PACKET SEPARATION IN WIRELESS AD-HOC NETWORKS BY KNOWN MODULUS ALGORITHMS

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We consider an asynchronous ad-hoc network with multiple users transmitting packets at the same time. The signal of interest is modulated by a known amplitude variation. This allows the corresponding multichannel receiver to estimate the beamformer weights that will suppress the interfering sources. We introduce "known modulus algorithms" (KMAs) to achieve this, and illustrate the throughput improvements that can be expected.

1. INTRODUCTION

A key limiting factor on the throughput of wireless networks is packet collisions among uncoordinated transmitters. Conventionally, medium access control (MAC) protocols are used to schedule transmissions either in a deterministic fashion (e.g., TDMA, FDMA or CDMA) or by random access protocols such as Aloha and CSMA. For ad-hoc networks, however, the absence of base stations and the necessity of distributed medium access control requires some form of random access, and avoiding collisions is difficult. Even more challenging is the so-called hidden/exposed terminal problem that severely limits the effectiveness of techniques based on carrier sensing. Although the use of CTS-RTS exchange along with busy-tone [1] can eliminate collisions [2], such protocols are vulnerable to interference from other services.

Recent advances in antenna array processing and space-time coding challenge the fundamental premise of the classical approach to MAC that prohibits the simultaneous transmission of different users. Various algorithms have been developed in the past decade that allow the separation of multiple signals, even without prior knowledge of the propagation channel [3]. This calls for new approaches in MAC protocols that exploit the new abilities [4].

Signal separation was first applied to the design of MAC protocols in [5] where an N-fold collision is resolved by a special retransmission protocol. This technique is only applicable in cellular networks. In [6], the problem of packet separation is formulated as one of signal separation in a MIMO system. While this technique is applicable in ad-hoc networks, it is restricted to a slot-synchronized network, which means that the network cannot cover a large area.

In this paper, we present a new technique that allows packet separation in asynchronous ad-hoc networks. As illustrated in figure 1, the user of interest transmits a constant modulus signal multiplied by an amplitude modulating code known at the receiver. This unique "color code" allows the antenna array at the receiver to detect and filter out the desired user among the other interfering signals that may or may not have a similar structure. The modulation code can be a random binary sequence determined either by the transmitter or the receiver, or it can be a common pseudo-random long code with different offsets for different users. The separating beamformer is computed using one of the known modulus algorithms (KMAs) developed in this paper. In general, KMA requires neither slot synchronization nor any coordination among transmitters, which makes its application in an uncontrolled environment such as wireless LAN particularly attractive.

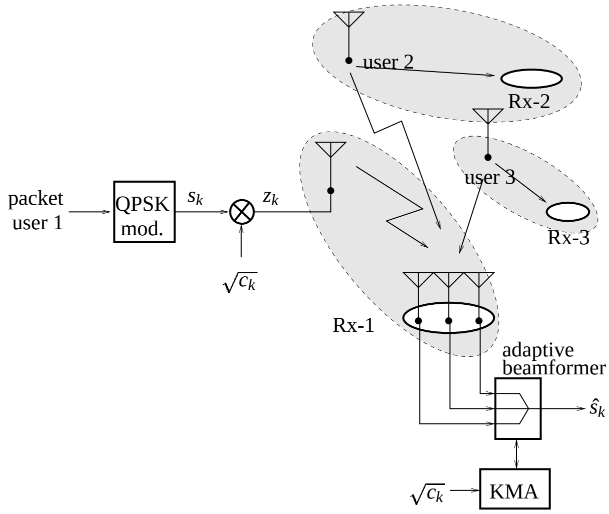

Figure 1. Wireless ad-hoc communication scenario. $\sqrt{c_k}$ is a known modulus variation used to recognize user-1.

From a source separation point of view, several techniques could play a role. We consider blind approaches, as channel estimation using training sequences has disadvantages in asynchronous systems. General blind techniques such as ACMA [7] are applicable, but not efficient since we are interested in only one user. Several modulation approaches have been proposed, such as "transmitter induced cyclostationarity" [8] which has recently been extended to multi-user convolutive channels [9] and OFDM [10]. Our objective here is to derive a system that is simpler than ACMA etc, does not reduce the capacity, and finds only the desired user.

2. DATA MODEL

Scenario We assume the situation in figure 1 where several users occupy a common wireless channel. For simplicity, the channel is assumed to be narrowband; in the case of OFDM this can easily be generalized to wider bands. The potential number of users is unlimited, but the offered network load is fixed. User 1 is the desired user, it is supposed to be received by receiver 1, but there will be interference from the other users. To suppress the interference, the receiver is equipped with an antenna array of M elements.

The transmission is modeled by a linear data model of the form

$$\mathbf{x}_k = \sum_{q=1}^{\infty} \mathbf{a}_q s_k^{(q)} + \mathbf{n}_k , \qquad (1)$$

where $\mathbf{x}_k \in \mathbb{C}^M$ is the data vector received by the array of M anten-

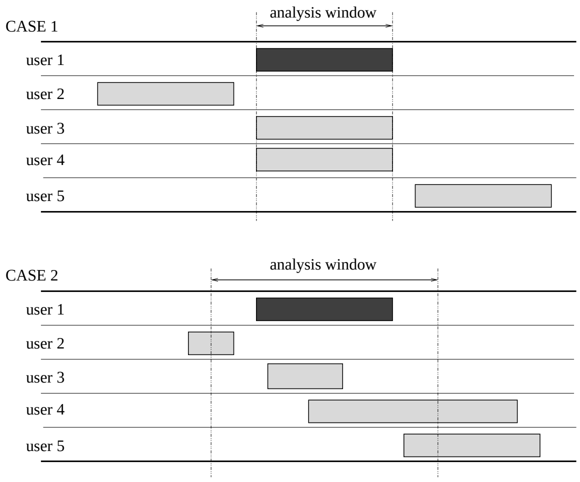

Figure 2. Slot structure

nas at time k, $\mathbf{a}_q$ is the signature vector of source q and $s_k^{(q)} \in \mathbb{C}$ its transmitted symbol at time k, and $\mathbf{n}_k \in \mathbb{C}^M$ an additive noise vector. In our traffic model, each source is assumed to transmit only once a data packet, and for the rest to be silent. Hence each $s^{(q)}$ has finite support. A physical user with several data packets counts as several independent sources, each with independent **a**-vectors, hence the model allows for a slowly changing (fading) channel.

The modulation of source 1 is assumed to be constant modulus, i.e. $|s_k^{(1)}| = 1$. The modulation of the other users is arbitrary.

Slot structure We will consider two types of transmission scenarios (see figure 2):

1. *slotted, with fixed slot length L.* The situation in a slot is stationary: the number of active users is constant inside a slot, and their spatial signature vectors are constant.
2. *unslotted, with fixed or variable packet lengths.* Packets can have arbitrary starting times, hence the number of active users changes throughout the slot. The packet length of user 1 is denoted by L.

In both cases, we assume that we are synchronized to the user of interest: the start time and length of his packet is known. We collect N samples in a data matrix $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_N] : M \times N$. In case 1, we take $N = L$ and $\mathbf{x}_1$ contains the first sample of the packet. In case 2, we take a slightly larger analysis window, $N \geq L$ samples, and center the packet of user 1 so that the first sample of his packet is in $\mathbf{x}_{(N-L)/2}$.

Let d be the maximal number of active users in the analysis window, and assume for notational simplicity that these are users 1 to d. Defining $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_d] : M \times d$, $\mathbf{S} = [s_k^{(q)}] : d \times N$ and $\mathbf{N} = [\mathbf{n}_1, \cdots, \mathbf{n}_N] : M \times N$, we obtain

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}. \qquad (2)$$

A, **S** and **N** are unknown. The objective is to reconstruct the nonzero part of $\mathbf{s}^{(1)}$ using linear beamforming, i.e., to find a beamformer $\mathbf{w}$ such that $\hat{s}_k = \mathbf{w}^{\mathrm{H}}\mathbf{x}_k$ approximates $s_k^{(1)}$, $k = 1, \cdots, N$.

Known modulus variation There are several algorithms for source separation that are applicable at this point (e.g., CMAs), but they all have the problem that they cannot distinguish one user from another. To distinguish the desired source, we give it a "color code", in the form of a known pseudo-random modulus variation. Instead of transmitting s_k, we transmit $z_k = s_k\sqrt{c_k}$, where $c_k = 1 \pm \varepsilon$ is a real and positive scaling that induces a small modulus variation, without changing the average transmission power. For notational convenience, we assume that $c_k = 0$ outside the support of the packet. The data model (2) is replaced by $\mathbf{X} = \mathbf{A}\mathbf{Z} + \mathbf{N}$.

Recall that we assume that $|s_k|^2 = 1$, so that $|z_k|^2 = c_k$. Similar to the CMA, the objective of the beamformer will be to recover z_k based on its modulus, i.e., such that

$$|\mathbf{w}^{\mathrm{H}}\mathbf{x}_k|^2 = |\hat{z}_k|^2 = c_k, \qquad k = 1, \cdots, N.$$

With noise, we try to minimize the difference and can obviously recover the source only approximatively.

3. KNOWN MODULUS ALGORITHMS

3.1. Iterative solutions

The usual CMA can easily be adapted for the present case, but apart from the usual stability and initialization issues, the resulting algorithm would not be very useful for the current purpose since we prefer to have a block solution. This is provided by an alternating projection algorithm: iterate until convergence

$$\begin{bmatrix} \mathbf{y} := \mathbf{w}^{\mathrm{H}}\mathbf{X} & \\ \hat{z}_k := \frac{y_k}{|y_k|}\sqrt{c_k}, & k = 1, \cdots, N \\ \mathbf{w} := (\hat{\mathbf{z}}\mathbf{X}^{\dagger})^{\mathrm{H}} & \end{bmatrix}$$

Note that a candidate solution $\hat{\mathbf{z}}$ is alternatingly projected onto the row span of $\mathbf{X}$ (via the projection $\mathbf{X}^{\dagger}\mathbf{X}$), and entry-wise scaled to fit the modulus condition. This algorithm is stable and converges usually nicely, but also needs an initial point.

3.2. AKMA for case 1

We will now set out to derive an algebraic closed-form solution, in the style of ACMA [7]. This can be used to obtain an initial point. We try to minimize

$$\hat{\mathbf{w}}_1 = \operatorname*{argmin}_{\mathbf{w}} \sum_{k=1}^{N} (|\mathbf{w}^{\mathrm{H}}\mathbf{x}_k|^2 - c_k)^2 = \operatorname*{argmin}_{\mathbf{w}} \|\mathbf{P}(\bar{\mathbf{w}} \otimes \mathbf{w}) - \mathbf{c}\|^2,$$

where $\mathbf{c} = [c_1, \cdots, c_N]^T$, $\mathbf{P} = (\bar{\mathbf{X}} \circ \mathbf{X})^{\mathrm{H}}$ and $\circ$ denotes a column-wise Kronecker product: $\bar{\mathbf{X}} \circ \mathbf{X} = [\bar{\mathbf{x}}_1 \otimes \mathbf{x}_1, \cdots, \bar{\mathbf{x}}_N \otimes \mathbf{x}_N]$. We follow the strategy of ACMA and split this optimization into two steps (hence suboptimal),

$$\begin{aligned} \hat{\mathbf{y}} &= \operatorname{argmin} \|\mathbf{P}\mathbf{y} - \mathbf{c}\|^2 \\ \hat{\mathbf{w}}_1 &= \operatorname{argmin} \|\hat{\mathbf{y}} - \bar{\mathbf{w}} \otimes \mathbf{w}\|^2. \end{aligned}$$

If $\mathbf{P}$ would have full column rank, the first problem has a unique solution in terms of the pseudo-inverse $\mathbf{P}^{\dagger}$:

$$\hat{\mathbf{y}} = \mathbf{P}^{\dagger}\mathbf{c}.$$

With this solution and setting $\hat{\mathbf{Y}} = \operatorname{unvec}(\hat{\mathbf{y}})$, where "unvec" denotes an unstacking of a vector into a square matrix, we can solve the second problem as

$$\hat{\mathbf{w}}_1 = \operatorname{argmin} \|\hat{\mathbf{y}} - \bar{\mathbf{w}} \otimes \mathbf{w}\|^2 = \operatorname{argmin} \|\hat{\mathbf{Y}} - \mathbf{w}\mathbf{w}^{\mathrm{H}}\|^2,$$

the solution of which is given in terms of the dominant eigenvector of $\mathbf{Y}$, scaled by the square root of the corresponding eigenvalue.

We thus see that, if $\mathbf{P}$ is full rank, the algorithm becomes particularly simple, and in the noise-free case will produce the exact separating beamformer to recover the desired packet. If $\mathbf{P}$ is not of full column rank, then there will exist additional solutions $\mathbf{y}_0$ to $\mathbf{P}\mathbf{y}_0 = \mathbf{0}$ which will add to the desired solution $\mathbf{y} = \bar{\mathbf{w}}_1 \otimes \mathbf{w}_1$, producing a result that cannot be factored. We thus need to study the rank properties of $\mathbf{P}$. We do this for the noise-free case.

First note that $\mathbf{X} = \mathbf{A}\mathbf{Z}$. To recover $\mathbf{Z}$ using linear beamforming, we need $\mathbf{A}$ to be tall: $d \leq M$. In this case, $\mathbf{X}$ has rank d. $\mathbf{P}$ has size $N\times M^2$. Since $\mathbf{P}^{\mathrm{H}} = (\bar{\mathbf{A}}\otimes\mathbf{A})(\bar{\mathbf{Z}}\circ\mathbf{Z})$, the rank of $\mathbf{P}$ is seen not to exceed d^2. A necessary condition for $\mathbf{P}$ to have rank d^2 is $d^2 \le N$.

$\mathbf{P}$ can be made full rank by a prefiltering step. Compute the SVD of $\mathbf{X}$, i.e., $\mathbf{X} = \mathbf{U\Sigma V}$, where $\mathbf{U}: M\times d$ orthogonal, $\mathbf{\Sigma}: d\times d$ positive diagonal, $\mathbf{V}: d\times N$ orthogonal, then we can replace $\mathbf{X}$ by

$$\underline{\mathbf{X}} := (\sqrt{N})\mathbf{\Sigma}^{-1}\mathbf{U}^{\mathrm{H}}\mathbf{X} = (\sqrt{N})\mathbf{V}$$

which has d rows and is of full rank. Note that due to the prewhitening, $\underline{\mathbf{X}}$ satisfies a model $\underline{\mathbf{X}} = \underline{\mathbf{A}}\mathbf{Z}$, where $\underline{\mathbf{A}}$ is $d\times d$ and asymptotically *unitary* (for large N). From now on, we assume that the prewhitening has been performed and that $d = M$ (we omit the underscore from the notation).

Even after the prefiltering, there are cases where $\mathbf{P}$ is singular, namely when sources are constant modulus (or equal-modulus). Indeed, if $\mathbf{z}^{(2)} = \mathbf{w}_2^{\mathrm{H}}\mathbf{X}$ and $\mathbf{z}^{(3)} = \mathbf{w}_3^{\mathrm{H}}\mathbf{X}$ are constant-modulus, then $\mathbf{P}(\bar{\mathbf{w}}_2\otimes\mathbf{w}_2) = \mathbf{1}$, $\mathbf{P}(\bar{\mathbf{w}}_3\otimes\mathbf{w}_3) = \mathbf{1}$, and

$$\mathbf{P}(\bar{\mathbf{w}}_2\otimes\mathbf{w}_2 - \bar{\mathbf{w}}_3\otimes\mathbf{w}_3) = \mathbf{0}\,.$$

To avoid this nullspace solution, all sources (except perhaps one) should have amplitude modulations.

We can show that if the sources are statistically independent constant modulus sources, all modulated by binary random power modulations $1\pm\varepsilon$, then $\frac{1}{N}\mathbf{P}^{\mathrm{H}}\mathbf{P}$ converges to its expected value

$$\begin{aligned}
\mathbf{C}_x &:= \mathrm{E}\{(\bar{\mathbf{x}}_k\otimes\mathbf{x}_k)(\bar{\mathbf{x}}_k\otimes\mathbf{x}_k)^{\mathrm{H}}\} = (\bar{\mathbf{A}}\otimes\mathbf{A})\mathbf{C}_z(\bar{\mathbf{A}}\otimes\mathbf{A})^{\mathrm{H}}\,,\\
\mathbf{C}_z &:= \mathrm{E}\{(\bar{\mathbf{z}}_k\otimes\mathbf{z}_k)(\bar{\mathbf{z}}_k\otimes\mathbf{z}_k)^{\mathrm{H}}\}\\
&= \mathbf{I} + \mathrm{vec}(\mathbf{I})\mathrm{vec}(\mathbf{I})^{\mathrm{H}} - (\mathbf{I}\circ\mathbf{I})(\mathbf{I}\circ\mathbf{I})^{\mathrm{H}} + \varepsilon^2(\mathbf{I}\circ\mathbf{I})(\mathbf{I}\circ\mathbf{I})^{\mathrm{H}}\,.
\end{aligned}$$

The eigenvalues of $\mathbf{C}_z$ are

$$\mathrm{eig}(\mathbf{C}_z) = \{d+\varepsilon^2,\ \underbrace{1,\cdots,1}_{d^2-d},\ \underbrace{\varepsilon^2,\cdots,\varepsilon^2}_{d-1}\}\,. \qquad (3)$$

These are also the eigenvalues of $\mathbf{C}_x$ since $\mathbf{A}$ is asymptotically unitary after prewhitening. Thus, the smallest eigenvalue of $\mathbf{C}_z$ is raised by the modulation to ε^2. If ε is not too small, $\mathbf{P}$ will be left invertible, so that $\mathbf{y} = \mathbf{P}^{\dagger}\mathbf{c}$ will lead to the correct solution.

3.3. AKMA for case 2

In case 2 there are additional situations where $\mathbf{P}$ becomes singular, namely when two sources are non-overlapping in time. Indeed, suppose $\mathbf{z}^{(2)} = \mathbf{w}_2^{\mathrm{H}}\mathbf{X}$, $\mathbf{z}^{(3)} = \mathbf{w}_3^{\mathrm{H}}\mathbf{X}$ are such that $z_k^{(2)}z_k^{(3)} = 0$, $\forall k$. Then $\mathbf{w}_2^{\mathrm{H}}\mathbf{x}_k\mathbf{x}_k^{\mathrm{H}}\mathbf{w}_3^{\mathrm{H}} = 0$, $\forall k$, hence

$$\mathbf{P}(\bar{\mathbf{w}}_2\otimes\mathbf{w}_3) = \mathbf{0}\,, \qquad \mathbf{P}(\bar{\mathbf{w}}_3\otimes\mathbf{w}_2) = \mathbf{0}\,.$$

Thus, the solution to $\mathbf{Py} = \mathbf{c}$ gives rise to

$$\mathbf{y} = \bar{\mathbf{w}}_1\otimes\mathbf{w}_1 + \lambda_{23}(\bar{\mathbf{w}}_2\otimes\mathbf{w}_3) + \lambda_{32}(\bar{\mathbf{w}}_3\otimes\mathbf{w}_2)$$

for unknown scalars $\lambda_{23},\lambda_{32}$, and $\mathbf{y}$ cannot be factored into $\bar{\mathbf{w}}_1\otimes\mathbf{w}_1$. We see two solutions for this problem. Firstly, we can write $\mathbf{Y} = \mathrm{unvec}(\mathbf{y})$ as

$$\mathbf{Y} = \begin{bmatrix}\mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3\end{bmatrix}\begin{bmatrix}1 & & \\ & \lambda_{32} & \\ & & \lambda_{23}\end{bmatrix}\begin{bmatrix}\mathbf{w}_1^{\mathrm{H}}\\ \mathbf{w}_3^{\mathrm{H}}\\ \mathbf{w}_2^{\mathrm{H}}\end{bmatrix} = \mathbf{W}\mathbf{\Lambda}_1\mathbf{M}^{\mathrm{H}}$$

where $\mathbf{M}$ is a permutation of $\mathbf{W}$. Similarly, if we take a basis $\{\mathbf{y}_2,\mathbf{y}_3\}$ of the null space, it can be written as

$$\mathbf{Y}_2 = \mathbf{W}\mathbf{\Lambda}_2\mathbf{M}^{\mathrm{H}}\,, \qquad \mathbf{Y}_3 = \mathbf{W}\mathbf{\Lambda}_3\mathbf{M}^{\mathrm{H}}$$

where $\mathbf{\Lambda}_2$, $\mathbf{\Lambda}_3$ are diagonal matrices (with their first entry equal to 0). The problem boils down to a joint diagonalization of unsymmetric matrices, or a joint Schur decomposition, which can be solved using Jacobi iterations [7].

Alternatively, we try to avoid the joint diagonalization step. If we have N sufficiently large and do prewhitening, then $\mathbf{A}$ is approximately unitary, and the $\mathbf{w}_i$ are orthogonal to each other. Hence, the desired solution $\bar{\mathbf{w}}_1\otimes\mathbf{w}_1$ is orthogonal to the null space of $\mathbf{P}$. In this case, we can simply set

$$\mathbf{y} = \mathbf{P}^{\dagger}\mathbf{c} \approx \bar{\mathbf{w}}_1\otimes\mathbf{w}_1\,.$$

With noise, $\mathbf{P}$ will not be exactly singular, and we will have to set a threshold on the pseudo-inverse. As is clear from equation (3), the threshold on the singular values of $\mathbf{P}$ should be smaller than $\varepsilon\sqrt{N}$. Figure 3 lists the algorithm as used in the simulations.

1. SVD: $\mathbf{X} =: \mathbf{U\Sigma V}$
   Estimate rank and truncate to $\mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}_s$
   Prefiltering: $\underline{\mathbf{X}} := \sqrt{L}\cdot\mathbf{\Sigma}_s^{-1}\mathbf{U}_s^{\mathrm{H}}\mathbf{X} = \sqrt{L}\cdot\mathbf{V}_s$
2. $\mathbf{P} = (\bar{\underline{\mathbf{X}}}\circ\underline{\mathbf{X}})^{\mathrm{H}}$
   $\mathbf{y} = \mathbf{P}^{\dagger}\mathbf{c}$, with pseudo-inverse threshold $\frac{1}{2}\varepsilon\sqrt{L}$
3. $\mathbf{Y} = \mathrm{unvec}(\mathbf{y})$
   $\underline{\mathbf{w}}$ = dominant eigenvector of $\mathbf{Y}$
4. $\mathbf{w} = \sqrt{L}\cdot\mathbf{U}_s\mathbf{\Sigma}_s^{-1}\underline{\mathbf{w}}$
   $\hat{\mathbf{z}} = \mathbf{w}^{\mathrm{H}}\mathbf{X}$
5. *optional:* alternating projection iterations

Figure 3. Summary of AKMA

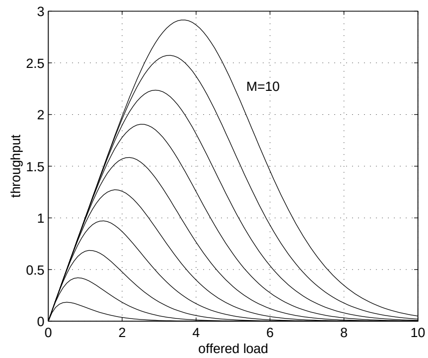

Figure 4. Throughput of the Aloha network with nodes using KMA. Throughput increases with M.

4. THROUGHPUT ANALYSIS

To gain some insight into the behavior of the network throughput, we use a simple analysis making the assumption that the packet arrival times are Poisson distributed and that Aloha is used as the random access protocol. The approach follows that of Abramson [11].

We shall assume that an unknown number (possibly infinite) of users may transmit packets asynchronously, and, without loss of generality, all packets have the same size $L = 1$. The packet arrival process that includes both the new arrivals and retransmissions is assumed to be Poisson with offered load λ. Given M antenna elements, a packet P will be successfully received if and only if there are no more than M users transmitting within a duration of $2L$ that begins L samples before P and ends at the end of P. We assume that all nodes use the same KMA. It then follows that the throughput, i.e., the average number of successfully received packets per unit time, is given by

$$T = \lambda e^{-2\lambda}\sum_{i=0}^{M-1}\frac{(2\lambda)^i}{i!}\,. \qquad (4)$$

It is evident that T increases with M as shown in figure 4. In the limit, $T = \lambda$, indicating a complete collision resolution.

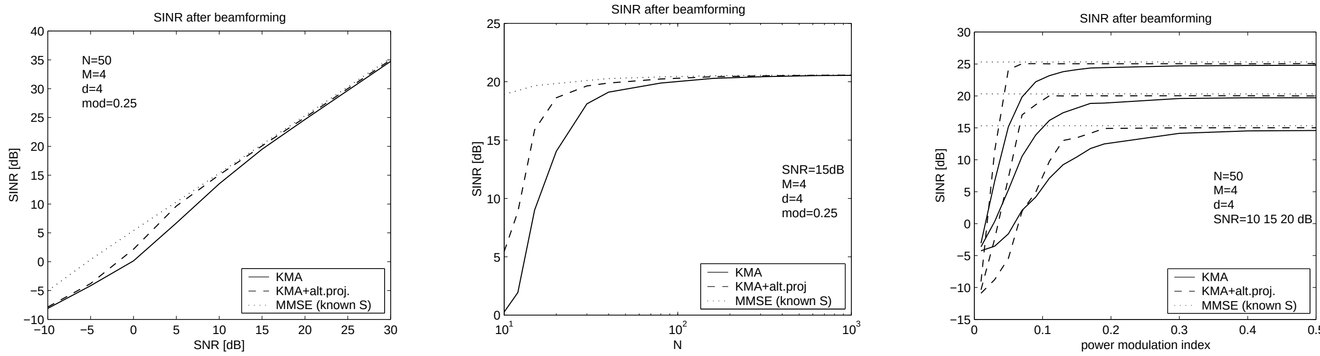

Figure 5. Case 1 beamformer performance: SINR of user 1 after beamforming.

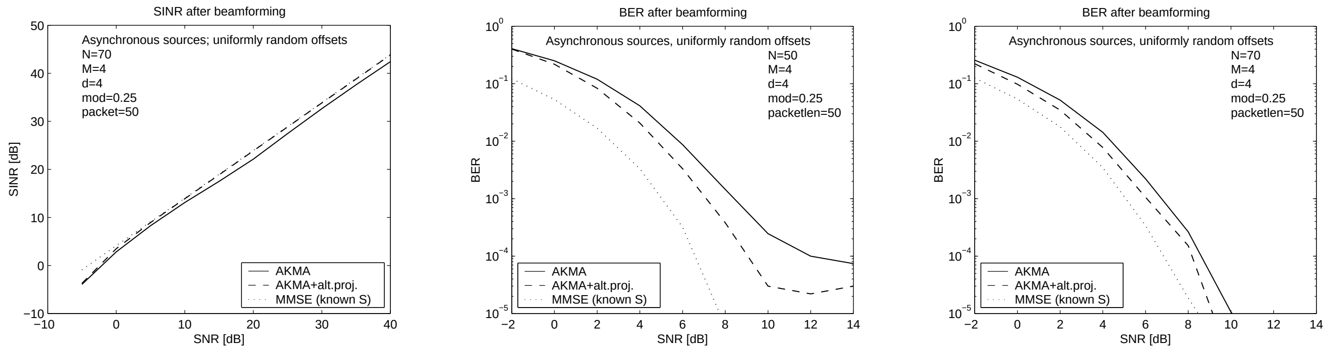

Figure 6. Case 2 beamformer performance: SINR and BER of user 1 after beamforming. Asynchronous sources with equal-length packets.

5. SIMULATIONS

Case 1 Figure 5 shows SINR performance plots of the beamformer of the first source for a simulation with $d = 4$ sources, $M = 4$ antennas in a uniform linear array, equal source powers, and source angles $-20°, 20°, 40°, -40°$, for varying SNR, packet length $L(= N)$, and power modulation index ε. All sources are modulated constant modulus sources. The reference line is the performance of the MMSE receiver with knowledge of $\mathbf{z}^{(1)}$, namely $\mathbf{w} = (\mathbf{z}^{(1)}\mathbf{X}^{\dagger})^{\mathrm{H}}$. The solid line is the performance of AKMA, the dashed line the performance of 15 iterations of the alternating projection algorithm, initialized by the AKMA. It is seen that the performance of the AKMA is generally quite good, but that it can be improved for small modulation indices and small N (i.e., $N < 2d^2$). This is due to the squaring involved in the construction of $\mathbf{P}$, and the presence of additional kernel solutions for small modulations.

Case 2 Figure $6(a)$ shows the performance in case 2, where users are unsynchronized but have equal packet lengths $L = 50$. The performance is virtually identical to that in case 1. The bit-error rate curve in fig. $6(b)$ shows some kind of jump for large SNRs. This is because the head and tail of the desired packet can be disturbed by the tail of another source, but with insufficient samples present to estimate that source reliably. A solution is to increase the analysis window N (see figure (c) where $N = 70$).

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