepsilon_A = \rho E \leqslant E$ is the transmitted bit energy in band A. The data symbols, $\{s[n]\}_n$, are drawn from a BPSK constellation with unit energy and are assumed independent and identically distributed. As illustrated in Fig. 1, relay R_1, which monitors frequency band A, receives

$$\boldsymbol{r}_1[i] = h_{sr_1}\sqrt{\varepsilon_A}\boldsymbol{s}[i] + \boldsymbol{z}_1[i], \quad (1)$$

where h_{sr_1} is the slow varying fading channel between the source and relay R_1, and $\boldsymbol{z}_1[i]$ is the noise vector with each entry being a complex circular Gaussian random variable with variance $N_0/2$ per dimension. Provided that the relay acquires the channel h_{sr_1} perfectly, the decision vector for $\boldsymbol{s}[i]$ with maximum likelihood decoding is

$$\boldsymbol{x}_1[i] = \boldsymbol{r}_1[i] / \big(\sqrt{\varepsilon_A} h_{sr_1}\big).$$

The relay R_1 quantizes $\boldsymbol{x}_1[i]$ in order to obtain an estimate of $\boldsymbol{s}[i]$, which can be written as $\hat{\boldsymbol{s}}[i] = [\hat{s}[2i], \hat{s}[2i+1]]^{\mathrm{T}} = 2\,\mathrm{sgn}\big(\boldsymbol{x}_1[i]\big) - 1$. The probability of error at the relay R_1 is $P_{r_1} := Q\big(\sqrt{2\varepsilon_A |h_{sr_1}|^2 / N_0}\big)$. During the next time slot, i.e, time slot $i+1$, the source and the relay transmit in band B using an Alamouti-type space-time code [5]. The source trans-

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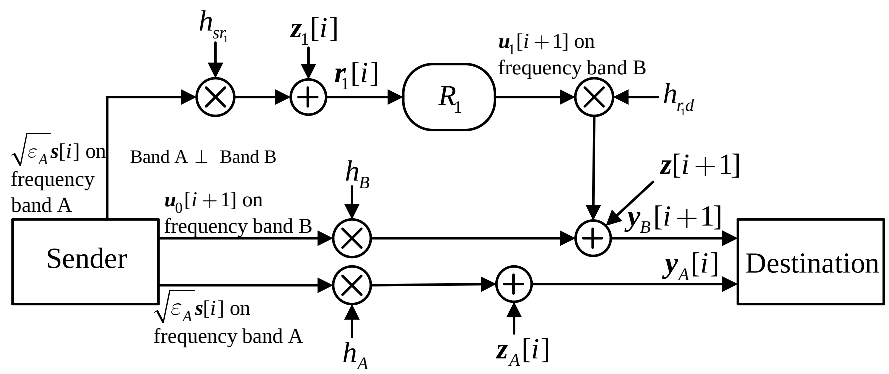

Fig. 1. Discrete-time equivalent relay channel with the half-duplex constraint. The signals $\boldsymbol{y}_A[i]$ and $\boldsymbol{y}_B[i+1]$ are received at the destination on the non-overlapping frequency bands A and B, respectively.

mits $\boldsymbol{u}_0[i+1] = \sqrt{\varepsilon_B}\boldsymbol{s}[i]$, which is the same as the block transmitted in band A during the previous time slot except for the transmit energy $\varepsilon_B = E - \varepsilon_A$. The source transmits continuously in band A as well as in band B. The transmissions from the source in band B are a delayed version of its transmissions in band A. The relay R_1 transmits

$$\boldsymbol{u}_1[i+1] = \sqrt{\alpha_{r_1}}\left[\hat{s}^*[2i+1], -\hat{s}^*[2i]\right]^{\mathrm{T}},$$

where the transmit energy at the relay is $\alpha_{r_1} \leqslant E_{r_1}$. While transmitting $\boldsymbol{u}_1[i+1]$, the relay receives $\boldsymbol{r}_1[i+1]$ from the source to update the information symbols needed for the relay-source cooperation in the next time-slot.

If we assume that the transmissions in band B from the relay and the source reach the destination at the same time, we can write the signals received at the destination in bands A and B as

$$\boldsymbol{y}_A[i] = \left[y_A[2i], y_A[2i+1]\right]^{\mathrm{T}} = h_A\sqrt{\varepsilon_A}\boldsymbol{s}[i] + \boldsymbol{z}_A[i], \quad (2)$$

$$\begin{aligned}\boldsymbol{y}_B[i+1] &= \left[y_B[2(i+1)], y_B[2(i+1)+1]\right]^{\mathrm{T}} \\ &= h_{r_1d}\boldsymbol{u}_1[i+1] + h_B\boldsymbol{u}_0[i+1] + \boldsymbol{z}[i+1].\end{aligned} \quad (3)$$

Notice from Fig. 1 that $\boldsymbol{y}_A[i]$ is received in band A and $\boldsymbol{y}_B[i+1]$ is received in band B. We assume that $\boldsymbol{z}_A[i]$ and $\boldsymbol{z}[i] := [z[2i],\ z[2i+1]]^{\mathrm{T}}$ are mutually independent noise vectors with each entry being a complex circular Gaussian random variable with zero mean and variance $N_0/2$ perd dimension. We also assume that the effect of the slowly time-varying flat fading is captured by the independent random variables h_{sr_1}, h_{r_1d}, h_A, and h_B. The destination uses an Alamouti decoder followed by a maximum ratio combiner receiver, which is optimum only if the relay decodes perfectly the information symbols received from the destination.

2. A FULL DIVERSITY RECEIVER

It is possible to show that the diversity performance of the DSTC system is poor without knowledge of P_{r_1} at the destination [3,4]. It turns out that this is not the case if we assume perfect knowledge at the destination of the error probability at the relay. In this paper we propose a system that can recover the full diversity of the relay channel. The idea is to assign less weight to the relay path (by varying α_{r_1}) when the channel between the source and the relay is in deep fade. We start by focusing on the amplification energy α_{r_1} that maximizes the signal to interference and noise ratio (SINR) at the destination, which we denote with $\gamma(\alpha_{r_1})$, under the constraint that $\alpha_{r_1} \leqslant E_{r_1}$. It is possible to prove that irrespective of the channel parameters and transmission energies, the maximum SINR is $\gamma^* = \max\left\{\gamma(0), \gamma(E_{r_1})\right\}$. We only use the SINR to reduce the set of acceptable amplifications at the relay from the interval $[0, E_{r_1}]$ to a set of cardinality 2, i.e., $\{0, E_{r_1}\}$, and we conjecture that $\alpha_{r_1} = 0$ and $\alpha_{r_1} = E_{r_1}$ are "good" amplifications. In other words, the maximum SINR approach identifies the two choices for the amplification at the relay, but does not specify when to switch between them. The bit error rate (BER) at the destination determines when the relay reverts from idle to full power. We propose the following amplification at the relay:

$$\alpha_{r_1}^{(1)} := \underset{\alpha_{r_1}\in\{0,E_{r_1}\}}{\arg\min}\ P(\alpha_{r_1}), \quad (4)$$

where

$$\begin{aligned}P(\alpha_{r_1}) =& (1-P_{r_1})^2 Q\left(\frac{\alpha_s}{\sqrt{N_1}}\right) + (1-P_{r_1})P_{r_1} Q\left(\frac{\alpha_d}{\sqrt{N_1}}\right) \\ &+0.5(1-P_{r_1})P_{r_1}\left[Q\left(\frac{\alpha_s+\beta_{1B}}{\sqrt{N_1}}\right) + Q\left(\frac{\alpha_s-\beta_{1B}}{\sqrt{N_1}}\right)\right] \\ &+0.5P_{r_1}^2\left[Q\left(\frac{\alpha_d+\beta_{1B}}{\sqrt{N_1}}\right) + Q\left(\frac{\alpha_d-\beta_{1B}}{\sqrt{N_1}}\right)\right]\end{aligned} \quad (5)$$

is the system's BER for amplification α_{r_1} at the relay, and where $\alpha_s := \alpha_{r_1}|h_{r_1d}|^2 + \varepsilon_B|h_B|^2 + \varepsilon_A|h_A|^2$, $\alpha_d := -\alpha_{r_1}|h_{r_1d}|^2 + \varepsilon_B|h_B|^2 + \varepsilon_A|h_A|^2$, $\beta_{1B} := 2\sqrt{\alpha_{r_1}\varepsilon_B}\,\Re\{h_{r_1d}h_B^*\}$, and $N_1 := \alpha_s N_0/2$.

Proposition 1. *The diversity gain of a DSTC system with α_{r_1} as in* (4) *is $t_A + t_B + min\{t_{sr_1}, t_{r_1d}\}$, where t_A, t_B, t_{sr_1}, and t_{r_1d} are the diversity orders of the channels h_A, h_B, h_{sr_1}, and h_{r_1d}, respectively.*

A sketch of the proof is presented in the Appendix. The result of Proposition 1 is to be contrasted with $\bar{P}(\alpha_{r_1})$, which achieves at most the diversity of the channel h_{sr_1}.

Designing a DSTC system with α_{r_1} as in (4) requires solving two problems. First, the destination has to know P_{r_1} (or equivalently, $\varepsilon_A|h_{sr_1}|$) in order to compute $P(\alpha_{r_1})$ and determine which α_{r_1} to use. Second, α_{r_1} has to be transmitted from the destination to the relay. Because the feedback channel is bandwidth consuming, one may favor a system that does not require transmissions from the destination to the relay. To achieve this goal we look for an α_{r_1} that is a continuous function of P_{r_1}. We choose

$$\alpha_{r_1}^{(2)} := \frac{(1-2P_{r_1})E_{r_1}}{1+4P_{r_1}\left[\varepsilon_B + (1-P_{r_1})E_{r_1}\right]\bar{\gamma}_{r_1d}}, \qquad (6)$$

which maximizes an approximate function of the SINR. In (6), $\bar{\gamma}_{r_1d} := E\left[|h_{r_1d}|^2/N_0\right]$ is the average channel quality between the relay and the destination. In deriving $\alpha_{r_1}^{(2)}$ we have constrained the relay to only cooperate with nearby sources, so that the path loss from the relay to the destination is approximately equal to the path loss from the source to the destination. In addition, if R_1 is a fixed relay, $\bar{\gamma}_{r_1d}$ varies slowly in time and can be communicated to the relay during a calibration phase.

3. PERFORMANCE ASSESSMENT

We consider a DSTC system with one relay. We assume that all the channels in the system are affected by Rayleigh fading and their average power is equal to one, i.e., $E[|h_A|^2] = E[|h_B|^2] = E[|h_{sr_1}|^2] = E[|h_{r_1d}|^2] = 1$. We select the transmit energies at the source $\varepsilon_A = \varepsilon_B = E/2$, and the maximum amplification at the relay equal to $E_{r_1} = E/2$. We plot in Fig. 2(a), $\bar{P}(E_{r_1})$, which is the error performance of the DSTC system without knowledge of P_{r_1} at the destination. We observer that the diversity slope of $\bar{P}(E_{r_1})$ is -1. We can also see from Fig. 2(a) that there is a large performance gap between $\bar{P}(E_{r_1})$ and the performance of a DSTC with a perfect relay. The orthogonal transmissions cooperative (OTC) system propose in [6] offers a good lower bound on the DSTC designs (if we ignore the 50% excess bandwidth of the OTC system) since it is an interference-free cooperative system that takes into account the errors at the relay. When we compare in Fig. 2(a) the error performance of a DSTC system that uses the relay amplification in (4), i.e., $\bar{P}\left(\alpha_{r_1}^{(1)}\right)$, with the error performance of the OTC we observe that the difference is less than 2dB at 10^{-5}. If no feedback channel is present in the DSTC system then one should expect a degradation in performance. We can see from Fig. 2(a) that if the relay amplifies the regenerated symbols with $\alpha_{r_1}^{(2)}$ it losses about 3 dB. Nevertheless, $\bar{P}\left(\alpha_{r_1}^{(2)}\right)$ shows considerable improvement when compared to $\bar{P}(E_{r_1})$.

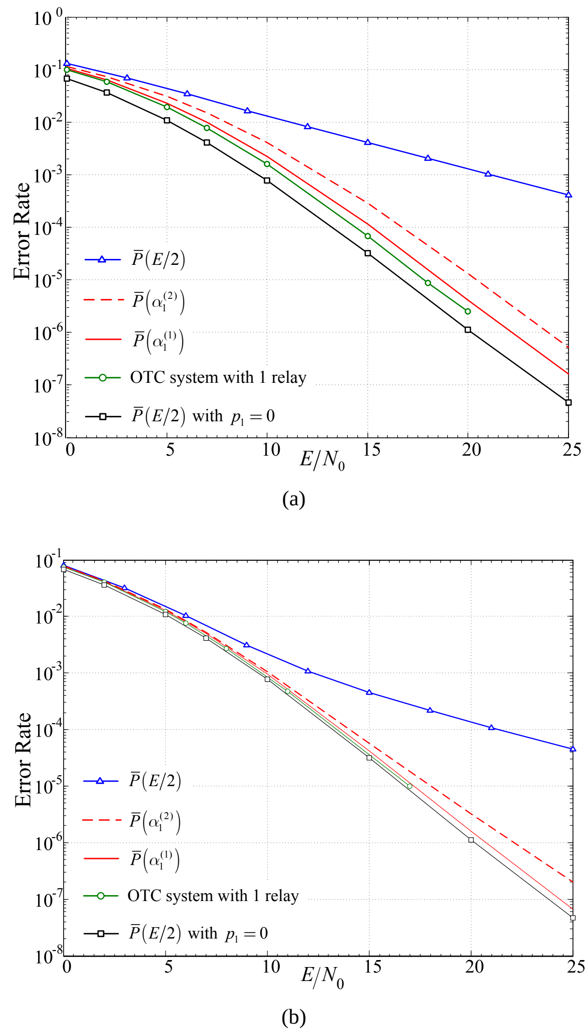

Fig. 2. Error performance of the DSTC systems with 1 relay: (a) comparison with the OTC system for equally balanced channels (b) comparison with the OTC system for unequally balanced channels

In general, it is expected from a relay to cooperate only with nearby sources (e.g., mobile users crossing the coverage area of the relay). Consequently, the channel from the source to the relay is on average better than the channel from the relay to the destination. For example, if the source is twice closer to the relay than the destination and if we consider a path loss coefficient of $\log_2(10) \approx 3.32$, we obtain that $E[|h_{sr_1}|^2]/E[|h_{r_1d}|^2] = 10$. Let us see how this situation affects the performance of the DSTC system. We select the same channel parameters as in previous example with the exception of h_{sr_1}, which has $E[|h_{sr_1}|^2] = 10$. When the quality of channel h_{sr_1} increases, the relay makes less errors, and $\bar{P}\left(\alpha_{r_1}^{(1)}\right)$ and $\bar{P}\left(\alpha_{r_1}^{(2)}\right)$ come closer to the error performance of the OTC system than in the previous example. We see from Fig. 2(b) that $\bar{P}(\alpha_{r_1}^{(1)})$ is almost indistinguishable from the error performance of the OTC system. Even though $\bar{P}(E_{r_1})$ seems to follow the lower bounds at low SINR, it is performing poorly at high SINR due to its diversity problem. Notice that $\bar{P}(\alpha_{r_1}^{(2)})$ is losing diversity too. However, it is a slower process and it happens at a higher SINR.

4. CONCLUSIONS

In this paper we have proposed a novel DSTC scheme with one relay that achieves full diversity by switching between cooperation using the Alamouti design and one-hop transmissions from the source to the destination based on the minimum error probability at the destination. We have also proposed a feedback-free DSTC system, which improves on the standard Alamouti design by allowing the amplification at the relay to depend on the relay's own error rate. The design guidelines for a one relay system can be extended to a two relay system. In addition, it is possible to shown that both the one and the two relay schemes perform close to the error probability lower bound obtained by considering error-free relays.

APPENDIX

Proof of Proposition 1. When $\alpha_{r_1} = 0$ the relay does not transmit any information and if $\gamma_A := |h_A|^2/N_0$, $\gamma_B := |h_B|^2/N_0$ then $P(0) = Q\left(\sqrt{2(\varepsilon_A\gamma_A + \varepsilon_B\gamma_B)}\right)$, which can be trivially upper bounded as $P(0) < 4Q\left(\sqrt{2(\varepsilon_A\gamma_A + \varepsilon_B\gamma_B)}\right)$. Finding a useful bound for $P(E_{r_1})$ is only a bit more laborious. With $\gamma_{r_1d} := |h_{r_1d}|^2/N_0$, the first term of $P(E_{r_1})$ is

$$(1-P_{r_1})^2 Q\left(\alpha_s/\sqrt{N_1}\right) = (1-2P_{r_1}+P_{r_1}^2)Q\left(\sqrt{2\alpha_s/N_0}\right) < (1-P_{r_1}+P_{r_1}^2)Q\left(\sqrt{2E_{r_1}\gamma_{r_1d}}\right),$$

where we used $\alpha_s \leqslant E_{r_1}|h_{r_1d}|^2$ to establish the inequality. Using a similar approach and the fact that $Q(x) < 1$, the second, third and fourth terms in $P(E_{r_1})$ can be upper bounded by $\left(P_{r_1} - P_{r_1}^2\right)\left[1 - Q\left(E_{r_1}|h_{r_1d}|^2/\sqrt{N_1}\right)\right]$, $\left(P_{r_1} - P_{r_1}^2\right) \cdot \left[Q\left(\sqrt{2E_{r_1}\gamma_{r_1d}}\right)+1\right]$, and $P_{r_1}^2\left[1 - Q\left(E_{r_1}|h_{r_1d}|^2/\sqrt{N_1}\right)+1\right]$ respectively. After canceling out opposite terms we obtain

$$P(E_{r_1}) < Q(\sqrt{2E_{r_1}\gamma_{r_1d}}) + 2P_{r_1}.$$

Using the definition of P_{r_1}, i.e., $P_{r_1} = Q\left(\sqrt{2\varepsilon_A\gamma_{sr_1}}\right)$, we can easily establish that $P(E_{r_1}) < 4Q\left(\sqrt{2\min\{\varepsilon_A\gamma_{sr_1}, E_{r_1}\gamma_{r_1d}\}}\right)$. Hence,

$$\min\{P(0), P(E_{r_1})\} < 4Q\left(\sqrt{2\max\left\{\varepsilon_A\gamma_A + \varepsilon_B\gamma_B, \min\{E_{r_1}\gamma_{r_1d}, \varepsilon_A\gamma_{sr_1}\}\right\}}\right).$$

Note that the last bound is 4 times the probability of error of a system with one-hop BPSK transmissions and SNR $\xi_t := \max\left\{\varepsilon_A\gamma_A + \varepsilon_B\gamma_B, \min\{E_{r_1}\gamma_{r_1d}, \varepsilon_A\gamma_{sr_1}\}\right\}$. Using the high SINR approximation developed in [7] for the symbol error rate of one-hop systems, we can show that the diversity gain of a system with SNR $\min\{E_{r_1}\gamma_{r_1d}, \varepsilon_A\gamma_{sr_1}\}$ is $\min\{t_{sr_1}, t_{r_1d}\}$. Our conclusion follows from the fact ξ_t can be interpreted as the SNR at the output of a selection combiner preceded by a maximum ratio combiner (see also Proposition 4 and its corollary in [7]).

5. REFERENCES

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