Adaptive Channel Estimation for OFDM Systems with Doppler spread

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***Abstract*— In this paper, we propose adaptive channel estimation for Orthogonal Frequency Division Multiplexing (OFDM) in fast time-varying (TV) channels. A Basis Expansion Model (BEM) approach is used to capture the time variation of the channel within each OFDM block, and to reduce the estimator dimensionality. Capitalizing on the BEM structure and on a frequency domain training, two adaptive approaches are proposed, based on Kalman filtering and Recursive Least Squares (LS) methods, which exploit the time correlation of the channel between successive blocks and do not require any a-priori knowledge of the channel statistics. Simulation results show that, compared to classical Least Squares and statistically-aided Linear Minimum Mean Squared Error (LMMSE) approaches, the two proposed techniques effectively estimate the channel, adapt fast to its non stationary changes, thus enabling efficient TV channel equalization of the inter-carrier interference (ICI) induced in OFDM systems by high Doppler spreads.**

I. Introduction

The wireless channel that affects Orthogonal Frequency-Division Multiplexing (OFDM) [1] in high-mobility scenarios, cannot be considered time-invariant (TI) over an entire OFDM block. A time-varying (TV) channel destroys the orthogonality among subcarriers and introduces intercarrier interference (ICI), which drastically reduces the equalization performance of single-tap equalizers [2] [3] that are classically used in OFDM systems. Thus, more complex equalizers are required, such as those recently proposed [4] [5] [6] [7] [8]. All these approaches require the knowledge of the channel variation within each OFDM block and, consequently, TV channel estimation plays a crucial role influencing the ultimate BER performance. Either basis expansion models (BEM) [9] or reduced rank estimation techniques [10] [11], coupled with training based approaches, are commonly used in order to parsimoniously model and rapidly estimate the channel unknowns in a time-limited observation. In this framework deterministic least squares (LS) and statistically-aided linear minimum mean squared error (LMMSE) channel estimators have been recently considered for both single and multiple OFDM block observations [8] [12] [13] [14] [15]. However, in practical applications we may lack statistical information and, moreover, the channel could not be wide-sense stationary (WSS). We propose two adaptive estimation techniques, based on Kalman filtering and RLS approaches, which are able to exploit the time correlation of the channel over consecutive OFDM blocks, and can also track the statistical variations of the channel in the absence of any a-priori statistical information. Our approach is different from similar works on the subject in that:

- several papers, such as [16] [17] [18], assume the channel to be time-invariant inside each OFDM block (e.g. a block fading channel) and try only to adaptively estimate the approximated flat-fading channel for each subcarrier. Differently, we are also interested in estimating the time variation of the channel inside each OFDM block.
- other papers, such as [10] [19], assume a time domain Kalman filter approach with a scalar-observation. Differently, we rely on a frequency-domain Kalman filtering approach with a vector observation.

Specifically, our approach capitalizes on a BEM channel model where the BEM bases capture the channel variation within each OFDM block, and adaptively estimates the BEM coefficients from one OFDM block to another. This significantly reduces the Kalman filter and RLS complexity compared to batch methods, similar to the channel modes tracking proposed in [10].

II. System Model

Due to user mobility, the Doppler spread causes the channel to be modelled in the discrete domain by a TV Finite Impulse Response (FIR) filter. In the following subsections we will show the relations between the TV-FIR coefficients and the physical channel model, and we will subsequently describe how to approximate the TV channel through a BEM.

A. Channel Statistics

Denoting by $h[n,l]$ the l^{th} channel tap at the n^{th} time interval, under the assumption of WSS uncorrelated scattering we can write

$$E(h[n,l]\, h^*[n-m,l-p]) = r[m,l]\delta[p] \tag{1}$$

where $\delta[\cdot]$ is the Kronecker delta. Assuming finite delay-spread L, we can write $r[m,l]=0,\ \ for\ \ l \notin [0,\cdots,L]$.

Assuming a Jakes' model [20], which however is not crucial in the paper, we further have

$$r[m,l] = \sigma_l^2 J_0(2\pi v_D m/N), \tag{2}$$

where J_0 denotes the 0^{th}-order Bessel function of the first kind, σ_l^2 the power of the l^{th} channel tap, and

$$v_D = \frac{v\, f_C}{c\, \Delta_f} = \frac{f_D}{\Delta_f} \tag{3}$$

is the normalized Doppler frequency, with v the speed of the mobile in m/s, f_C the carrier frequency in Hz, c the speed of light, f_D the actual Doppler frequency, and Δ_f the subcarrier spacing. For very low speeds v, the time variation is generally not perceptible in an OFDM block and we can assume to deal with a time-invariant purely frequency-selective channel such as in [17] [18]. On the contrary, when the speed increases, this assumption no longer holds true.

B. Basis Expansion Model

OFDM systems work on data blocks of length N and each OFDM block is transmitted through the $(L+1)$-tap TV-FIR channel. Since each tap varies in time, the total number of parameters to estimate would be $(L+1)N$ (we need to know $h[n,l]$, for $n=0,...,N-1$, $l=0,...,L$), whereas the number of transmitted and received symbols per block is only N. In order to reduce the number of unknown parameters needed to represent the channel, a useful approach is to

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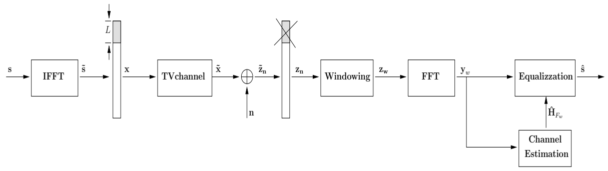

Fig. 1. OFDM System: windowed channel equalization

approximate the channel time variation by the superposition of $2Q+1$ basis functions $\lambda_q[n]$, $q=-Q,\cdots,Q$

$$h[n;l] \approx \sum_{q=-Q}^{Q} h_{q,l}\lambda_q[n], \qquad n=0,\cdots,N-1, \tag{4}$$

or, equivalently, in matrix form

$$\mathbf{h}_{tl} = \mathbf{B}\,\mathbf{h}_{cl}, \tag{5}$$

where $\mathbf{h}_{tl} = [h[0;l],\cdots,h[N-1;l]]^T$, $\mathbf{h}_{cl} = [h_{-Q,l},\cdots,h_{Q,l}]^T$, $\mathbf{B} = [\boldsymbol{\lambda}_{-Q},\cdots,\boldsymbol{\lambda}_Q]$, and $\boldsymbol{\lambda}_q = [\lambda_q[0],\cdots,\lambda_q[N-1]]^T$. The basis functions $\boldsymbol{\lambda}_q$ are fixed, therefore we only need to determine the $(L+1)(2Q+1)$ expansion coefficients $h_{q,l}$ to approximate the channel within each OFDM block. It is now clear that it is possible to solve the estimation problem if we choose Q to satisfy $(L+1)(2Q+1) \le N_p \le N$, where N_p is the number of known pilots that cannot obviously exceed the OFDM block length N. Usually, a small set of basis functions is sufficient to get a good approximation, according to the empirical rule given by $Q \ge \lceil v_D \rceil$. It follows that, for realistic mobile speeds and carrier frequencies up to several Ghz, practical values of Q are very small ($Q = 1,2,3$). Among the different alternatives, we focus on the Generalized Complex Exponential BEM (GCE-BEM) (other BEMs have been widely analyzed in [21]). Denoting by $\mathbf{B}_{GCE-BEM}$ the definition given in [22], we will use the following basis expansion model

$$\mathbf{B} = \mathbf{Q}_B\mathbf{D}_w\mathbf{B}_{GCE-BEM}, \tag{6}$$

where $\mathbf{D}_w = diag(\mathbf{w})$ is the diagonal matrix with $\mathbf{w} = [w[0],\cdots,w[N-1]]^T$ an opportune window function, and $\mathbf{Q}_B$ a square matrix that makes the columns of $\mathbf{B}$ orthonormal.

C. Data Model

Consider the OFDM system depicted in Fig. 1. The data symbols are first modulated by N orthogonal subcarriers through the IFFT block, a cyclic prefix (CP) of length L is appended to the IFFT output to induce a circular convolution with the channel and to prevent intersymbol interference from adjacent blocks (if the CP length L is greater than the channel delay spread). At the receiver side, after CP elimination, the data are reshaped by a time-domain window $\mathbf{w} = [w_0,...,w_{N-1}]^T$ (which helps to reduce Doppler effects [8]) and are demodulated by the FFT block. Assuming $\mathbf{s} = [s[0],\cdots,s[N-1]]^T$ represents the stacked data symbols, the received data $\mathbf{z}_w = [z_w[0],\cdots,z_w[N-1]]^T$ can be written as

$$\mathbf{z}_w = \mathbf{D}_w\mathbf{H}_T\mathbf{F}^H\mathbf{s} + \mathbf{D}_w\mathbf{n}_T, \tag{7}$$

where $\mathbf{F}$ is the unitary DFT matrix, $\mathbf{n}_T = [n[0],\cdots,n[N-1]]^T$ is an additive white gaussian noise (AWGN), and $\mathbf{D}_w\mathbf{H}_T$ is the $N\times N$ time-domain windowed channel matrix

$$\mathbf{D}_w\mathbf{H}_T = \sum_{q=-Q}^{Q} \boldsymbol{\Lambda}_q\mathbf{H}_q, \tag{8}$$

where $\mathbf{H}_q$ is a circulant matrix generated by the N dimensional vector $[h_{q,0},\cdots,h_{q,L},0,\cdots,0]^T$ and $\boldsymbol{\Lambda}_q = diag(\boldsymbol{\lambda}_q)$.

After the FFT demodulation stage, the input-output relation is

$$\begin{aligned}\mathbf{y} = \mathbf{F}\mathbf{z}_w &= \mathbf{H}_F\,\mathbf{s} + \mathbf{n}_F = \mathbf{F}\mathbf{D}_w\mathbf{H}_T\mathbf{F}^H\mathbf{s} + \mathbf{n}_F \\ &= \sum_{q=-Q}^{Q} \mathbf{C}_q\boldsymbol{\Delta}_q\mathbf{s} + \mathbf{n}_F\end{aligned} \tag{9}$$

where $\mathbf{n}_F := \mathbf{F}\mathbf{D}_w\mathbf{n}_T$, $\mathbf{C}_q := \mathbf{F}\boldsymbol{\Lambda}_q\mathbf{F}^H$ is a circulant matrix with entries

$$[\mathbf{C}_q]_{k,m} = \frac{1}{N}\sum_{n=0}^{N-1}\lambda_q[n]e^{-i\frac{2\pi(k-m)n}{N}}, \tag{10}$$

and

$$\boldsymbol{\Delta}_q := \mathbf{F}\mathbf{H}_q\mathbf{F}^H = diag(\mathbf{F}_L\mathbf{h}_q), \tag{11}$$

where $\mathbf{h}_q = [h_{q,0},\cdots,h_{q,L}]^T$, and $\mathbf{F}_L$ is the matrix containing the first $L+1$ columns of $\sqrt{N}\,\mathbf{F}$.
We remark that (9) implicitly subsumes the expression for TI channels, in which case $Q=0$, $\mathbf{C}_q$ is a scaled identity matrix, and $\boldsymbol{\Delta}_q$ is a diagonal matrix, resulting in a diagonal channel matrix. The more the channel is TV, the more the channel matrix departs from a diagonal one, which means that ICI is introduced, and the subcarrier orthogonality is destroyed.

III. Channel Estimation

To estimate the channel, we will rely on Pilot Symbol Assisted Modulation (PSAM) where, as suggested in [23], known pilots will be grouped in P blocks each of length L_P and interleaved with the information data to form $\mathbf{s} = [\mathbf{d}^{(1)^T},\mathbf{p}^{(1)^T},\cdots,\mathbf{d}^{(P)^T},\mathbf{p}^{(P)^T},\mathbf{d}^{(P+1)^T}]^T$ as shown in Fig 2.

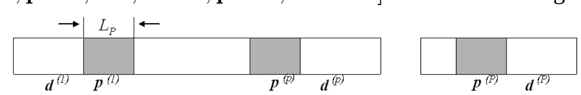

Fig. 2. Pilot placement

By inspecting the structure of the $\mathbf{H}_F$ matrix we can say that it is almost banded with bandwidth $B+1$, and that the residual values out of the band can be neglected. Intuitively, B can be interpreted as an index of the amount of ICI: the higher the Doppler spread, the larger will be the bandwidth B.

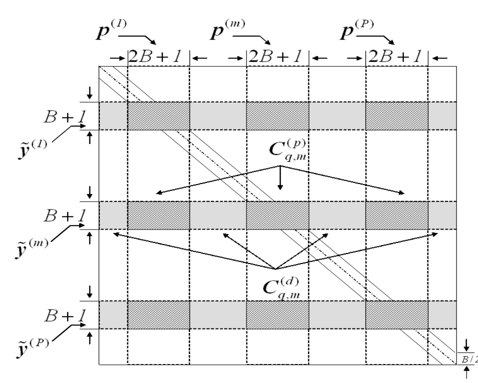

Fig. 3. Structure of the matrix $\mathbf{C}_q$

Assuming $B+1$ is the bandwidth of the matrix, we set $L_P = 2B+1$ and select the corresponding $B+1$ received samples (see Fig. 3). By fixing the total number of pilots to $N_p = P(2B+1)$ (see [21] for further details), and denoting $\mathbf{p} = [\mathbf{p}^{(1)^T},\cdots,\mathbf{p}^{(P)^T}]^T$, $\mathbf{d} = [\mathbf{d}^{(1)^T},\cdots,\mathbf{d}^{(P+1)^T}]^T$, and $N_d = N - N_p$ the total number of data symbols, we can write the received vector $\tilde{\mathbf{y}}^{(m)}$ related to the m^{th} selected block as

$$\tilde{\mathbf{y}}^{(m)} = \sum_{q=-Q}^{Q} \mathbf{C}_{q,m}^{(p)} \, \boldsymbol{\Delta}_q^{(p)} \; \mathbf{p} + \underbrace{\sum_{q=-Q}^{Q} \mathbf{C}_{q,m}^{(d)} \, \boldsymbol{\Delta}_q^{(d)} \; \mathbf{d}}_{\mathbf{i}_d} + \mathbf{n}_F^{(m)} \quad , \tag{12}$$

where $\mathbf{C}_{q,m}^{(p)}$ and $\mathbf{C}_{q,m}^{(d)}$ are the $(B+1) \times N_p$ and $(B+1) \times N_d$ matrices representing the hatched and the shaded parts of $\mathbf{C}_q$ in Fig. 3, respectively; $\boldsymbol{\Delta}_q^{(p)}$ and $\boldsymbol{\Delta}_q^{(d)}$ are $N_p \times N_p$ and $N_d \times N_d$ diagonal matrices which are carved out of $\boldsymbol{\Delta}_q$, corresponding to the pilot subcarriers and symbol subcarriers, respectively. Moreover, $\mathbf{n}_F^{(m)}$ is the corresponding part of $\mathbf{n}_F$, and $\mathbf{i}_d$ of (12) represents the interference that comes from the information data $\mathbf{d}$, due to the fact that $\mathbf{H}_F$ is not really banded. Now, rewriting the above equation as a function of $\mathbf{h} = [\mathbf{h}_{-Q}^T, \cdots, \mathbf{h}_Q^T]^T$, we have

$$\tilde{\mathbf{y}}^{(m)} = \mathbf{C}^{(m)} \mathbf{V}_p \mathbf{h} + \underbrace{\mathbf{D}^{(m)} \mathbf{V}_d \mathbf{h}}_{\mathbf{i}_d} + \mathbf{n}_F^{(m)}, \tag{13}$$

with $\mathbf{C}^{(m)} = [\mathbf{C}_{-Q,m}^{(p)}, \cdots, \mathbf{C}_{Q,m}^{(p)}]$, $\mathbf{V}_p = \mathbf{I}_{2Q+1} \otimes diag(\mathbf{p}) \; \mathbf{F}_l^{(p)}$, $\mathbf{D}^{(m)} = [\mathbf{C}_{-Q,m}^{(d)}, \cdots, \mathbf{C}_{Q,m}^{(d)}]$ and $\mathbf{V}_d = \mathbf{I}_{2Q+1} \otimes diag(\mathbf{d}) \; \mathbf{F}_l^{(d)}$. Here, $\mathbf{F}_l^{(p)}$ and $\mathbf{F}_l^{(d)}$ collect the rows of $\mathbf{F}_L$ corresponding to the positions of the pilots and the information symbols, respectively.

Now, stacking all the data in the column vector $\tilde{\mathbf{y}} = \left[\tilde{\mathbf{y}}^{(1)^T} \cdots \tilde{\mathbf{y}}^{(P)^T}\right]^T$, we obtain

$$\begin{aligned} \tilde{\mathbf{y}} &= \begin{bmatrix} \mathbf{C}^{(1)} \mathbf{V}_p \\ \vdots \\ \mathbf{C}^{(P)} \mathbf{V}_p \end{bmatrix} \mathbf{h} + \begin{bmatrix} \mathbf{D}^{(1)} \mathbf{V}_d \\ \vdots \\ \mathbf{D}^{(P)} \mathbf{V}_d \end{bmatrix} \mathbf{h} + \begin{bmatrix} \mathbf{n}_F^{(1)} \\ \vdots \\ \mathbf{n}_F^{(P)} \end{bmatrix} \\ &= \mathbf{P}\mathbf{h} + \mathbf{D}\mathbf{h} + \tilde{\mathbf{n}}_F. \end{aligned} \tag{14}$$

Different approaches can be pursued to estimate the channel coefficients $\mathbf{h}$. Herein, after a brief summary of the LMMSE estimator, we will focus on Kalman filter and MRLS. In order to compare the estimation performance of the proposed approaches we will refer to the Normalized Mean Square Error (NMSE) of the true windowed channel taps $\mathbf{H}_{tw} = \mathbf{D}_w [\mathbf{h}_{t0}, \cdots, \mathbf{h}_{tL}]$ with the estimated windowed channel taps $\hat{\mathbf{H}}_{tw} = [\hat{\mathbf{h}}_{tw0}, \cdots, \hat{\mathbf{h}}_{twL}]$, defined as

$$\begin{aligned} &\text{NMSE} = \frac{E\big(||\mathbf{H}_{tw} - \hat{\mathbf{H}}_{tw}||^2\big)}{tr(\mathbf{R}_{\text{Doppler}}^{(t)} \otimes \mathbf{R}_{\text{Multipath}})} = \frac{E\big(||\mathbf{H}_{tw} - \mathbf{B}\hat{\mathbf{h}}_{cw}||^2\big)}{tr(\mathbf{R}_{\text{Doppler}}^{(t)} \otimes \mathbf{R}_{\text{Multipath}})}, \\ &\hat{\mathbf{H}}_{cw} = [\hat{\mathbf{h}}_{cw0}, \cdots, \hat{\mathbf{h}}_{cwL}] \end{aligned} \tag{15}$$

where $[\mathbf{R}_{\text{Doppler}}^{(t)}]_{m,n} = J_0(2\pi(m-n)v_D/N)[\mathbf{w}]_m[\mathbf{w}]_n$, $\mathbf{R}_{\text{Multipath}}$ is the covariance matrix of the channel taps, and $\mathbf{h}_{twl}$, $\mathbf{h}_{cwl}$ are vectors of the windowed channel $\mathbf{D}_w \mathbf{h}_{tl}$, analogously defined to those appearing in (5) for the unwindowed channel.

A. LMMSE Estimator

Assuming that i) the data $\mathbf{s}$ are zero mean i.i.d.random, variables, ii) the noise $\tilde{\mathbf{n}}_F$ the data $\mathbf{s}$ and the channel coefficients $\mathbf{h}$ are jointly uncorrelated, iii) the channel taps are statically modelled by Jakes' equations (2), and iv) the channel modelling error $\boldsymbol{\epsilon}$ can be neglected with respect to the noise, the LMMSE estimator is given by [14]

$$\begin{aligned} \hat{\mathbf{h}}_{\text{LMMSE}} &= \mathbf{R}_h \mathbf{P}^H (\mathbf{P}\mathbf{R}_h \mathbf{P}^H + \mathbf{R}_I)^{-1} \tilde{\mathbf{y}} \\ &= (\mathbf{P}^H \mathbf{R}_I^{-1} \mathbf{P} + \mathbf{R}_h^{-1})^{-1} \mathbf{P}^H \mathbf{R}_I^{-1} \tilde{\mathbf{y}}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\mathbf{R}_{\text{Doppler}} = \mathbf{B}^{\dagger} \mathbf{R}_{\text{Doppler}}^{(t)} \mathbf{B}^{\dagger H} \quad ; \quad \mathbf{R}_h = \mathbf{R}_{\text{Doppler}} \otimes \mathbf{R}_{\text{Multipath}}; \\ &\mathbf{R}_I = E(\mathbf{D}\mathbf{R}_h \mathbf{D}^H) + \mathbf{R}_{\tilde{n}_F} \end{aligned} \tag{17}$$

and $\mathbf{R}_{\tilde{n}_F}$ is the covariance matrix of the colored noise.

B. Kalman Filter

We assume that a 1^{st} order Gauss-Markov model is enough to model the variation of the BEM coefficients from one OFDM bock to the next one, as expressed by

$$\mathbf{h}_k = \mathbf{A}\mathbf{h}_{k-1} + \mathbf{v}_k, \tag{18}$$

where $\mathbf{h}_k$ are the BEM coefficients that model the channel during the k^{th} OFDM block, $\mathbf{A}$ drives the model evolution, and $\mathbf{v}_k$ is the process noise characterized by

$$\begin{aligned} &E(\mathbf{v}_k) = \mathbf{0}_{(2Q+1)(L+1)\times 1} \quad ; \quad E(\mathbf{v}_k \mathbf{v}_{k-m}^H) = \mathbf{Q}\delta[m] \\ &E(\mathbf{h}_k \mathbf{v}_{k-m}^H) = \mathbf{0}_{(2Q+1)(L+1)}. \end{aligned} \tag{19}$$

Thanks to the ICI reduction induced by windowing, and to the almost orthogonal FDKD training pilots in [13], the interference $\mathbf{i}_{d,k} = \mathbf{D}_k \mathbf{h}_k$ in (14) can be neglected for the Kalman filter design, and thus for the k^{th} OFDM symbol we can write

$$\tilde{\mathbf{y}}_k = \mathbf{P}\mathbf{h}_k + \tilde{\mathbf{n}}_k, \tag{20}$$

where we omitted the subscript $_F$ for simplicity.

The Kalman filtering algorithm for the model described by (18) and (20)

can be summarized as follows [24]:

$$\begin{aligned} &\mathbf{M}_k^F = \mathbf{A}\mathbf{M}_{k-1}^A \mathbf{A}^H + \mathbf{Q}_k && \textit{Forward Error Covariance} \\ &\mathbf{K}_k = \mathbf{M}_k^F \mathbf{P}^H (\mathbf{R}_{\tilde{n}} + \mathbf{P}\mathbf{M}_k^F \mathbf{P}^H)^{-1} && \textit{Kalman Gain} \\ &\hat{\mathbf{h}}_k = \mathbf{A}\hat{\mathbf{h}}_{k-1} + \mathbf{K}_k(\tilde{\mathbf{y}}_k - \mathbf{P}\mathbf{A}\hat{\mathbf{h}}_{k-1}) && \textit{A-Posteriori Estimation} \\ &\mathbf{M}_k^A = (\mathbf{I} - \mathbf{K}_k \mathbf{P})\mathbf{M}_k^F && \textit{A-Posteriori Error Covariance} \end{aligned} \tag{21}$$

From (18) and (19), and based on the Yule-Walker equations, for TI channel statistics is easy to find:

$$\begin{aligned} &\mathbf{A} = \mathbf{R}_{h_{cross}} \mathbf{R}_h^{-1}; \qquad \mathbf{Q} = \mathbf{R}_h - \mathbf{A}\mathbf{R}_{h_{cross}} \\ &\mathbf{R}_{h_{cross}} = E(\mathbf{h}_k \mathbf{h}_{k-1}^H); \quad \mathbf{R}_h = E(\mathbf{h}_k \mathbf{h}_k^H). \end{aligned} \tag{22}$$

However, since both $\mathbf{A}$ and $\mathbf{Q}$ depend on $\mathbf{R}_{h_{cross}} = \mathbf{R}_{h_{cross},k}$ and $\mathbf{R}_h = \mathbf{R}_{h,k}$ at step k, it is clear that they change with the Doppler spread. Hence, we propose to compute (22) by recursively estimating $\mathbf{R}_{h_{cross},k}$ with the aid of an exponential window forgetting factor λ, that is

$$\hat{\mathbf{R}}_{h_{cross},k} = \lambda \hat{\mathbf{R}}_{h_{cross},k-1} + (1-\lambda)\hat{\mathbf{h}}_k \hat{\mathbf{h}}_{k-1}^H, \tag{23}$$

and, exploiting the same RLS upadate rule, we update the inverse covariance $\boldsymbol{\Phi}_k = \mathbf{R}_{h,k}^{-1}$ directly from the estimated channel coefficients $\hat{\mathbf{h}}_k$ [24], as summarized in last equation of (27).

Moreover, for a given estimate $\hat{\mathbf{h}}_k$, $\hat{\mathbf{A}}_k$ and $\hat{\mathbf{h}}_{k-1}$, similarly to (23), we can find $\hat{\mathbf{Q}}_k$ as

$$\begin{aligned} &\hat{\mathbf{v}}_k = \hat{\mathbf{h}}_k - \hat{\mathbf{A}}_k \hat{\mathbf{h}}_{k-1} \\ &\hat{\mathbf{Q}}_k = \lambda \hat{\mathbf{Q}}_{k-1} + (1-\lambda)\hat{\mathbf{v}}_k \hat{\mathbf{v}}_k^H. \end{aligned} \tag{24}$$

It is well known that the memory factor λ determines the importance of the new entries $\hat{\mathbf{v}}_k$ at step k, for the update of the old matrices $\hat{\mathbf{R}}_{h_{cross},k-1}$ and $\hat{\boldsymbol{\Phi}}_{k-1}$ at step $k-1$. It it is usually chosen into the range $[0.9, 1)$ trading accuracy ($\lambda \to 1$) for convergence speed ($\lambda \to 0.9$).

A good trade off would be to make λ time-varying *decreasing* λ when the estimation error is high to get faster convergence, while *increasing* λ when the convergence is reached to further filter the noise and reduce the estimation error. To decide whether to increase or decrease λ, we can exploit the *A-Posteriori* error covariance matrix $\mathbf{M}_k^A$, which is embedded in the Kalman filter structure and provides a measure of the estimation error power. By normalizing $\mathbf{M}_k^A$ to the noise and the channel coefficients powers $\hat{\sigma}_{h,k-1}^2 = tr(\hat{\mathbf{R}}_{h,k-1})$, we find an empirical update rule for λ, which is the firs equation of (27).

Summarizing, the Kalman filter is composed of

1) *Initialization*

$$
\begin{array}{lcl}
\hat{\mathbf{h}}_0 = \mathbf{0}_{(2Q+1)(L+1)\times 1} & ; & \tilde{\mathbf{y}}_0 = \mathbf{0}_{P(B+1)\times 1} \\
\hat{\mathbf{A}}_0 = \mathbf{R}_{h_{cross},0}\,\mathbf{R}_{h,0}^{-1} & ; & \mathbf{M}_0^A = \mathbf{R}_{h,0}; \\
\hat{\mathbf{Q}}_0 = \mathbf{R}_{h,0} - \hat{\mathbf{A}}_0\,\mathbf{R}_{h_{cross},0} & ; & \hat{\mathbf{\Phi}}_0 = \mathbf{R}_{h,0}^{-1} \\
\hat{\mathbf{R}}_{h_{cross},0} = \mathbf{R}_{h_{cross},0} & ; & \hat{\sigma}^2_{h,0} = tr(\mathbf{R}_{h,0})
\end{array}
\tag{25}
$$

2) *Recursion*

$$
\begin{array}{l}
\mathbf{M}_k^F = \hat{\mathbf{A}}_{k-1}\mathbf{M}_{k-1}^A\hat{\mathbf{A}}_{k-1}^H + \hat{\mathbf{Q}}_{k-1} \\
\mathbf{K}_k = \mathbf{M}_k^F\mathbf{P}^H(\mathbf{R}_{\tilde{n}} + \mathbf{P}\mathbf{M}_k^F\mathbf{P}^H)^{-1} \\
\hat{\mathbf{h}}_k = \hat{\mathbf{A}}_{k-1}\hat{\mathbf{h}}_{k-1} + \mathbf{K}_k(\tilde{\mathbf{y}}_k - \mathbf{P}\hat{\mathbf{A}}_{k-1}\hat{\mathbf{h}}_{k-1}) \\
\mathbf{M}_k^A = (\mathbf{I} - \mathbf{K}_k\mathbf{P})\mathbf{M}_k^F
\end{array}
\tag{26}
$$

3) *Model Update*

$$
\begin{array}{l}
\lambda_k = 1 - \frac{tr(\mathbf{M_k}^A)}{\hat{\sigma}^2_{h,k-1}tr(\mathbf{R}_{\tilde{n}})} \\
\hat{\sigma}^2_{h,k} = \lambda_k\hat{\sigma}^2_{h,k-1} + (1-\lambda_k)\hat{\mathbf{h}}_k^H\hat{\mathbf{h}}_k \\
\hat{\mathbf{R}}_{h_{cross},k} = \lambda_k\hat{\mathbf{R}}_{h_{cross},k-1} + (1-\lambda_k)\hat{\mathbf{h}}_k\hat{\mathbf{h}}_{k-1}^H \\
\hat{\mathbf{A}}_k = \hat{\mathbf{R}}_{h_{cross},k}\hat{\mathbf{\Phi}}_{k-1} \\
\hat{\mathbf{v}}_k = \hat{\mathbf{h}}_k - \hat{\mathbf{A}}_k\hat{\mathbf{h}}_{k-1} \\
\hat{\mathbf{Q}}_k = \lambda_k\hat{\mathbf{Q}}_{k-1} + (1-\lambda_k)\hat{\mathbf{v}}_k\hat{\mathbf{v}}_k^H \\
\hat{\mathbf{\Phi}}_k = \lambda_k^{-1}\hat{\mathbf{\Phi}}_{k-1} - \lambda_k^{-2}\frac{\hat{\mathbf{\Phi}}_{k-1}\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\hat{\mathbf{\Phi}}_{k-1}}{(1-\lambda_k)^{-1}+\lambda_k^{-1}\hat{\mathbf{h}}_k^H\hat{\mathbf{\Phi}}_{k-1}\hat{\mathbf{h}}_k}.
\end{array}
\tag{27}
$$

The algorithm may be initialized with the theoretical matrices $\mathbf{R}_{h_{cross},0}$ and $\mathbf{R}_{h,0}$, assuming Jakes' model for a given initial mobile speed. However, simulations show this is not crucial.

The complexity ($O([(2Q+1)(L+1)]^3)$) is increased wrt to LMMSE due to the matrix inversion computation that is required at each stept by the Kalman gain $\mathbf{K}_k$.

C. Modified Recursive Least Squares Estimator

To reduce the complexity involved with the Kalman filtering procedure we can rely on a RLS approach [24]. Under the same assumptions made for (20) and with classical derivations that we omit due to lack of space, we obtain a Modified RLS estimator expressed by

1) *Initialization*

$$
\begin{array}{lcl}
\lambda_0 = 1; & ; & \tilde{\mathbf{y}}_0 = \mathbf{0}_{P(B+1)\times 1} \\
\hat{\mathbf{h}}_0 = \mathbf{0}_{(L+1)(2Q+1)\times 1} & ; & \hat{\mathbf{\Psi}}_0 = \mathbf{I}_{P(B+1)}
\end{array}
\tag{28}
$$

2) *Recursion*

$$
\begin{array}{l}
\lambda_k = \alpha\lambda_{k-1} + (1-\alpha)\ \ 1 - K\varepsilon_k^H\varepsilon_k \\
\hat{\mathbf{\Psi}}_k = \lambda_k^{-1}\hat{\mathbf{\Psi}}_{k-1} - \lambda_k^{-2}\frac{\hat{\mathbf{\Psi}}_{k-1}\,\tilde{\mathbf{y}}_k\tilde{\mathbf{y}}_k^H\,\hat{\mathbf{\Psi}}_{k-1}}{(1-\lambda_k)^{-1}+\lambda_k^{-1}\tilde{\mathbf{y}}_k^H\,\hat{\mathbf{\Psi}}_{k-1}\,\tilde{\mathbf{y}}_k} \\
\hat{\mathbf{h}}_k = \mathbf{P}^{\dagger}(\mathbf{I}_{P(B+1)} - \mathbf{R}_{\tilde{n}}\hat{\mathbf{\Psi}}_k)\tilde{\mathbf{y}}_k.
\end{array}
\tag{29}
$$

Since no matrix inversion is involved in the above equations, this algorithm has a lower computational cost with respect to the modified Kalman filter. Differently from the Kalman filter, we cannot rely on the *A-Posteriori* error covariance matrix $\mathbf{M}_k^A$ to adjust the memory factor λ at each step. We note however that, if the estimation performance is good, the estimation error $\xi_k = \mathbf{h}_{k-1} - \hat{\mathbf{h}}_{k-1}$ has a low power expressed by $\mathrm{tr}(E(\xi_k\xi_k^H))$. Thus, the modelling error defined by $\varepsilon_k = \tilde{\mathbf{y}}_{k-1} - \mathbf{P}\hat{\mathbf{h}}_{k-1}$ has an average power expressed by

$$
\sigma^2_{\varepsilon_k} = tr(E(\varepsilon_k\varepsilon_k^H)) = tr(\mathbf{P}E(\xi_k\xi_k^H)\mathbf{P}^H) + \sigma^2_{\tilde{n}} \approx \sigma^2_{\tilde{n}}. \tag{30}
$$

We consequently propose the first equation of (29), where α is chosen to set how fast λ_k should vary, while K controls the influence of the estimation error on the update. Since we want $\lambda \in [0.9, 1)$ we set $K = 1/(100\sigma^2_{\tilde{n}})$, such that for good estimation scenarios the quantity $K\varepsilon_k^H\varepsilon_k$ is around 10^{-2} and does not change significantly λ_k in the first equation of (29).

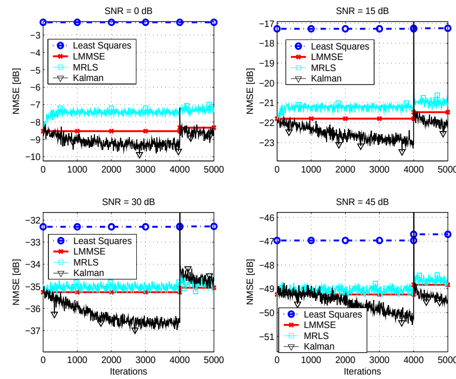

Fig. 4. Convergence Analysis: $\alpha = 0.5$, $v_D = 0.25 \rightarrow 0.30$ at time $k = 4001$

IV. Simulation Results

We consider an OFDM system (fig. 1) with N_A active carriers, $N_V = N - N_A$ zero frequency guard bands, and a Jakes' channel with constant power delay profile. The data symbols are carved from a QPSK alphabet with $E(\mathbf{d}\mathbf{d}^H) = \sigma_d^2\mathbf{I}_{N_d}$, whereas the pilots are chosen according to the FDKD scheme in [13]: each block $\mathbf{p}^{(p)}$ is formed by a single pilot, $b = \sqrt{\sigma_p^2(2B+1)}$ (σ_p^2 is the power of each pilot block)surrounded by B zeros at each edge. To reshape the received data (as well as the BEM basis functions), we use the MBAE-SOE-2 window $\mathbf{w}$ proposed in [7].

The parameters used for all the simulations, are as follows

- $N = 256$; $N_A = 244$; $Q = 2$;
- $B = 2Q = 4$; $L = 4$; $P = L + 1 = 5$;
- $\sigma_d^2 = \sigma_p^2 = 1$; $\sigma^2_{h_{tl}} = 1$ for $l = 0, \cdots, L$;

To study the convergence properties of the adaptive algorithms, we initialize the two estimators assuming the knowledge of the true channel statistics for $v_D = 0.25$, in such a way they can quickly reach the steady state regime. After 4000 time steps (e.g. OFDM blocks), we suddenly change the Doppler spread to $v_D = 0.3$ (20%) and look what happens for the next 1000 time steps. We underscore that this is quite a hard test for the algorithms because in reality they have to face smoother changes of the channel statistics (e.g mobile speed and direction). In order to analyze performance, we average over 1000 channel realizations for each of the 5000 time steps. Figure 4 shows the results for different values of the SNR (SNR = 0, 15, 30, 45 dB). As expected, the Kalman filter always reaches a better performance. Figure 5 shows the mean value of the memory factor for both models: as v_D changes, the estimation error increases and λ_k decreases to make the estimators' equations more influenced by the new data and speed up convergence. The change in the statistics is more penalizing for the Kalman filter, which exhibits a peak in the NMSE; however the adaptation speed is so fast that the convergence is reached in few steps. Figure 6 compares the two estimators with the LMMSE (always aided by perfect statistical channel information) when the convergence is reached. The multi-block LMMSE proposed in [15] is also plotted to have a bound for the Kalman filter. For this approach, three consecutive blocks are considered. The MRLS algorithm exhibits very good results, getting close to the LMMSE performance. Of course, the strong influence of the noise at low SNR translates in a higher loss of estimation accuracy. This is due to the fact that the instantaneous noise values make the inverse covariance matrix $\mathbf{\Psi}_k$ to continuously oscillate around the true theoretical value. The Kalman filter, exploiting the channel correlation among successive OFDM blocks outperforms the single-block LMMSE, getting close to the multi-block LMMSE based on three consecutive blocks.

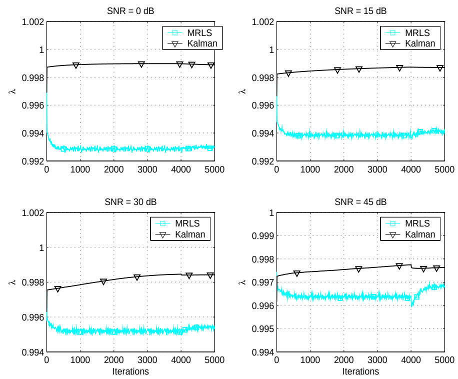

Fig. 5. Memory Factor: $\alpha = 0.5$, $v_D = 0.25 \rightarrow 0.30$ at time $k = 4001$

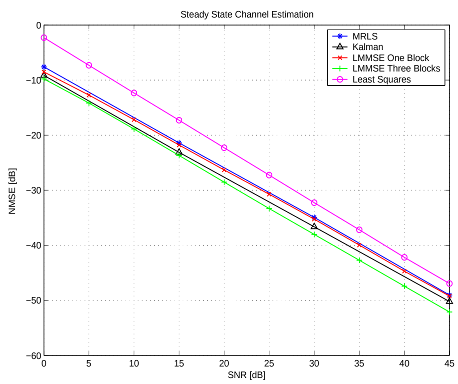

Fig. 6. Estimation Comparison: $\alpha = 0.5$, $v_D = 0.256$

V. Conclusions

In this paper, two novel adaptive channel estimators have been proposed to weapon OFDM equalizers against Doppler effects. Starting from the LMMSE estimator we derived a Kalman filter and a Modified RLS algorithm. Simulation results showed that both the estimators present good performances with a reasonable complexity, due to the reduction in unknowns obtained by a BEM approach. Since they are adaptive in nature and independent of the channel statistics, these estimators find their application when the channel statistics are not known or cannot be exactly estimated. Future directions are the investigation of a data-aided channel tracking mode to reduce the information rate loss associated with training, as well as suboptimal Kalman filter solutions to further reduce the complexity.

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