SEMI-BLIND CHANNEL ESTIMATION FOR RAPIDLY TIME-VARYING CHANNELS

Geert Leus

Delft University of Technology, Fac. EEMCS, Mekelweg 4, 2628CD Delft, The Netherlands

ABSTRACT

In this paper, we discuss a semi-blind channel estimation algorithm for rapidly time-varying channels, relying on a complex exponential basis expansion model (CE-BEM) for the channel. However, whereas the original CE-BEM approach models a rectangularly windowed version of the channel, the proposed CE-BEM approach models a smoothly windowed version of the channel. This allows for a much better fit, and leads to better channel estimates. The obtained semi-blind channel estimates are subsequently used to construct a recently developed CE-BEM serial decision-feedback equalizer for CE-BEM channels. Simulation results are carried out to validate the proposed ideas.

1. INTRODUCTION

Doppler shifts due to high mobility cause a major impediment for some of today's wireless systems, such as digital video broadcasting applications. To combat these time-varying distortions, nontrivial equalization techniques are required. Such equalizers can be designed in a direct fashion or based on channel estimation. We focus on the latter in this paper. The channel estimation method we present is a semi-blind method that combines a training-based criterion with a blind criterion. As channel model we employ a complex exponential basis expansion model (CE-BEM). Our semi-blind method will basically combine the training-based method of [1, 2] with the blind method of [3]. However, instead of modeling a rectangularly windowed version of the channel by a CE-BEM as in the previous papers, we model a smoothly windowed version of the channel by a CE-BEM [4]. This leads to a much better fit and thus to better channel estimates. Note that other blind channel estimation methods for CE-BEM channels have been developed in [5, 6, 7, 8]. However, they are either stochastic in nature, or they require many receive antennas for blind identifiability, whereas the blind method of [4] is deterministic and only requires two receive antennas for blind identifiability.

Once the CE-BEM channel is estimated, we use it to design a recently proposed CE-BEM serial decision feedback equalizer [9, 10], for which the feedforward and feedback filter have a CE-BEM structure that is related to the CE-BEM used to model the channels. Note that we only implement this equalizer in the flat region of the window, since this is the region of interest. In the simulation results section, we compare our algorithm with a serial decision feedback equalizer that is designed in a direct fashion. More specifically, we consider an adaptive serial decision feedback equalizer that is first updated in a training-based fashion and then in a decision-directed fashion. Note that direct CE-BEM serial equalizer designs based on batch processing, such as the one proposed in [11], only apply to linear equalizers and not to decision feedback equalizers (unless we consider an iterative approach of course). That is why we choose for the more traditional adaptive serial decision feedback equalizer as our reference method.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ represent complex conjugate, transpose, and complex conjugate transpose (Hermitian), respectively. $(\cdot)^\dagger$ stands for the pseudo-inverse. We write the Kronecker delta as $\delta[n]$ and the Kronecker product as $\otimes$. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Further, diag$\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on its diagonal. Finally, we reserve $\|\cdot\|$ for the Frobenius norm.

2. SYSTEM DESCRIPTION

We consider a baseband description of a wireless system with a single transmit and R receive antennas. Transmitting a data symbol sequence $x[n]$ at symbol rate $1/T$, the baseband-equivalent received signal at the rth receive antenna can generally be written as

$$y^{(r)}(t) = \sum_{n=-\infty}^{\infty} g^{(r)}(t; t - nT)x[n] + w^{(r)}(t),$$

where $g^{(r)}(t; t-nT)$ is the baseband-equivalent time-varying channel at the rth receive antenna and $w^{(r)}(t)$ is the baseband-equivalent additive noise at the rth receive antenna. Sampling each receive antenna at symbol rate, the received sequence at the rth receive antenna $y^{(r)}[n] := y^{(r)}(nT)$ can be written as

$$y^{(r)}[n] = \sum_{\nu=-\infty}^{\infty} g^{(r)}[n; \nu]x[n - \nu] + w^{(r)}[n], \tag{1}$$

where $g^{(r)}[n; \nu] := g^{(r)}(nT; \nu T)$ and $w^{(r)}[n] := w^{(r)}(nT)$.

3. CHANNEL MODEL

As channel model, we will make use of the complex exponential basis expansion model (CE-BEM). However, instead of using a CE-BEM to model a rectangularly windowed version of the channel, we will use it to model a smoothly windowed version of the channel.

Assume the windowed channel at the rth receive antenna is defined as $\breve{g}^{(r)}[n; \nu] := d[n]g^{(r)}[n; \nu]$, where $d[n]$ is a smooth window given by

$$d[n] := \begin{cases} s[n] & \text{if } n \in [0, M) \\ 1 & \text{if } n \in [M, N - M) \\ s[N - 1 - n] & \text{if } n \in [N - M, N) \\ 0 & \text{elsewhere} \end{cases}.$$

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As long as the window is smooth, the exact shape does not influence performance much. Hence, we will always assume a cosine window taper: $s[n] = (1 - \cos(\pi n/M))/2$. We model $\breve{g}^{(r)}[n;\nu]$ in the interval $n \in [0, N)$ as:

$$\breve{g}^{(r)}[n;\nu] \approx h^{(r)}[n;\nu], \quad n \in [0, N), \tag{2}$$

where $h^{(r)}[n;\nu]$ is a CE-BEM with period N:

$$h^{(r)}[n;\nu] = \frac{1}{N}\sum_{l=0}^{L}\delta[\nu - l]\sum_{q=-Q/2}^{Q/2} h_{l,q}^{(r)} e^{j2\pi qn/N},$$

with L and Q chosen such that LT and $Q/(2NT)$ are larger than the channel delay and Doppler spread, respectively: $LT \geq \tau_{\max}$ and $Q/(2NT) \geq f_{\max}$. The best possible fit of this CE-BEM is obtained by selecting $\{h_{l,q}^{(r)}\}_{q=-Q/2}^{Q/2}$ as the $Q+1$ samples around zero from the critically sampled Doppler spectrum of the lth windowed channel tap $\breve{g}^{(r)}[n;l]$. Existing channel estimation approaches based on the CE-BEM only consider a rectangular window [1, 2, 3]. However, since only a limited Doppler range of the windowed channel taps is considered in the above fitting procedure, it is beneficial to reduce the sidelobes by means of a smooth window, as shown in [4].

To observe a windowed channel at the rth receive antenna, we obviously have to window the received sequence at the rth receive antenna, leading to $\breve{y}^{(r)}[n] := d[n]y^{(r)}[n]$. From (1) and (2), we then obtain

$$\begin{aligned}\breve{y}^{(r)}[n] &= \sum_{\nu=-\infty}^{\infty}\breve{g}^{(r)}[n;\nu]x[n-\nu] + \breve{w}^{(r)}[n] \\ &\approx \sum_{\nu=-\infty}^{\infty} h^{(r)}[n;\nu]x[n-\nu] + \breve{w}^{(r)}[n], \quad n \in [0, N),\end{aligned} \tag{3}$$

where $\breve{w}^{(r)}[n]$ is similarly defined as $\breve{y}^{(r)}[n]$.

4. BLOCK DATA MODEL

It is convenient to rewrite (3) on a block level. This block data model will form the basis of our semi-blind channel estimation algorithm.

Defining the $(N+L)\times 1$ data symbol block as $\mathbf{x} := [x[-L], \ldots, x[N-1]]^T$, the $N \times 1$ windowed received sample block at the rth receive antenna $\breve{\mathbf{y}}^{(r)} := [\breve{y}^{(r)}[0], \ldots, \breve{y}^{(r)}[N-1]]^T$ can be written as

$$\breve{\mathbf{y}}^{(r)} = \mathbf{H}^{(r)}\mathbf{x} + \breve{\mathbf{w}}^{(r)}, \tag{4}$$

where $\breve{\mathbf{w}}^{(r)}$ is similarly defined as $\breve{\mathbf{y}}^{(r)}$, and $\mathbf{H}^{(r)}$ is the $N \times (N+L)$ band matrix given by

$$\mathbf{H}^{(r)} = \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)}\mathbf{D}_q\mathbf{Z}_l, \tag{5}$$

with $\mathbf{D}_q := \mathrm{diag}\{[1, e^{j2\pi q/N}, \ldots, e^{j2\pi q(N-1)/N}]^T\}$, and $\mathbf{Z}_l := [\mathbf{0}_{N\times(L-l)}, \mathbf{I}_N, \mathbf{0}_{N\times l}]$. Substituting (5) in (4), the $N \times 1$ windowed received sample block at the rth receive antenna can be written as

$$\breve{\mathbf{y}}^{(r)} = \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)}\mathbf{D}_q\mathbf{Z}_l\mathbf{x} + \breve{\mathbf{w}}^{(r)}. \tag{6}$$

Based on this block data model we will now derive a training-based and blind criterion, which will subsequently be combined to obtain a semi-blind criterion. The approach is reminiscent of the semi-blind methods that have been proposed for time-invariant channels [12].

5. TRAINING-BASED CRITERION

Let us start by rewriting (6) as $\breve{\mathbf{y}}^{(r)T} = \mathbf{h}^{(r)T}\underline{\mathbf{X}} + \breve{\mathbf{w}}^{(r)T}$, where $\underline{\mathbf{X}} = [\mathbf{D}_{-Q/2}\mathbf{Z}_0\mathbf{x}, \ldots, \mathbf{D}_{-Q/2}\mathbf{Z}_L\mathbf{x}, \ldots, \mathbf{D}_{Q/2}\mathbf{Z}_L\mathbf{x}]^T$ and $\mathbf{h}^{(r)} := [h_{-Q/2,0}^{(r)}, \ldots, h_{-Q/2,L}^{(r)}, \ldots, h_{Q/2,L}^{(r)}]^T$. Suppose now that training packets are inserted in $\mathbf{x}$. Then some columns of $\underline{\mathbf{X}}$ are known, which can be stacked into $\underline{\mathbf{X}}_{tr}$. Stacking the corresponding entries of $\breve{\mathbf{y}}^{(r)}$ into $\breve{\mathbf{y}}_{tr}^{(r)}$, we obtain $\breve{\mathbf{y}}_{tr}^{(r)T} = \mathbf{h}^{(r)T}\underline{\mathbf{X}}_{tr} + \breve{\mathbf{w}}_{tr}^{(r)T}$, where $\breve{\mathbf{w}}_{tr}^{(r)}$ is similarly defined as $\breve{\mathbf{y}}_{tr}^{(r)}$. Stacking the R vectors $\breve{\mathbf{y}}_{tr}^{(r)}$ as $\breve{\mathbf{y}}_{tr} = [\breve{\mathbf{y}}_{tr}^{(1)T}, \ldots, \breve{\mathbf{y}}_{tr}^{(R)T}]^T$, we finally obtain $\breve{\mathbf{y}}_{tr}^T = \mathbf{h}^T(\mathbf{I}_R \otimes \underline{\mathbf{X}}_{tr}) + \breve{\mathbf{w}}_{tr}^T$, where $\breve{\mathbf{w}}_{tr}$ is similarly defined as $\breve{\mathbf{y}}_{tr}$ and $\mathbf{h} := [\mathbf{h}^{(1)T}, \ldots, \mathbf{h}^{(R)T}]^T$. A training-based channel estimate can then be obtained by solving

$$\hat{\mathbf{h}}_{tr} = \arg\min_{\mathbf{h}} \|\breve{\mathbf{y}}_{tr}^T - \mathbf{h}^T(\mathbf{I}_R \otimes \underline{\mathbf{X}}_{tr})\|^2. \tag{7}$$

The solution is given by $\hat{\mathbf{h}}_{tr} = (\mathbf{I}_R \otimes \underline{\mathbf{X}}_{tr}^{T\dagger})\breve{\mathbf{y}}_{tr}$. Identifiability is guaranteed if $\underline{\mathbf{X}}_{tr}$ has full row rank, which can be satisfied by design.

6. BLIND CRITERION

To construct a blind criterion, we first have to mold the block data model (6) into a special form in order to better expose the structure of the CE-BEM.

For $l' = 0, \ldots, L'$ and $q' = -Q'/2, \ldots, Q'/2$, we premultiply $\breve{\mathbf{y}}^{(r)}$ with $\bar{\mathbf{D}}_{q'}\bar{\mathbf{Z}}_{l'}$, where $\bar{\mathbf{D}}_{q'} := \mathrm{diag}\{[1, e^{j2\pi q'/N}, \ldots, e^{j2\pi q(N-L'-1)/N}]^T\}$, and $\bar{\mathbf{Z}}_{l'} := [\mathbf{0}_{(N-L')\times(L'-l')}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times l'}]$. In other words, we compute a number of time- and frequency-shifted versions of $\breve{\mathbf{y}}^{(r)}$. Using the property $\bar{\mathbf{Z}}_{l'}\mathbf{D}_q = e^{j2\pi q(L'-l')/N}\bar{\mathbf{D}}_q\bar{\mathbf{Z}}_{l'}$, we then obtain

$$\begin{aligned}\mathbf{y}_{q',l'}^{(r)} &:= \bar{\mathbf{D}}_{q'}\bar{\mathbf{Z}}_{l'}\breve{\mathbf{y}}^{(r)} \\ &= \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} e^{j2\pi q(L'-l')/K}\bar{\mathbf{D}}_{q'}\bar{\mathbf{D}}_q\bar{\mathbf{Z}}_{l'}\mathbf{Z}_l\mathbf{x} + \mathbf{w}_{q',l'}^{(r)} \\ &= \sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} e^{j2\pi q(L'-l')/K} h_{q,l}^{(r)}\bar{\mathbf{D}}_{q+q'}\tilde{\mathbf{Z}}_{l+l'}\mathbf{x} + \mathbf{w}_{q',l'}^{(r)},\end{aligned}$$

where $\mathbf{w}_{q',l'}^{(r)}$ is similarly defined as $\mathbf{y}_{q',l'}^{(r)}$, and $\tilde{\mathbf{Z}}_k := [\mathbf{0}_{(N-L')\times(L+L'-k)}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times k}]$. Introducing $k := l + l'$ and $p := q + q'$, and defining $\mathbf{x}_{p,k} := \bar{\mathbf{D}}_p\tilde{\mathbf{Z}}_k\mathbf{x}$, we can also write this as

$$\mathbf{y}_{q',l'}^{(r)} = \sum_{k=0}^{L+L'}\sum_{p=-(Q+Q')/2}^{(Q+Q')/2} e^{j2\pi(p-q')(L'-l')/K} h_{p-q',k-l'}^{(r)}\mathbf{x}_{p,k} + \mathbf{w}_{q',l'}^{(r)}.$$

Stacking the $(Q'+1)(L'+1)$ vectors $\mathbf{y}_{q',l'}^{(r)}$ as $\mathbf{Y}^{(r)} := [\mathbf{y}_{-Q'/2,0}^{(r)}, \ldots, \mathbf{y}_{-Q'/2,L'}^{(r)}, \ldots, \mathbf{y}_{Q'/2,L'}^{(r)}]^T$, we get $\mathbf{Y}^{(r)} = \boldsymbol{\mathcal{H}}^{(r)}\mathbf{X} + \mathbf{W}^{(r)}$,

$$\boldsymbol{\mathcal{U}}_i^{(r)} := \begin{bmatrix} \boldsymbol{\Omega}_{0:L}^{-Q/2}\boldsymbol{\mathcal{U}}_{i,-Q'/2}^{(r)}\boldsymbol{\Omega}_{-L':L}^{Q/2} \cdots \boldsymbol{\Omega}_{0:L}^{-Q/2}\boldsymbol{\mathcal{U}}_{i,Q'/2}^{(r)}\boldsymbol{\Omega}_{-L':L}^{Q/2} & & \mathbf{0} \\ & \ddots & \ddots \\ \mathbf{0} & & \boldsymbol{\Omega}_{0:L}^{Q/2}\boldsymbol{\mathcal{U}}_{i,-Q'/2}^{(r)}\boldsymbol{\Omega}_{-L':L}^{-Q/2} \cdots \boldsymbol{\Omega}_{0:L}^{Q/2}\boldsymbol{\mathcal{U}}_{i,Q'/2}^{(r)}\boldsymbol{\Omega}_{-L':L}^{-Q/2} \end{bmatrix}$$

where $\mathbf{W}^{(r)}$ is similarly defined as $\mathbf{Y}^{(r)}$, $\mathbf{X} := [\mathbf{x}_{-(Q+Q')/2,0}, \ldots, \mathbf{x}_{-(Q+Q')/2,L+L'}, \ldots, \mathbf{x}_{(Q+Q')/2,L+L'}]^T$, and $\boldsymbol{\mathcal{H}}^{(r)}$ is the $(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ block Toeplitz matrix given by

$$\boldsymbol{\mathcal{H}}^{(r)} := \begin{bmatrix} \boldsymbol{\Omega}_{-L':0}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} \cdots \boldsymbol{\Omega}_{-L':0}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(r)} & & \mathbf{0} \\ & \ddots & \ddots \\ \mathbf{0} & & \boldsymbol{\Omega}_{-L':0}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} \cdots \boldsymbol{\Omega}_{-L':0}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(r)} \end{bmatrix},$$

with $\boldsymbol{\mathcal{H}}_q^{(r)}$ the $(L'+1) \times (L+L'+1)$ Toeplitz matrix given by

$$\boldsymbol{\mathcal{H}}_q^{(r)} := \begin{bmatrix} h_{q,0}^{(r)} \cdots h_{q,L}^{(r)} & & 0 \\ & \ddots & \ddots \\ 0 & & h_{q,0}^{(r)} \cdots h_{q,L}^{(r)} \end{bmatrix},$$

and $\boldsymbol{\Omega}_{n_1:n_2} := \mathrm{diag}\{[e^{-j2\pi n_1/N}, \ldots, e^{-j2\pi n_2/N}]^T\}$.

Stacking the R matrices $\mathbf{Y}^{(r)}$ as $\mathbf{Y} := [\mathbf{Y}^{(1)T}, \ldots, \mathbf{Y}^{(R)T}]^T$, we finally obtain $\mathbf{Y} = \boldsymbol{\mathcal{H}}\mathbf{X} + \mathbf{W}$, where $\mathbf{W}$ is similarly defined as $\mathbf{Y}$ and $\boldsymbol{\mathcal{H}}$ is the $R(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ matrix given by $\boldsymbol{\mathcal{H}} := [\boldsymbol{\mathcal{H}}^{(1)T}, \ldots, \boldsymbol{\mathcal{H}}^{(R)T}]^T$.

Let us first assume there is no noise. Hence, we get $\mathbf{Y} = \boldsymbol{\mathcal{H}}\mathbf{X}$. Further, let us assume that $\boldsymbol{\mathcal{H}}$ is tall, which requires $R(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$, and that $\mathbf{X}$ is wide, which requires $N-L' \geq (Q+Q'+1)(L+L'+1)$. These conditions can be satisfied by design. Note that in order to satisfy the first condition, we need more than one receive antenna, i.e., $R > 1$. Under the above assumptions the matrix $\mathbf{Y}$ has at most rank $(Q+Q'+1)(L+L'+1)$, and thus there are at least $I = R(Q'+1)(L'+1) - (Q+Q'+1)(L+L'+1)$ left singular vectors of $\mathbf{Y}$ with a zero singular value. Suppose these singular vectors are denoted as $\mathbf{u}_1, \ldots, \mathbf{u}_I$. Then we can write $\mathbf{u}_i^H\boldsymbol{\mathcal{H}} = \mathbf{0}_{1\times(Q+Q'+1)(L+L'+1)}$. Let us rewrite $\mathbf{u}_i$ as $\mathbf{u}_i := [\mathbf{u}_i^{(1)T}, \ldots, \mathbf{u}_i^{(R)T}]^T$, with $\mathbf{u}_i^{(r)} := [u_{i,-Q'/2,0}^{(r)}, \ldots, u_{i,-Q'/2,L'}^{(r)}, \ldots, u_{i,Q'/2,L'}^{(r)}]^T$. We can then reformulate the above equation as $\boldsymbol{\mathcal{U}}_i^H\mathbf{h} = \mathbf{0}_{(Q+Q'+1)(L+L'+1)\times 1}$, where $\mathbf{h}$ is defined as in Section 5. In this formula, $\boldsymbol{\mathcal{U}}_i$ is the $R(Q+1)(L+1) \times (Q+Q'+1)(L+L'+1)$ matrix given by $\boldsymbol{\mathcal{U}}_i := [\boldsymbol{\mathcal{U}}_i^{(1)T}, \ldots, \boldsymbol{\mathcal{U}}_i^{(R)T}]^T$, where $\boldsymbol{\mathcal{U}}_i^{(r)}$ is the $(Q+1)(L+1) \times (Q+Q'+1)(L+L'+1)$ matrix given at the top of this page, with $\boldsymbol{\mathcal{U}}_{i,q'}^{(r)}$ the $(L+1) \times (L+L'+1)$ Toeplitz matrix given by

$$\boldsymbol{\mathcal{U}}_{i,q'}^{(r)} = \begin{bmatrix} u_{i,q',0}^{(r)} \cdots u_{i,q',L'}^{(r)} & & 0 \\ & \ddots & \ddots \\ 0 & & u_{i,q',0}^{(r)} \cdots u_{i,q',L'}^{(r)} \end{bmatrix}.$$

Stacking the results for the I left singular vectors, we obtain $\boldsymbol{\mathcal{U}}^H\mathbf{h} = \mathbf{0}_{I(Q+Q'+1)(L+L'+1)\times 1}$, where $\boldsymbol{\mathcal{U}}$ is the $R(Q+1)(L+1) \times I(Q+Q'+1)(L+L'+1)$ matrix given by $\boldsymbol{\mathcal{U}} := [\boldsymbol{\mathcal{U}}_1, \ldots, \boldsymbol{\mathcal{U}}_I]$.

In the presence of noise, we compute the I left singular singular vectors of $\mathbf{Y}$ with the smallest singular values. We denote them as $\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_I$ and define $\hat{\boldsymbol{\mathcal{U}}}$ based on $\{\hat{\mathbf{u}}_i\}_{i=1}^I$ in a similar fashion as we defined $\boldsymbol{\mathcal{U}}$ based on $\{\mathbf{u}_i\}_{i=1}^I$. A blind channel estimate can then obtained by solving

$$\hat{\mathbf{h}}_{bl} = \arg\min_{\mathbf{h}} \|\hat{\boldsymbol{\mathcal{U}}}^H\mathbf{h}\|^2. \tag{8}$$

The solution is given by the left singular vector of $\hat{\boldsymbol{\mathcal{U}}}$ corresponding to the smallest singular value. Blind identifiability, i.e., identifiability up to a complex scaling ambiguity, is guaranteed if $\boldsymbol{\mathcal{H}}$ has full column rank and $\mathbf{X}$ has full row rank. These conditions can not fully be guaranteed by design. However, we solve this in the next section.

7. SEMI-BLIND CRITERION

To resolve possible blind identifiability problems and the complex scaling ambiguity of the blind criterion (8), we now combine it with the training-based criterion (7), leading to a semi-blind channel estimation method. In other words, a semi-blind channel estimate can be obtained by solving

$$\hat{\mathbf{h}}_{sb} = \arg\min_{\mathbf{h}} \|\breve{\mathbf{y}}_{tr}^T - \mathbf{h}^T(\mathbf{I}_R \otimes \underline{\mathbf{X}}_{tr})\|^2 + \alpha\|\hat{\boldsymbol{\mathcal{U}}}^H\mathbf{h}\|^2,$$

where α is some weighting factor. The solution is given by

$$\hat{\mathbf{h}}_{sb} = [\mathbf{I}_R \otimes (\underline{\mathbf{X}}_{tr}^*\underline{\mathbf{X}}_{tr}^T) + \alpha\hat{\boldsymbol{\mathcal{U}}}\hat{\boldsymbol{\mathcal{U}}}^H]^\dagger(\mathbf{I}_R \otimes \underline{\mathbf{X}}_{tr}^*)\breve{\mathbf{y}}_{tr}.$$

As for the training-based criterion, identifiability is guaranteed if $\underline{\mathbf{X}}_{tr}$ has full row rank, which can be satisfied by design.

The choice of the weighting factor α is important. Ideally, one would want to optimize α with respect to the channel mean square error (MSE), but this is not an easy problem. In practice, one can simply select an α based on some initial simulation results making some assumptions about the statistics of the channel.

8. DECISION FEEDBACK EQUALIZATION

Recently, we have proposed a CE-BEM serial decision feedback equalizer for CE-BEM channels [9, 10]. The basic idea is to employ a CE-BEM for the feedforward and feedback filters, which is related to the CE-BEM of the channel. This allows one to construct the CE-BEM coefficients of the serial decision feedback equalizer based on the CE-BEM coefficients of the channel, thereby avoiding to compute a new serial decision feedback equalizer at each time-instant, which is computationally intensive. In the simulation results we will use the semi-blind CE-BEM channel estimate to construct such a CE-BEM serial decision feedback equalizer, but only for the flat region of the window.

9. SIMULATION RESULTS

In this section, we carry out a few simulations to validate the proposed approach. We consider a transmission scheme where a training packet is sent in between every 148 data packets. Each packet is assumed to contain 10 symbols, modulated using QPSK.

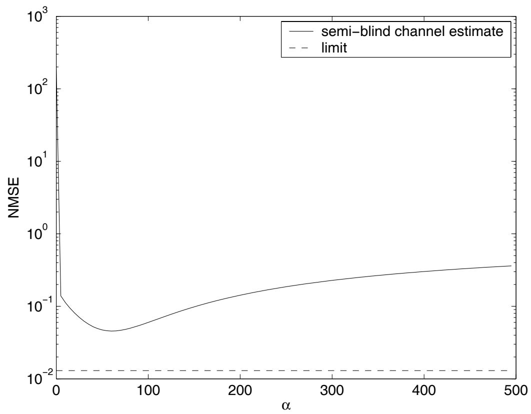

Fig. 1. NMSE of the semi-blind channel estimate as a function of α for a received SNR of 15 dB.

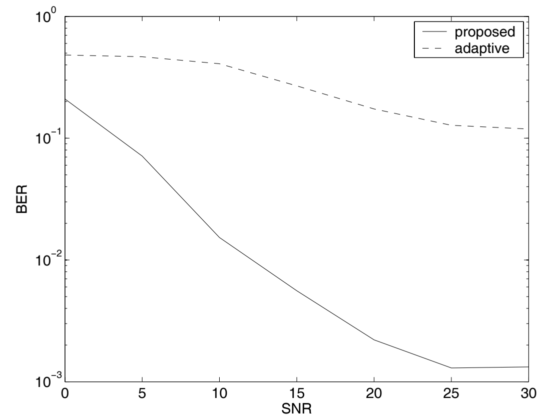

Fig. 2. Performance comparison between the proposed approach and an adaptive approach.

Note that such a sparse training structure for instance appears in the ATSC DTV standard. We assume a system with 1 transmit and $R = 2$ receive antennas. The $R = 2$ channels are modeled as an FIR filter with 2 i.i.d. channel taps, each of which has a Jakes' Doppler spectrum with a normalized Doppler spread of $f_{\max}T = 5.10^{-4}$. As block size N we take $N = 2000$, and as window taper length we take $M = 250$. Hence, if we center this window around one group of 148 data packets, the flat region of the window contains this group of 148 data packets together with one training packet on each side of the group. From the above parameters, we can accurately model the channels adopting a CE-BEM with $L = 1$ and $Q = 2$. For the semi-blind channel estimation method, we consider $L' = 5$ and $Q' = 4$.

In Figure 1, we show the normalized MSE (NMSE) of the semi-blind channel estimate as a function of α for a received SNR of 15 dB. Also shown is the NMSE of the best possible fit for the considered CE-BEM. Note that only the error in the flat region of the window is considered, since this is the region we are interested in. We clearly observe that the purely training-based method ($\alpha = 0$) is useless in this case, because the space in between the two exploited training packets is too large. The semi-blind method, on the other hand, does a pretty good job.

Figure 2 shows the BER versus the received SNR of a CE-BEM serial decision feedback equalizer designed using the semi-blind CE-BEM channel estimate for $\alpha = 100$. Note that this design is only carried out for the flat region of the window. The feedforward filter consists of 6 taps that are modeled by 5 complex exponential basis functions, whereas the feedback filter consists of 3 taps that are modeled by 7 complex exponential basis functions. We also compare this equalizer with an adaptive serial decision feedback equalizer that is first adapted in a training-based fashion and then in a decision-directed fashion. We use NLMS to update the filter coefficients and employ a step-size of $\mu = 0.1$. Again a 6-tap feedforward and a 3-tap feedback filter are considered. Since this equalizer is not able to converge within a training packet of 10 symbols, we considered a training period of 100 symbols after which convergence is always achieved. To decode the remaining symbols in the 148 data packets, we switch to a decision-directed mode during which we compute the BER. Clearly the adaptive serial decision feedback equalizer is outperformed by the proposed approach, because it is not capable of tracking the time-variations of the channel over such a large period.

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