Time-Varying FIR Equalization for Doubly Selective Channels

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***Abstract*—In this paper, we propose a time-varying (TV) finite impulse response (FIR) equalizer for doubly selective (time- and frequency-selective) channels. We use a basis expansion model (BEM) to approximate the doubly selective channel and to design the TV FIR equalizer. This allows us to turn a complicated equalization problem into an equivalent simpler equalization problem, containing only the BEM coefficients of both the doubly selective channel and the TV FIR equalizer. The minimum mean-square error (MMSE) as well as the zero-forcing (ZF) solutions are considered. Comparisons with the block linear equalizer (BLE) are made. The TV FIR equalization we propose here unifies and extends many previously proposed serial equalization approaches. In contrast to the BLE, the proposed TV FIR equalizer allows a flexible tradeoff between complexity and performance. Moreover, through computer simulations, we show that the performance of the proposed MMSE TV FIR equalizer comes close to the performance of the ZF and MMSE BLE, at a point where the design as well as the implementation complexity are much lower.**

***Index Terms*—Doubly selective channels, equalization, fading channels, orthogonal frequency division multiplexing (OFDM), time-varying finite impulse response (TV FIR), two-dimensional (2-D) equalization, wireless communications.**

I. Introduction

THE wireless communications industry has experienced rapid growth in recent years, and digital cellular systems are currently being designed to provide high-speed multimedia services such as voice, Internet access, and video conferencing. These services require access speeds ranging from a few hundred kilobits per second for high-mobility users up to a few megabits per second for low-mobility users. Such high data rates give rise to frequency-selective propagation, while mobility and carrier offsets introduce time selectivity. This results in so-called doubly selective channels.

To combat these doubly selective channel effects, equalizers are required. For frequency-selective channels, such equalizers have been extensively studied in literature. We can distinguish between block equalizers and serial equalizers. Block linear equalizers (BLEs) for frequency-selective channels only require a single receive antenna for the zero-forcing (ZF) solution to exist [1]. They are usually complex to design, since inversion of a large matrix is required, and to implement, since multiplication with a large matrix is required. However, since a frequency-selective channel can be diagonalized by means of the discrete Fourier transform (DFT), the design and implementation complexity can be reduced, at the cost of a slight decrease in performance. On the other hand, serial linear equalizers (SLEs), more specifically, finite impulse response (FIR) equalizers, for frequency-selective channels generally require at least two receive antennas for the ZF solution to exist, but allow for a flexible tradeoff between complexity and performance [2], [3].

Recently, equalizers have also been developed for doubly selective channels [4]. As for frequency-selective channels, BLEs for doubly selective channels only require a single receive antenna for the ZF solution to exist. However, since a doubly selective channel cannot be diagonalized by means of a channel-independent transformation (such as the DFT), they cannot be simplified and hence are always complex to design and implement. This motivates us to look at SLEs, more specifically, FIR equalizers, for doubly selective channels, which we expect to allow for a flexible tradeoff between complexity and performance. Until now, only time-invariant (TIV) FIR equalizers for doubly selective channels have been introduced [4][1]. However, TIV FIR equalizers for doubly selective channels require many receive antennas for the ZF solution to exist. In this paper, we introduce time-varying (TV) FIR equalizers for doubly selective channels. We use a basis expansion model (BEM) to approximate the doubly selective channel and to design the TV FIR equalizer. This allows us to turn a complicated equalization problem (a TV one-dimensional (1-D) deconvolution problem) into an equivalent simpler equalization problem (a TIV two-dimensional (2-D) deconvolution problem), containing only the BEM coefficients of both the doubly selective channel and the TV FIR equalizer. We focus on the minimum mean-square error (MMSE) as well as the ZF solution. It turns out that TV FIR equalizers generally require at least two receive antennas for the

Manuscript received December 19, 2002; revised August 4, 2003; accepted October 27, 2003. The editor coordinating the review of this paper and approving it for publication is H. Boelcskei. This work was supported by the Belgian State Prime Minister's Office-Federal Office for Scientific, Technical and Cultural Affairs-Interuniversity Poles under Attraction Programme (2002–2007)-IUAP P5/22 ("Dynamical Systems and Control: Computation, Identification and Modeling") and Programme P5/11 ("Mobile multimedia communication systems and networks"), the Concerted Research Action GOA-MEFISTO-666 (Mathematical Engineering for Information and Communication Systems Technology) of the Flemish Government, and Research Project FWO G.0196.02 ("Design of efficient communication techniques for wireless time-dispersive multi-user MIMO systems"). The work of I. Barhumi was supported in part by the Palestinian European Academic Cooperation in Education (PEACE) Programme.

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Digital Object Identifier 10.1109/TWC.2004.840204

[1]Although [4] considers a set of TIV FIR equalizers, each of which reconstructs the transmitted sequence with a different frequency offset, we will only consider one of these TIV FIR equalizers.

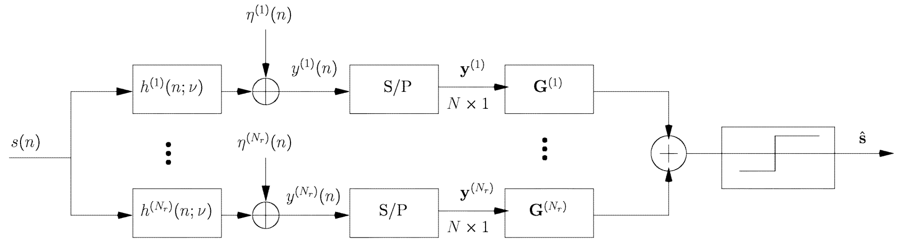

Fig. 1. BEM block diagram.

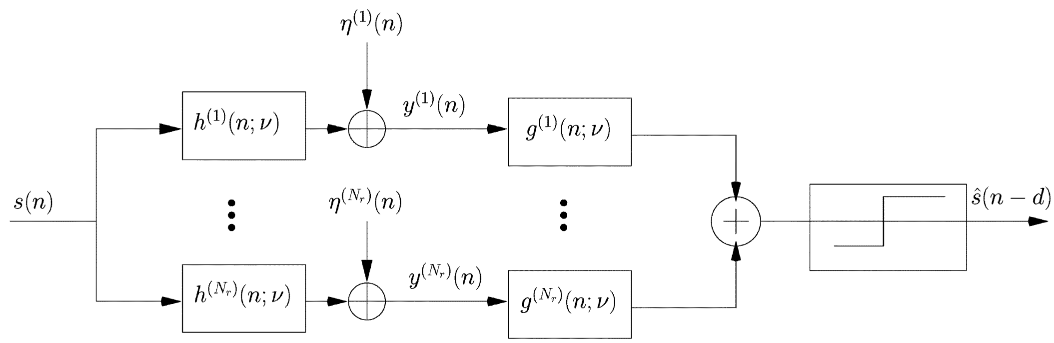

Fig. 2. Block linear equalization.

ZF solution to exist. However, this is generally much lower than the number of receive antennas that TIV FIR equalizers require. Note that TV FIR decision feedback equalizers have been investigated in [5].

Although we could consider a nonprecoded transmission scheme, we consider a cyclic prefix-based transmission scheme for the sake of simplicity. Zero padding-based transmission schemes have been considered in [6] and [7]. Note also that more elaborated precoded transmission schemes have recently been developed for doubly selective channels to improve performance [8] (see also [9] for the multiuser scenario).

This paper is organized as follows. In Section II, we give a brief description of the wireless doubly selective channel model. We then switch to a block data model, which is described in Section III. We review block linear equalization in Section IV. TV FIR equalization (MMSE and ZF) is introduced in Section V. Section VI compares block linear equalization and TV FIR equalization with respect to the existence of the ZF solution and complexity. In Section VII, we show that our framework unifies and extends many previously proposed linear serial equalizers. Application to orthogonal frequency division multiplexing (OFDM) is discussed in Section VIII. In Section IX, we then compare through computer simulations the performance of block linear equalization with TV FIR equalization. Finally, our conclusions are drawn in Section X.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote the 1-D and 2-D Kronecker delta as δ_n and $\delta_{n,m}$, respectively. The Kronecker product is denoted by $\otimes$. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Finally, $\operatorname{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on the diagonal, and $\operatorname{tr}\{\mathbf{X}\}$ denotes the trace of matrix $\mathbf{X}$.

II. Wireless Doubly Selective Channel Model

The system under consideration is depicted on the left side of Figs. 1 and 2. We assume a single-input multiple-output (SIMO) system with N_r receive antennas.[2] Focusing on a baseband-equivalent description, when transmitting a symbol sequence $s(n)$ and sampling each receive antenna at the symbol rate, the received sample sequence at the rth receive antenna can be written as

$$y^{(r)}(n) = \sum_{\nu=-\infty}^{\infty} h^{(r)}(n;\nu)s(n-\nu) + \eta^{(r)}(n)$$

where $\eta^{(r)}(n)$ is the additive noise at the rth receive antenna and $h^{(r)}(n;\nu)$ is the doubly selective (time- and frequency-selective) channel corresponding to the rth receive antenna, which includes the physical channel as well as the transmit and receive filters.

In this paper, we model the doubly selective channel $h^{(r)}(n;\nu)$ using a basis expansion model (BEM). Many BEMs have been introduced in the past [10]–[13], [8], [9]. In this study, we use the BEM of [13], [8], and [9], which has been shown to accurately model realistic channels. In this BEM (which we simply call the BEM from now on), the channel is modeled as a TV FIR filter, where each tap is expressed as a superposition of complex exponential basis functions with frequencies on a DFT grid, as described next.

Let us first make the following assumptions:

A1) The delay spread is bounded by $\tau_{\max}$.

A2) The Doppler spread is bounded by $f_{\max}$.

[2]The extension to multiple-input multiple-output (MIMO) systems is straightforward.

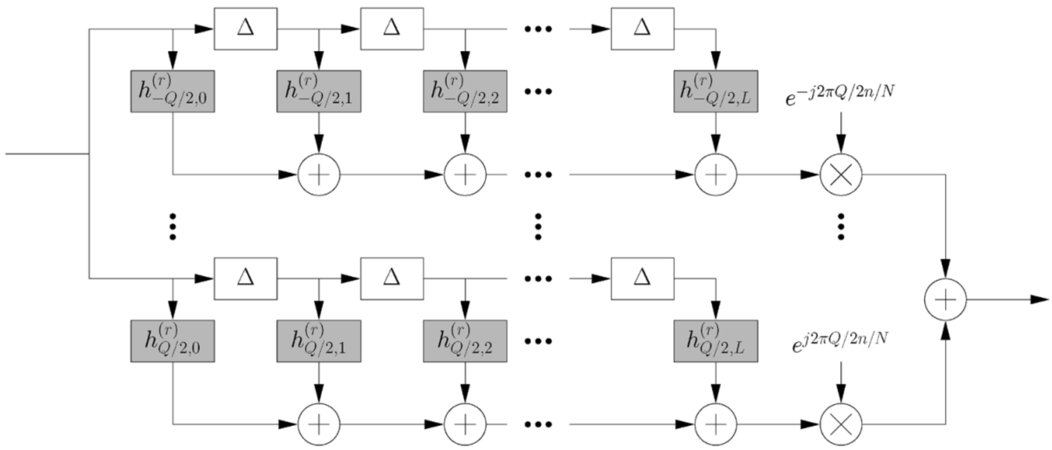

Fig. 3. TV FIR equalization.

Under assumptions A1) and A2), it is possible to accurately model the doubly selective channel $h^{(r)}(n;\nu)$ for $n \in \{0, 1, \ldots, N-1\}$ as (see Fig. 3)

$$h^{(r)}(n;\nu) = \sum_{l=0}^{L} \delta_{\nu-l} \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/N} h_{q,l}^{(r)} \qquad (1)$$

where L and Q satisfy the following conditions:

C1) $LT \geq \tau_{\max}$;

C2) $Q/(NT) \geq 2f_{\max}$;

where T is the symbol period. In this expansion model, L represents the discrete delay spread (expressed in multiples of T, the delay resolution of the model), and $Q/2$ represents the discrete Doppler spread (expressed in multiples of $1/(NT)$, the Doppler resolution of the model). Note that the coefficients $h_{q,l}^{(r)}$ remain invariant for $n \in \{0, 1, \ldots, N-1\}$ and hence are the BEM coefficients of interest.

III. Block Data Model

Using the BEM to model the doubly selective channel, we can write $y^{(r)}(n)$ for $n \in \{0, 1, \ldots, N-1\}$ as

$$y^{(r)}(n) = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/N} h_{q,l}^{(r)} s(n-l) + \eta^{(r)}(n),$$

where, for convenience, we assume that $s(n) = s(N+n)$ for $n \in \{-L, \ldots, -1\}$. Note that this corresponds to the conventional cyclic prefix insertion at the transmitter and removal at the receiver [1]. Defining the $N \times 1$ symbol block as $\mathbf{s} := [s(0), \ldots, s(N-1)]^T$, the $N \times 1$ received sample block at the rth receive antenna $\mathbf{y}^{(r)} := [y^{(r)}(0), \ldots, y^{(r)}(N-1)]^T$ can then be written as

$$\mathbf{y}^{(r)} = \mathbf{H}^{(r)}\mathbf{s} + \boldsymbol{\eta}^{(r)} \qquad (2)$$

where $\boldsymbol{\eta}^{(r)}$ is similarly defined as $\mathbf{y}^{(r)}$, and $\mathbf{H}^{(r)}$ is the $N \times N$ matrix given by

$$\mathbf{H}^{(r)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} \mathbf{D}_q \mathbf{Z}_l \qquad (3)$$

where $\mathbf{D}_q := \operatorname{diag}\{[1, \ldots, e^{j2\pi q(N-1)/N}]^T\}$, and $\mathbf{Z}_l$ is the $N \times N$ circulant matrix with $[\mathbf{Z}_l]_{n,n'} = \delta_{(n-n'-l) \bmod N}$. Substituting (3) into (2), we can write

$$\mathbf{y}^{(r)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} \mathbf{D}_q \mathbf{Z}_l \mathbf{s} + \boldsymbol{\eta}^{(r)}. \qquad (4)$$

Stacking the N_r (number of receive antennas) received sample blocks: $\mathbf{y} := [\mathbf{y}^{(1)T}, \ldots, \mathbf{y}^{(N_r)T}]^T$, we finally obtain

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta}$$

where $\boldsymbol{\eta}$ is similarly defined as $\mathbf{y}$, and $\mathbf{H} := [\mathbf{H}^{(1)T}, \ldots, \mathbf{H}^{(N_r)T}]^T$. In the following, we assume perfect channel knowledge at the receiver. In practice, the BEM coefficients have to be estimated. This can be done blindly [11], [14] or by training [15]. This generally requires the channel to be underspread, i.e., the product of the delay spread $\tau_{\max}$ (or L) and the Doppler spread $f_{\max}$ (or $Q/2$) to be smaller than $1/2$ (or $N/2$) [16], [17]. However, under the assumption of perfect channel knowledge, the channel spread factor does not play any role in our equalizer design.

IV. Block Linear Equalizers

In this section, we review conventional block linear equalizers (BLEs). We focus on the MMSE and ZF BLEs.

The idea is to apply a BLE $\mathbf{G}^{(r)}$ on the rth receive antenna, as depicted in Fig. 2. Hence, we estimate $\mathbf{s}$ as

$$\hat{\mathbf{s}} = \sum_{r=1}^{N_r} \mathbf{G}^{(r)}\mathbf{y}^{(r)} = \left(\sum_{r=1}^{N_r} \mathbf{G}^{(r)}\mathbf{H}^{(r)}\right)\mathbf{s} + \sum_{r=1}^{N_r} \mathbf{G}^{(r)}\boldsymbol{\eta}^{(r)}. \qquad (5)$$

Defining $\mathbf{G} := [\mathbf{G}^{(1)}, \ldots, \mathbf{G}^{(N_r)}]$, we obtain

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{G}\mathbf{H}\mathbf{s} + \mathbf{G}\boldsymbol{\eta}. \qquad (6)$$

We will now design BLEs according to the MMSE and ZF criteria.

A. MMSE BLE

The MMSE BLE is determined as

$$\mathbf{G}_{\mathrm{MMSE}} = \arg\min_{\mathbf{G}} \mathcal{E}\{\|\hat{\mathbf{s}} - \mathbf{s}\|^2\}$$

which leads to [18]

$$\mathbf{G}_{\mathrm{MMSE}} = \mathbf{R}_s\mathbf{H}^H(\mathbf{H}\mathbf{R}_s\mathbf{H}^H + \mathbf{R}_\eta)^{-1} \tag{7a}$$

$$= \left(\mathbf{H}^H\mathbf{R}_\eta^{-1}\mathbf{H} + \mathbf{R}_s^{-1}\right)^{-1}\mathbf{H}^H\mathbf{R}_\eta^{-1} \tag{7b}$$

where $\mathbf{R}_s := \mathcal{E}\{\mathbf{s}\mathbf{s}^H\}$ is the symbol covariance matrix, and $\mathbf{R}_\eta := \mathcal{E}\{\boldsymbol{\eta}\boldsymbol{\eta}^H\}$ is the noise covariance matrix. Note that (7b) is obtained from (7a) by applying the matrix inversion lemma. For white data and noise with variances σ_s^2 and σ_η^2, respectively ($\mathbf{R}_s = \sigma_s^2\mathbf{I}_N, \mathbf{R}_\eta = \sigma_\eta^2\mathbf{I}_N$), the MMSE BLE reduces to

$$\mathbf{G}_{\mathrm{MMSE}} = \left(\mathbf{H}^H\mathbf{H} + \sigma_\eta^2/\sigma_s^2\mathbf{I}_N\right)^{-1}\mathbf{H}^H. \tag{8}$$

B. ZF BLE

From (6), an (unbiased) ZF solution is obtained if

$$\mathbf{G}\mathbf{H} = \mathbf{I}_N. \tag{9}$$

Many ZF solutions exist. A simple ZF solution is obtained as

$$\mathbf{G}_{\mathrm{ZF}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H. \tag{10}$$

However, this does not necessarily lead to the minimum norm ZF solution, which is obtained by minimizing the quadratic cost function $\mathcal{E}\{(\mathbf{G}\mathbf{y})^H(\mathbf{G}\mathbf{y})\}$ subject to (9) or, equivalently, by setting the signal power to infinity in the MMSE solution [see (7b) and (8)]. This leads to [18]

$$\mathbf{G}_{\mathrm{ZF}} = \left(\mathbf{H}^H\mathbf{R}_\eta^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{R}_\eta^{-1}. \tag{11}$$

Note that (11) reduces to (10) in the white noise case.

V. TV FIR EQUALIZATION

In this section, we apply a TV FIR equalizer $g^{(r)}(n;\nu)$ on the rth receive antenna, as depicted in Fig. 3. We estimate $s(n)$ for $n \in \{0, 1, \ldots, N-1\}$ as

$$\hat{s}(n) = \sum_{r=1}^{N_r}\sum_{\nu=-\infty}^{\infty} g^{(r)}(n;\nu)y^{(r)}(n-\nu). \tag{12}$$

Since the doubly selective channel $h^{(r)}(n;\nu)$ was described by the BEM, it is also convenient to design the TV FIR equalizer $g^{(r)}(n;\nu)$ using the BEM. This approach will allow us to turn a complicated equalizer design problem into an equivalent simpler equalizer design problem, containing only the BEM coefficients of both the doubly selective channel and the TV FIR equalizer or, as we explain next, it will allow us to convert a complicated TV 1-D deconvolution problem into an equivalent simpler TIV 2-D deconvolution problem. More specifically, we design each TV FIR equalizer $g^{(r)}(n;\nu)$ for $n \in \{0, 1, \ldots, N-1\}$ to have $L'+1$ taps, where the time variation of each tap is modeled by $Q'+1$ complex exponential basis functions with frequencies on the same DFT grid as for the channel

$$g^{(r)}(n;\nu) = \sum_{l'=-d}^{L'-d}\delta_{\nu-l'}\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/N}g_{q',l'}^{(r)} \tag{13}$$

where d is the delay of the TV FIR equalizer. Note that a similar equalizer structure has been proposed in ([11, Sec. V-B]), but there the authors did not use a model for the time variation of the different equalizer taps. Note that the TV FIR equalizer in (13) has the same structure as the channel in (1). Hence, the TV FIR equalizer is designed as in Fig. 1 with L, Q, and $h_{q,l}^{(r)}$ replaced by L', Q', and $g_{q',l'}^{(r)}$, respectively.

Using the BEM to design the TV FIR equalizer, we estimate $s(n)$ for $n \in \{0, 1, \ldots, N-1\}$ as

$$\hat{s}(n) = \sum_{l'=-d}^{L'-d}\sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/N}g_{q',l'}^{(r)}y^{(r)}(n-l') \tag{14}$$

where, for convenience, we assume that $y^{(r)}(n) = y^{(r)}(n+N)$ for $n \in \{-L'+d, \ldots, d-1\}$. In other words, for each receive antenna, we virtually insert a cyclic prefix at the receiver, in order to obtain a circulant convolution on each branch of the TV FIR equalizer.

Instead of continuing to work on the sample level, it is more convenient to switch to the block level at this point. This will more clearly reveal the structure we impose on the TV FIR equalizer. On the block level, (14) corresponds to estimating the transmitted block $\mathbf{s}$ as in (5), but now with $\mathbf{G}^{(r)}$ constrained to

$$\mathbf{G}^{(r)} = \sum_{l'=-d}^{L'-d}\sum_{q'=-Q'/2}^{Q'/2} g_{q',l'}^{(r)}\mathbf{D}_{q'}\mathbf{Z}_{l'}. \tag{15}$$

An estimate of $\mathbf{s}$ is then obtained as

$$\hat{\mathbf{s}} = \sum_{r=1}^{N_r}\sum_{q'=-Q'/2}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)}\mathbf{D}_{q'}\mathbf{Z}_{l'}\sum_{q=-Q/2}^{Q/2}\sum_{l=0}^{L} h_{q,l}^{(r)}\mathbf{D}_q\mathbf{Z}_l\mathbf{s} + \sum_{r=1}^{N_r}\sum_{q'=-Q'/2}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)}\mathbf{D}_{q'}\mathbf{Z}_{l'}\boldsymbol{\eta}^{(r)}. \tag{16}$$

Defining $p := q+q', k := l+l'$, and using the property $\mathbf{Z}_{l'}\mathbf{D}_q = e^{-j2\pi ql'/N}\mathbf{D}_q\mathbf{Z}_{l'}$, (16) can be rewritten as

$$\hat{\mathbf{s}} = \sum_{p=-(Q+Q')/2}^{(Q+Q')/2}\sum_{k=-d}^{L+L'-d} f_{p,k}\mathbf{D}_p\mathbf{Z}_k\mathbf{s} + \sum_{r=1}^{N_r}\sum_{q'=-Q'/2}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)}\mathbf{D}_{q'}\mathbf{Z}_{l'}\boldsymbol{\eta}^{(r)} \tag{17}$$

where we have introduced the 2-D function

$$f_{p,k} := \sum_{r=1}^{N_r}\sum_{q'=-Q'/2}^{Q'/2}\sum_{l'=-d}^{L'-d} e^{-j2\pi(p-q')l'/N}g_{q',l'}^{(r)}h_{p-q',k-l'}^{(r)}. \tag{18}$$

It will be useful to rewrite (17) in matrix-vector form as

$$\hat{\mathbf{s}} = (\mathbf{f}^T \otimes \mathbf{I}_N)\mathbf{A}\mathbf{s} + \sum_{r=1}^{N_r} \left(\mathbf{g}^{(r)T} \otimes \mathbf{I}_N\right) \mathbf{B}^{(r)} \boldsymbol{\eta}^{(r)} \tag{19}$$

where $\mathbf{f} := [f_{-(Q+Q')/2,-d}, \dots, f_{-(Q+Q')/2,L+L'-d}, \dots, f_{(Q+Q')/2,L+L'-d}]^T$, $\mathbf{g}^{(r)} := [g^{(r)}_{-Q'/2,-d}, \dots, g^{(r)}_{-Q'/2,L'-d}, \dots, g^{(r)}_{Q'/2,L'-d}]^T$ and the matrices $\mathbf{A}$ and $\mathbf{B}^{(r)}$ are defined as

$$\mathbf{A} := \begin{bmatrix} \mathbf{D}_{-(Q+Q')/2}\mathbf{Z}_{-d} \\ \vdots \\ \mathbf{D}_{-(Q+Q')/2}\mathbf{Z}_{L+L'-d} \\ \vdots \\ \mathbf{D}_{(Q+Q')/2}\mathbf{Z}_{L+L'-d} \end{bmatrix}$$

$$\mathbf{B}^{(r)} := \begin{bmatrix} \mathbf{D}_{-Q'/2}\mathbf{Z}_{-d} \\ \vdots \\ \mathbf{D}_{-Q'/2}\mathbf{Z}_{L'-d} \\ \vdots \\ \mathbf{D}_{Q'/2}\mathbf{Z}_{L'-d} \end{bmatrix}.$$

Note that $\mathbf{B}^{(r)}$ does not depend on r. However, if different equalizer orders and delays were selected for the different receive antennas, $\mathbf{B}^{(r)}$ would depend on r. We can further rewrite (19) as

$$\hat{\mathbf{s}} = (\mathbf{f}^T \otimes \mathbf{I}_N)\mathbf{A}\mathbf{s} + (\mathbf{g}^T \otimes \mathbf{I}_N)\mathbf{B}\boldsymbol{\eta} \tag{20}$$

where

$$\mathbf{g} := \left[\mathbf{g}^{(1)T}, \dots, \mathbf{g}^{(N_r)T}\right]^T$$

and the matrix $\mathbf{B}$ is defined as

$$\mathbf{B} := \begin{bmatrix} \mathbf{B}^{(1)} & & \\ & \ddots & \\ & & \mathbf{B}^{(N_r)} \end{bmatrix}.$$

The term in $f_{p,k}$ [see (18)] corresponding to the rth receive antenna is related to a 2-D convolution of the BEM coefficients of the doubly selective channel for the rth receive antenna and the BEM coefficients of the TV FIR equalizer for the rth receive antenna. This allows us to derive a linear relationship between $\mathbf{f}$ and $\mathbf{g}$, as discussed next.

We first define the $(L'+1) \times (L'+L+1)$ block Toeplitz matrix

$$\mathcal{T}_{l,L'+1}\left(h^{(r)}_{q,l}\right) := \begin{bmatrix} h^{(r)}_{q,0} & \cdots & h^{(r)}_{q,L} & & 0 \\ & \ddots & & \ddots & \\ 0 & & h^{(r)}_{q,0} & \cdots & h^{(r)}_{q,L} \end{bmatrix}.$$

We then define $\boldsymbol{\mathcal{H}}^{(r)}_q := \boldsymbol{\Omega}_q \mathcal{T}_{l,L'+1}(h^{(r)}_{q,l})$, where $\boldsymbol{\Omega}_q := \mathrm{diag}\{[e^{-j2\pi q(-d)/N}, \dots, e^{-j2\pi q(-d+L')/N}]^T\}$, and introduce the $(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ block Toeplitz matrix

$$\mathcal{T}_{q,Q'+1}\left(\boldsymbol{\mathcal{H}}^{(r)}_q\right) := \begin{bmatrix} \boldsymbol{\mathcal{H}}^{(r)}_{-Q/2} & \cdots & \boldsymbol{\mathcal{H}}^{(r)}_{Q/2} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \boldsymbol{\mathcal{H}}^{(r)}_{-Q/2} & \cdots & \boldsymbol{\mathcal{H}}^{(r)}_{Q/2} \end{bmatrix}.$$

Introducing the definitions $\boldsymbol{\mathcal{H}}^{(r)} := \mathcal{T}_{q,Q'+1}(\boldsymbol{\mathcal{H}}^{(r)}_q)$ and $\boldsymbol{\mathcal{H}} := [\boldsymbol{\mathcal{H}}^{(1)T}, \dots, \boldsymbol{\mathcal{H}}^{(N_r)T}]^T$, we can then derive from (18) that

$$\mathbf{f}^T = \mathbf{g}^T \boldsymbol{\mathcal{H}}. \tag{21}$$

Substituting (21) into (20) finally leads to

$$\hat{\mathbf{s}} = (\mathbf{g}^T \boldsymbol{\mathcal{H}} \otimes \mathbf{I}_N)\mathbf{A}\mathbf{s} + (\mathbf{g}^T \otimes \mathbf{I}_N)\mathbf{B}\boldsymbol{\eta}. \tag{22}$$

Based on this equation, we will now design TV FIR equalizers according to the MMSE and ZF criteria.

A. MMSE TV FIR Equalizer

The BEM coefficients of the MMSE TV FIR are determined as

$$\mathbf{g}_{\text{MMSE}} = \arg\min_{\mathbf{g}} \mathcal{E}\{\|\hat{\mathbf{s}} - \mathbf{s}\|^2\}. \tag{23}$$

Using (22), the MSE can be written as

$$\begin{aligned} \mathcal{E}\{\|\hat{\mathbf{s}} - \mathbf{s}\|^2\} = {} & \mathrm{tr}\{(\mathbf{g}^T \boldsymbol{\mathcal{H}} \otimes \mathbf{I}_N)\mathbf{A}\mathbf{R}_s\mathbf{A}^H(\boldsymbol{\mathcal{H}}^H\mathbf{g}^* \otimes \mathbf{I}_N)\} \\ & + \mathrm{tr}\{(\mathbf{g}^T \otimes \mathbf{I}_N)\mathbf{B}\mathbf{R}_\eta\mathbf{B}^H(\mathbf{g}^* \otimes \mathbf{I}_N)\} \\ & - 2\Re\{\mathrm{tr}\{(\mathbf{g}^T \boldsymbol{\mathcal{H}} \otimes \mathbf{I}_N)\mathbf{A}\mathbf{R}_s\}\} + \mathrm{tr}\{\mathbf{R}_s\}. \end{aligned} \tag{24}$$

Let us now introduce the following properties:

$$\mathrm{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_N)\mathbf{V}\} = \mathbf{x}^T \mathrm{subtr}\{\mathbf{V}\}$$
$$\mathrm{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_N)\mathbf{X}(\mathbf{x}^* \otimes \mathbf{I}_N)\} = \mathbf{x}^T \mathrm{subtr}\{\mathbf{X}\}\mathbf{x}^*$$

for an arbitrary $k \times 1$ vector $\mathbf{x}$, a $kN \times N$ matrix $\mathbf{V}$, and a $kN \times kN$ matrix $\mathbf{X}$. The $\mathrm{subtr}\{\cdot\}$ operation splits the matrix up into $N \times N$ submatrices and replaces each submatrix by its trace. Let $\mathbf{A}$ be the $pN \times qN$ matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1q} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{p1} & \cdots & \mathbf{A}_{pq} \end{bmatrix}$$

where $\mathbf{A}_{ij}$ is the (i,j)th $N \times N$ submatrix of $\mathbf{A}$. The $p \times q$ matrix $\mathrm{subtr}\{\mathbf{A}\}$ is then given by

$$\mathrm{subtr}\{\mathbf{A}\} = \begin{bmatrix} \mathrm{tr}\{\mathbf{A}_{11}\} & \cdots & \mathrm{tr}\{\mathbf{A}_{1q}\} \\ \vdots & \ddots & \vdots \\ \mathrm{tr}\{\mathbf{A}_{p1}\} & \cdots & \mathrm{tr}\{\mathbf{A}_{pq}\} \end{bmatrix}.$$

Hence, $\text{subtr}\{\cdot\}$ reduces the row and column dimension by a factor N. The MSE can then be written as

$$\mathcal{E}\{\|\hat{\mathbf{s}}-\mathbf{s}\|^2\} = \text{tr}\{\mathbf{R}_s\} + \mathbf{g}^T\boldsymbol{\mathcal{H}}\mathbf{R}_A\boldsymbol{\mathcal{H}}^H\mathbf{g}^* + \mathbf{g}^T\mathbf{R}_B\mathbf{g}^* - 2\Re\{\mathbf{g}^T\boldsymbol{\mathcal{H}}\mathbf{r}_A\} \quad (25)$$

where $\mathbf{r}_A := \text{subtr}\{\mathbf{A}\mathbf{R}_s\}, \mathbf{R}_A := \text{subtr}\{\mathbf{A}\mathbf{R}_s\mathbf{A}^H\}$, and $\mathbf{R}_B := \text{subtr}\{\mathbf{B}\mathbf{R}_\eta\mathbf{B}^H\}$. The BEM coefficients of the MMSE TV FIR equalizer are now obtained by solving $\partial\mathcal{E}\{\|\hat{\mathbf{s}}-\mathbf{s}\|^2\}/\partial\mathbf{g} = \mathbf{0}$, which leads to

$$\begin{aligned}\mathbf{g}_{\text{MMSE}}^T &= \mathbf{r}_A^H\boldsymbol{\mathcal{H}}^H(\boldsymbol{\mathcal{H}}\mathbf{R}_A\boldsymbol{\mathcal{H}}^H + \mathbf{R}_B)^{-1} &(26a)\\ &= \mathbf{r}_A^H\mathbf{R}_A^{-1}\left(\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_A^{-1}\right)^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1} &(26b)\\ &= \mathbf{e}_d^T\left(\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_A^{-1}\right)^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1} &(26c)\end{aligned}$$

where (26b) is again obtained from (26a) by applying the matrix inversion lemma, and (26c) is obtained from (26b) by using the fact that $\mathbf{r}_A^H\mathbf{R}_A^{-1} = \mathbf{e}_d^T$, where $\mathbf{e}_d$ is a $(Q+Q'+1)(L+L'+1)\times 1$ unit vector with the 1 in the $(d(Q+Q'+1)+(Q+Q')/2+1)$st position. Substituting (26c) into (25), we can now write the MSE of the MMSE TV FIR equalizer as

$$\begin{aligned}\text{MSE}_{\text{MMSE}} &= \mathbf{e}_d^T\left(\mathbf{R}_A - \mathbf{R}_A\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}}\left(\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}} + \mathbf{R}_A^{-1}\right)^{-1}\right)\mathbf{e}_d\\ &= \mathbf{e}_d^T\left(\mathbf{R}_A^{-1} + \boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}}\right)^{-1}\mathbf{e}_d. \quad (27)\end{aligned}$$

Note that we have used the fact that $\text{tr}\{\mathbf{R}_s\} = \mathbf{e}_d^T\mathbf{R}_A\mathbf{e}_d$.

For white data and noise with variances σ_s^2 and σ_η^2, respectively ($\mathbf{R}_s = \sigma_s^2\mathbf{I}_N, \mathbf{R}_\eta = \sigma_\eta^2\mathbf{I}_N$), it is easy to show that $\mathbf{R}_A = N\mathbf{I}_{(Q+Q'+1)(L+L'+1)}$ and $\mathbf{R}_B = N\mathbf{I}_{N_r(Q'+1)(L'+1)}$, where we used the fact that $\text{tr}\{\mathbf{Z}_l\mathbf{D}_p\mathbf{Z}_{l'}\} = N\delta_{l-l'}$. Hence, the BEM coefficients of the MMSE TV FIR equalizer reduce to

$$\mathbf{g}_{\text{MMSE}}^T = \mathbf{e}_d^T\left(\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}} + \sigma_\eta^2/\sigma_s^2\mathbf{I}_{(Q+Q'+1)(L+L'+1)}\right)^{-1}\boldsymbol{\mathcal{H}}^H. \quad (28)$$

The MSE of the MMSE TV FIR equalizer then becomes

$$\text{MSE}_{\text{MMSE}} = \mathbf{e}_d^T\left(\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}} + \sigma_\eta^2/\sigma_s^2\mathbf{I}_{(Q+Q'+1)(L+L'+1)}\right)^{-1}\mathbf{e}_d.$$

B. ZF TV FIR Equalizer

From (22), an (unbiased) ZF solution is obtained if

$$(\mathbf{g}^T\boldsymbol{\mathcal{H}}\otimes\mathbf{I}_N)\mathbf{A} = \mathbf{I}_N. \quad (29)$$

As for the BLE, many ZF solutions exist. A simple ZF solution is obtained as

$$\mathbf{g}_{\text{ZF}}^T = \mathbf{e}_d^T(\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}})^{-1}\boldsymbol{\mathcal{H}}^H. \quad (30)$$

However, this does not necessarily lead to the minimum norm ZF solution, which is obtained by minimizing the quadratic cost function $\mathcal{E}\{[(\mathbf{g}^T\otimes\mathbf{I}_N)\mathbf{B}\mathbf{y}]^H(\mathbf{g}^T\otimes\mathbf{I}_N)\mathbf{B}\mathbf{y}\}$ subject to (29) or, equivalently, by setting the signal power equal to infinity in the MMSE solution [see (26c) and (28)]. This leads to

$$\mathbf{g}_{\text{ZF}}^T = \mathbf{e}_d^T\left(\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}}\right)^{-1}\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}. \quad (31)$$

Note that (31) reduces to (30) in the white noise case [in a similar fashion as (26c) reduces to (28)]. By setting the signal power to infinity in (27), we obtain the MSE of the ZF TV FIR equalizer as follows:

$$\text{MSE}_{\text{ZF}} = \mathbf{e}_d^T\left(\boldsymbol{\mathcal{H}}^H\mathbf{R}_B^{-1}\boldsymbol{\mathcal{H}}\right)^{-1}\mathbf{e}_d. \quad (32)$$

VI. Discussion and Comparisons

A. Existence of ZF Solution

The existence of the ZF BLE in (10) or (11) requires that $\mathbf{H}$ has full column rank, which happens with probability one regardless of N_r. On the other hand, the existence of the ZF TV FIR equalizer requires that $\boldsymbol{\mathcal{H}}$ has full column rank, which happens with probability one when $\boldsymbol{\mathcal{H}}$ has at least as many rows as columns, i.e., when $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$, which can always be obtained with a sufficiently large Q' and L' if $N_r \geq 2$. A more detailed existence result follows from a result for the existence of a ZF TIV FIR equalizer of a purely frequency-selective multiple-input multiple-output (MIMO) channel [19], since we can view $\boldsymbol{\mathcal{H}}$ as the channel matrix related to the purely frequency-selective MIMO channel $\boldsymbol{\mathcal{H}}_q := [\boldsymbol{\mathcal{H}}_q^{(1)T},\ldots,\boldsymbol{\mathcal{H}}_q^{(N_r)T}]^T$ of order $Q(q\in\{-Q/2,\ldots,Q/2\})$, which has $(L+L'+1)$ inputs and $N_r(L'+1)$ outputs. The following proposition gives a set of sufficient conditions for $\boldsymbol{\mathcal{H}}$ to be of full column rank [19]:

Proposition 1: The matrix $\boldsymbol{\mathcal{H}}$ has full column rank if:

C1) $Q' \geq Q(L+L'+1)$;

C2) $\boldsymbol{\mathcal{H}}(z) = \sum_{q=-Q/2}^{Q/2}\boldsymbol{\mathcal{H}}_q z^{-q}$ has full column rank $\forall z \notin \{0,\infty\}$;

C3) $\boldsymbol{\mathcal{H}}_{-Q/2}$ and $\boldsymbol{\mathcal{H}}_{Q/2}$ have full column rank.

The second and third conditions, which require $N_r(L'+1) \geq L+L'+1$, are usually known as the irreducible and column-reduced conditions, respectively.

Note that condition C1) puts a rather tough constraint on Q'. However, the conditions are sufficient and not necessary. In any case, as stated before, $\boldsymbol{\mathcal{H}}$ has full column rank with probability one when $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$, which can always be obtained with a sufficiently large Q' and L' if $N_r \geq 2$. This can be easily seen by solving for Q' for a fixed L' as

$$Q' \geq \frac{Q(L+L'+1)}{(N_r-1)(L'+1)-L} - 1, \quad Q' \geq 0. \quad (33)$$

Equation (33) implies the following necessary (not necessarily sufficient) conditions:

1) $N_r \geq 2$;
2) $L'+1 > (L/N_r - 1)$.

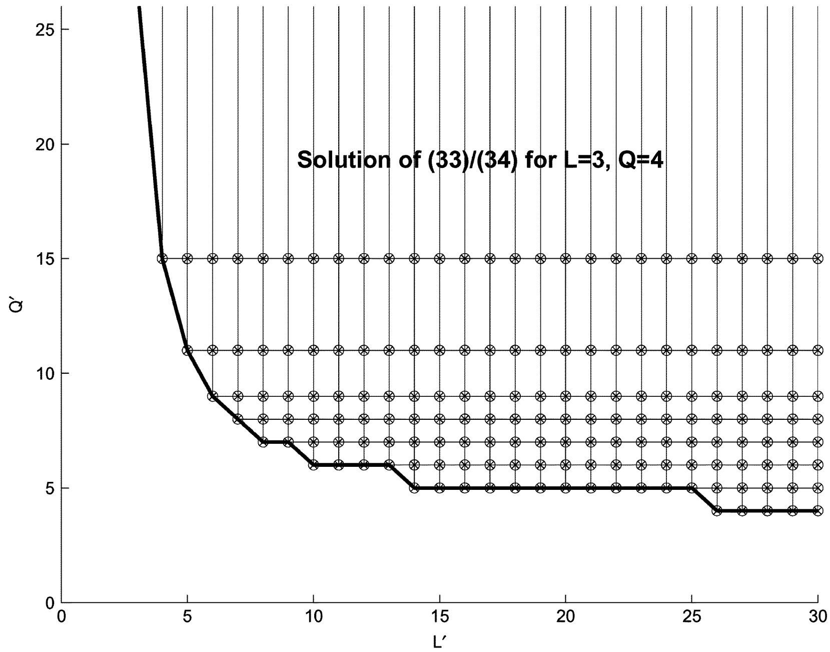

Fig. 4. Solution region for (33) and (34) for $L = 3, Q = 4$, and $N_r = 2$ receive antennas.

In a similar fashion, we can solve for L' for a fixed Q' as

$$L' \geq \frac{L(Q+Q'+1)}{(N_r-1)(Q'+1)-Q} - 1, \quad L' \geq 0. \tag{34}$$

Equation (34) implies the following necessary (not necessarily sufficient) conditions:

1) $N_r \geq 2$
2) $Q' + 1 > (Q/N_r - 1)$

For $N_r = 2$ receive antennas, $L = 3$ and $Q = 4$, the solution region of (33) and (34) is shown in Fig. 4.

Note that existence is not an issue when an MMSE equalizer is considered. However, it is clear that the performance of the MMSE equalizer will improve when it is designed in such a way that the corresponding ZF equalizer exists.

B. Complexity

In this section, we will compare the complexity of the BLE with the complexity of the TV FIR equalizer proposed in this paper. Two types of complexity can be considered: the design complexity and the implementation complexity. The bock size N will play an important role in these complexities. In general, we assume that N is large enough such that blind channel estimation performs well or the overhead of the training symbols for training based channel estimation does not decrease the data rate too much, which basically means that the channel should be far underspread, i.e., $LQ \ll N$.

Design Complexity: The design complexity is the complexity associated with computing the equalizer. To design the BLE, we have to compute the inverse of an $N \times N$ matrix. This requires $\mathcal{O}(N^3)$ flops. On the other hand, to design the TV FIR equalizer, we have to compute the inverse of a $K \times K$ matrix, where $K = (Q + Q' + 1)(L + L' + 1)$. This requires $\mathcal{O}(K^3)$ flops. Assuming that the channel is far underspread and L' and Q' are not much larger than L and Q, we may assume that K is less than the block size N, and thus the design complexity of the TV FIR equalizer is less than the design complexity of the BLE.

Implementation Complexity: The implementation complexity (run-time complexity) is computed as the number of multiply–add (MA) operations required to estimate the transmitted block. For the BLE, estimating the transmitted block requires N^2 MA operations per receive antenna. On the other hand, for the TV FIR equalizer, estimating the transmitted block requires $N(Q'+1)(L'+1)$ MA operations per receive antenna. Again, assuming the channel is far underspread and L' and Q' are not much larger than L and Q, we may assume that $(Q'+1)(L'+1)$ is less than the block size N, and thus the implementation complexity of the TV FIR equalizer is less than the implementation complexity of the BLE.

Note that our simulation results indicate that in order to obtain a performance close to the performance of the BLE, we do not have to take L' and Q' much larger than L and Q (at least what the practical MMSE case is concerned). As indicated above, in this case, the proposed approach leads to a lower design and implementation complexity.

VII. Unifying Framework

In this section, we show that the TV FIR equalizer proposed in this paper unifies and extends many previously proposed serial linear equalization approaches, as illustrated next.

First of all, TIV FIR equalization of a purely frequency-selective channel ($Q = 0$ and $Q' = 0$) and TV one-tap FIR equalization of a purely time-selective channel ($L = 0$ and $L' = 0$) can be viewed as special cases of our approach. In the first case, the existence of the ZF solution requires $L' \geq \lceil (L/Nr - 1) - 1 \rceil$,

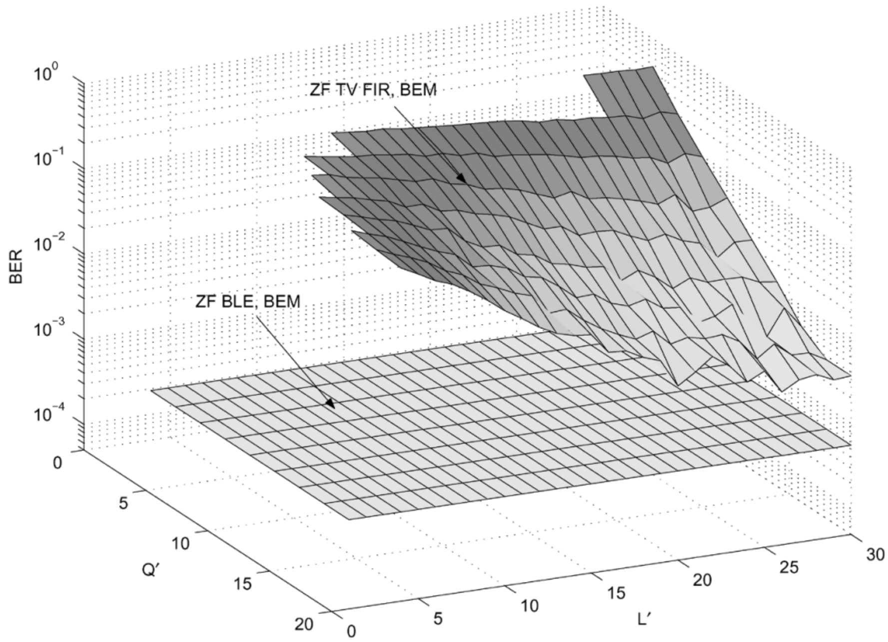

Fig. 5. BER as a function of (Q', L') for $N_r = 2$ receive antennas using a ZF TV FIR equalizer at an SNR of 16 dB.

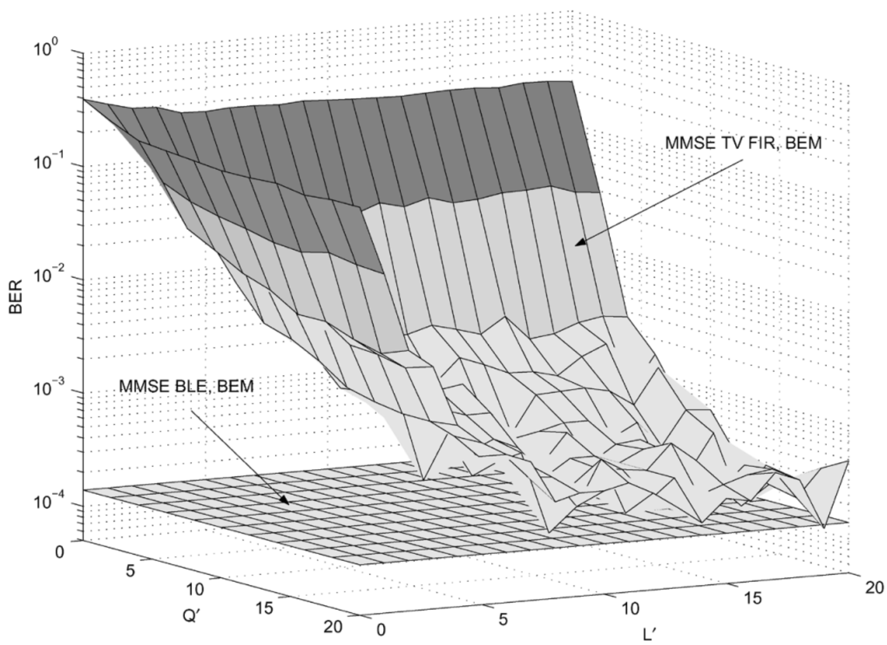

Fig. 6. BER as a function of (Q', L') for $N_r = 2$ receive antennas using an MMSE TV FIR equalizer at an SNR of 16 dB.

which coincides with the result obtained in [3]. On the other hand, in the second case, the existence of a ZF solution requires $Q' \geq \lceil (Q/Nr - 1) - 1 \rceil$, which is a novel observation.

It is also worth noting that our framework encompasses equalization of doubly selective channels using a TIV FIR equalizer or a TV one-tap FIR equalizer, provided enough receive antennas are available. For $N_r > Q + 1$, it is possible to perfectly equalize a doubly selective channel with a TIV FIR equalizer $(Q' = 0)$ if we choose $L' \geq \lceil ((Q+1)L/N_r - (Q+1)) - 1 \rceil$. This result coincides with the result obtained in [4]. On the other hand, for $N_r > L + 1$, it is possible to perfectly equalize a doubly selective channel with a TV one-tap FIR equalizer $(L' = 0)$ if we choose $Q' \geq \lceil ((L+1)Q/N_r - (L+1)) - 1 \rceil$, which is again a novel observation.

TABLE I
TV FIR EQUALIZER PARAMETERS

	ZF		MMSE	
Nr	Q'	L'	Q'	L'
1	NA	NA	20	20
2	20	20	12	12
4	8	8	8	8
6	4	4		

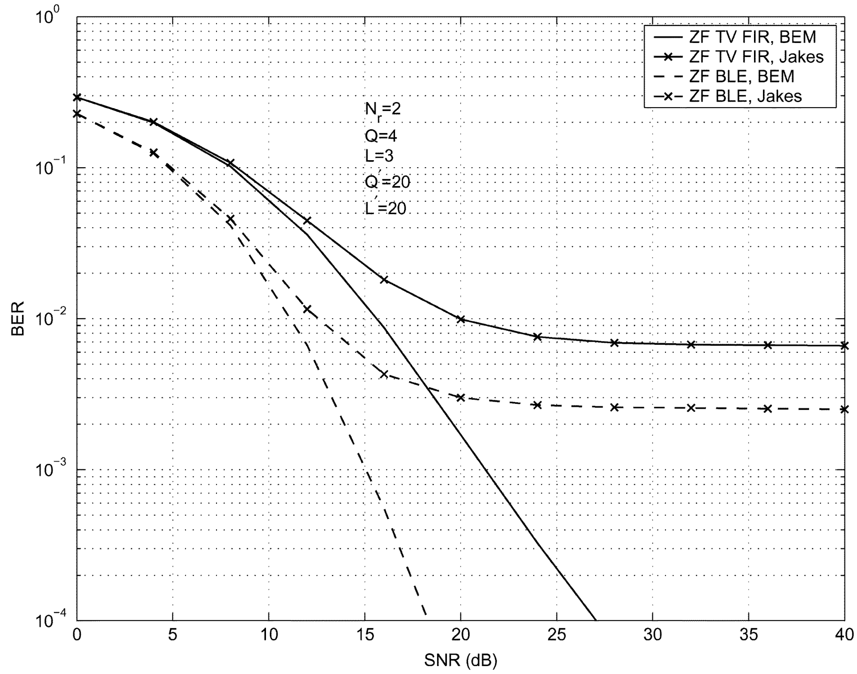

Fig. 7. BER versus SNR for ZF receivers ($N_r = 2$ receive antennas).

VIII. Application to OFDM

The approach presented in this paper is based on a cyclic extension to handle the edge effects, which makes it also applicable to OFDM systems, where the use of a cyclic extension is common practice.

Consider an OFDM system with N carriers. In such a system, the $N \times 1$ symbol block $\mathbf{s}$ is actually obtained by applying an N-point IDFT to an $N \times 1$ symbol block $\tilde{\mathbf{s}}$: $\mathbf{s} := \mathbf{F}^H \tilde{\mathbf{s}}$, where $\mathbf{F}$ is the $N \times N$ unitary DFT matrix. Suppose we have designed a TV FIR equalizer as before. An estimate of $\mathbf{s}$ is then obtained as

$$\hat{\mathbf{s}} = \sum_{l'=-d}^{L'-d} \sum_{q'=-Q'/2}^{Q'/2} g_{q,l}^{(r)} \mathbf{D}_{q'} \mathbf{Z}_{l'} \mathbf{y}^{(r)}.$$

An estimate of $\tilde{\mathbf{s}}$ is then obtained by applying an N-point DFT to $\hat{\mathbf{s}}$: $\hat{\tilde{\mathbf{s}}} := \mathbf{F}\hat{\mathbf{s}}$, which can be written as

$$\hat{\tilde{\mathbf{s}}} = \sum_{r=1}^{N_r} \sum_{l'=-d}^{L'-d} \sum_{q'=-Q'/2}^{Q'/2} g_{q,l}^{(r)} \mathbf{F} \mathbf{D}_{q'} \mathbf{Z}_{l'} \mathbf{F}^H \tilde{\mathbf{y}}^{(r)} \qquad (35)$$

where $\tilde{\mathbf{y}}^{(r)}$ is obtained by applying an N-point DFT to $\mathbf{y}^{(r)}$: $\tilde{\mathbf{y}}^{(r)} := \mathbf{F}\mathbf{y}^{(r)}$. Using the properties $\mathbf{F}\mathbf{Z}_{l'}\mathbf{F}^H = \mathbf{D}_{-l'}, \mathbf{F}\mathbf{D}_{q'}\mathbf{F}^H = \mathbf{Z}_{q'}$, and $\mathbf{Z}_{q'}\mathbf{D}_{-l'} = e^{j2\pi q' l'/N}\mathbf{D}_{-l'}\mathbf{Z}_{q'}$, (35) can then be rewritten as

$$\hat{\tilde{\mathbf{s}}} = \sum_{r=1}^{N_r} \sum_{l'=-d}^{L'-d} \sum_{q'=-Q'/2}^{Q'/2} g_{q',l'}^{(r)} e^{j2\pi q' l'/N} \mathbf{D}_{-l'} \mathbf{Z}_{q'} \tilde{\mathbf{y}}^{(r)}.$$

Hence, we can shift each TV FIR equalizer with $L' + 1$ taps, where the time variation of each tap is modeled by $Q' + 1$ complex exponential basis functions with frequencies on the N-point DFT grid, from the time domain to the frequency domain, where it can be interpreted as a frequency-varying (FV) finite exponential response (FER) equalizer with $Q' + 1$ taps, where the frequency variation of each tap is modeled by $L' + 1$ complex exponential basis functions with frequencies on the N-point DFT grid. Note that a similar equalizer structure has been proposed in [20] and [21], but there the authors did not use a model for the frequency variation of the different equalizer taps.

IX. Simulations

In our simulations, we investigate the performance of both ZF TV FIR equalizers and MMSE TV FIR equalizers. For the case of ZF TV FIR equalization, we consider a SIMO system with $N_r = 2$, 3, 4, and 6 receive antennas, while for the case of MMSE TV FIR equalization we consider a SISO system as well as a SIMO system with $N_r = 2$ and 4 receive antennas. Other parameters of the system are listed below:

- Doppler spread $f_{\max} = 100$ Hz;
- delay spread $\tau_{\max} = 75\ \mu$s;
- block size $N = 800$;
- symbol/sample period $T = 25\ \mu$s;
- discrete Doppler spread $Q/2 = \lceil f_{\max} NT \rceil = 2$;
- discrete delay spread $L = \lceil \tau_{\max}/T \rceil = 3$.

Note that the maximum Doppler spread of 100 Hz corresponds to a vehicle speed of 120 km/h and a carrier frequency of 900 MHz.

The channel taps are first simulated as i.i.d. random variables, correlated in time with a correlation function according to Jakes' model $r_{hh}(\tau) = J_0(2\pi f_{\max}\tau)$, where J_0 is the zeroth-order Bessel function of the first kind. The doubly selective channel is then approximated by the BEM. We only assume the knowledge of the BEM coefficients of the channel at the receiver and not the knowledge of the true Jakes' channel, which

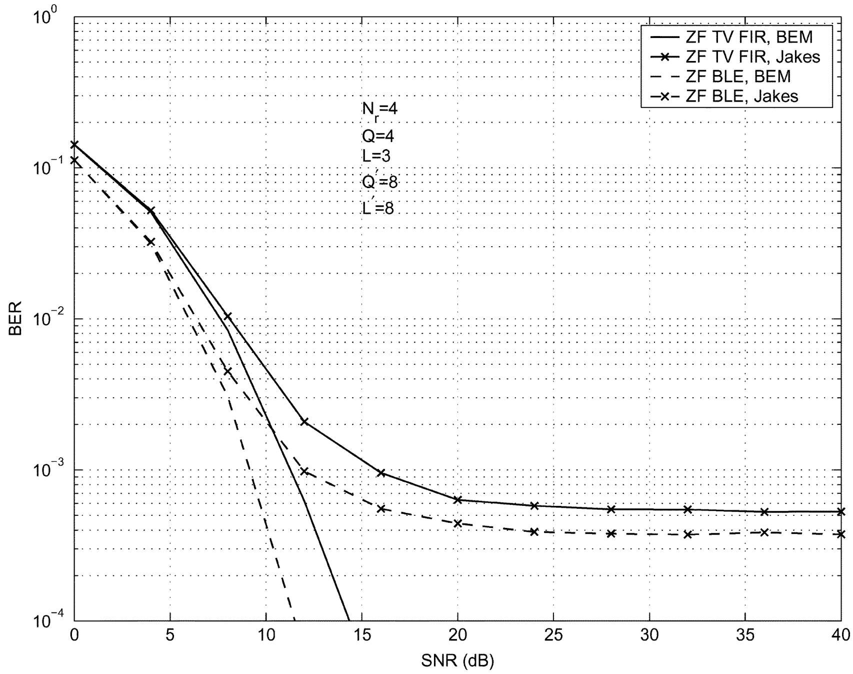

Fig. 8. BER versus SNR for ZF receivers ($N_r = 4$ receive antennas).

can never be obtained in practice. We use these BEM coefficients to design the BLE as well as the TV FIR equalizer. For the lth tap of the rth BEM channel, the BEM coefficient vector $\mathbf{h}_l^{(r)} = [h_{-Q/2,l}^{(r)}, \ldots, h_{Q/2,l}^{(r)}]^T$ is obtained by

$$\mathbf{h}_l^{(r)} = \boldsymbol{\mathcal{L}}^\dagger \mathbf{h}_{\text{Jakes}}^{(r)}(l)$$

where $\mathbf{h}_{\text{Jakes}}^{(r)}(l) = [h_{\text{Jakes}}(0;l), \ldots, h_{\text{Jakes}}(N-1;l)]^T$ is the lth tap of the rth time-varying channel modeled by Jakes' model over N symbol periods, and $\boldsymbol{\mathcal{L}}$ is an $N \times (Q+1)$ matrix with the $(q+Q/2+1)$th column given by $[1, e^{j2\pi q/N}, \ldots, e^{j2\pi q(N-1)/N}]^T$.

In order to illustrate the influence of the BEM modeling error on performance, we use both Jakes' model and the approximated BEM to simulate propagation. In all simulations, QPSK signaling is used and the synchronization delay is chosen as $d = \lfloor L + L'/2 \rfloor + 1$. The performance is measured in terms of bit error rate (BER) versus signal-to-noise ratio (SNR). We compare the performance of the TV FIR equalizer (ZF and MMSE) with the performance of the BLE (ZF and MMSE) for both Jakes' model and the BEM.

1) Design Parameters: To get a feeling on how to choose the TV FIR equalizer parameters, we measure the performance of the TV FIR equalizer for $N_r = 2$ receive antennas as a function of the TV FIR equalizer design parameters Q' and L' at a fixed SNR of 16 dB. We consider both the ZF and the MMSE cost function and assume the BEM to simulate propagation. As shown in Figs. 5 and 6, increasing the equalizer parameters, the performance of the ZF and MMSE TV FIR equalizers comes closer to the performance of the ZF and MMSE BLEs. One can also observe that a significant gain can be obtained by increasing the TV FIR equalizer parameters up to a certain threshold value at which point the gradient of the BER surface becomes very small. For the ZF case, this point might be too far away to be practical (it gets too complex), and hence we resort to $Q' = L' = 20$ in the simulations for $N_r = 2$. For the MMSE case, on the other hand, this point is situated around $Q' = L' = 12$, which is what we will use in the simulations for $N_r = 2$. Note that, for this channel setup, which is almost symmetric in Q and L, we also observe a performance symmetry in Q' and L', which means that there is no reason to choose Q' different from L'. This motivates us to take $Q' = L'$ in the sequel. The TV FIR equalizer parameters used in the simulations are listed in Table I.

2) ZF Receivers: As shown in Fig. 7, using $N_r = 2$ receive antennas, the ZF TV FIR equalizer with $Q' = 20$ and $L' = 20$ is outperformed by the ZF BLE. At a BER of 10^{-2}, we notice a 4-dB SNR loss using the BEM for propagation and a 7-dB SNR loss using the true Jakes' channel for propagation. However, using $N_r = 4$ receive antennas, the performance of the ZF TV FIR equalizer with $Q' = 8$ and $L' = 8$ comes much closer to the performance of the ZF BLE. At a BER of 10^{-2}, the gap is now reduced to 2 dB for both the BEM and the true Jakes' channel, as shown in Fig. 8. In Fig. 9, we use $N_r = 6$ receive antennas, which also allows us to equalize the channel using a TIV FIR equalizer (see also [4]). We can see that the ZF TV FIR equalizer with $Q' = 4$ and $L' = 4$ significantly outperforms the ZF TIV FIR equalizer with $Q' = 0$ and $L' = 20$. Note that, for this case, the design complexity of the ZF TV FIR equalizer is one fifth of the design complexity of the ZF TIV FIR equalizer. On the other hand, the implementation complexity of the ZF TV FIR equalizer is less than a quarter of the implementation complexity of the ZF TIV FIR equalizer. This leads us to conclude that incorporating time variation in the equalizer really pays off. Note that, due to the fact that we use the BEM channel to design the BLE and not the true Jakes' channel, the BLE performance for the true Jakes' channel suffers from an error floor, because

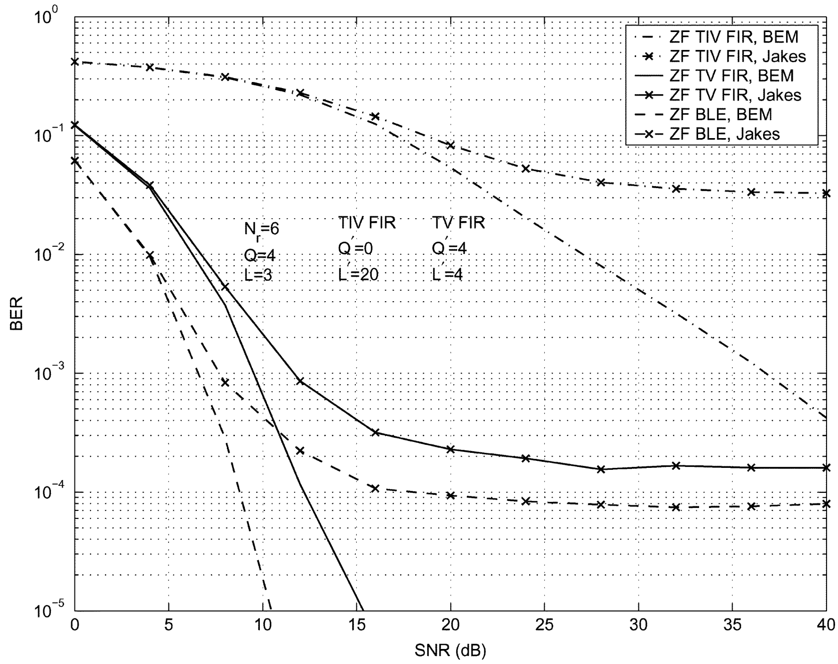

Fig. 9. BER versus SNR for ZF receivers ($N_r = 6$ receive antennas).

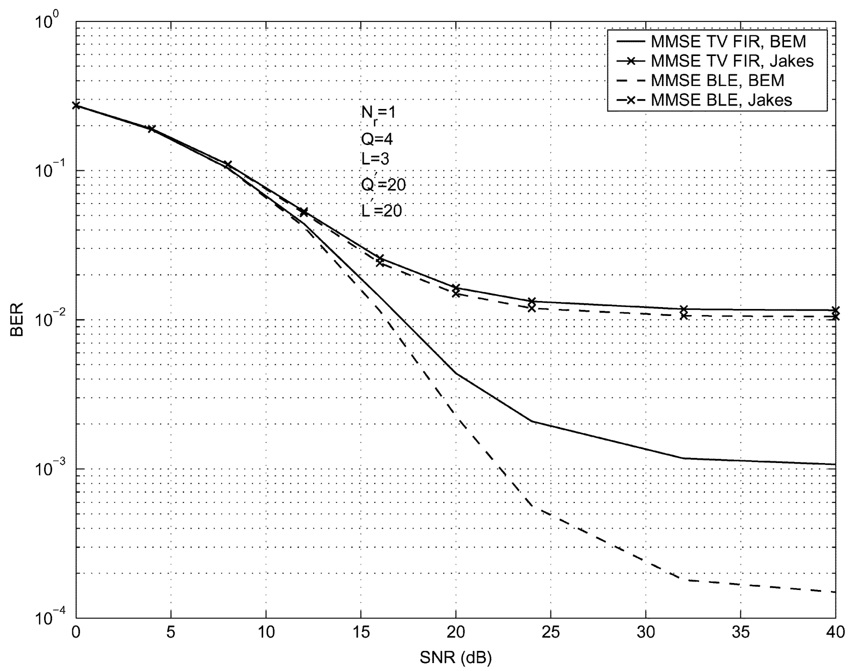

Fig. 10. BER versus SNR for MMSE receivers ($N_r = 1$ receive antenna).

of the mismatch between the BEM channel and the true Jakes' channel.

3) MMSE Receivers: In Fig. 10, we use $N_r = 1$ receive antenna and apply the MMSE TV FIR equalizer with $Q' = 20$ and $L' = 20$. We can see that the performance loss of the MMSE TV FIR equalizer compared to the MMSE BLE is less than 1 dB at a BER of 10^{-2} using the BEM for propagation. Note that, using the true Jakes' channel, both equalization approaches saturate at a BER above 10^{-2}. For $N_r = 2$ receive antennas, where $Q' = 12$ and $L' = 12$ is used, and $N_r = 4$ receive antennas, where $Q' = 8$ and $L' = 8$ is used, we observe that the performance of the MMSE TV FIR equalizer almost coincides with that of the MMSE BLE (for the BEM as well as the true Jakes' channel). These cases are shown in Figs. 11 and 12, respectively.

4) Complexity: To compute the BLE, we require $\mathcal{O}(N^3)$ flops, where $N^3 = 512\ 000\ 000$ for this channel setup. The implementation complexity associated with the BLE requires N^2 MA operations per receive antenna, where $N^2 = 640\ 000$. Note that this complexity analysis does not change with the number of receive antennas. On the other hand, the complexity associated with computing and implementing the TV FIR equalizer does change with the number of receive antennas, because we consider different values for Q' and L'. These complexities are shown in Table II for both the ZF and MMSE criterion.

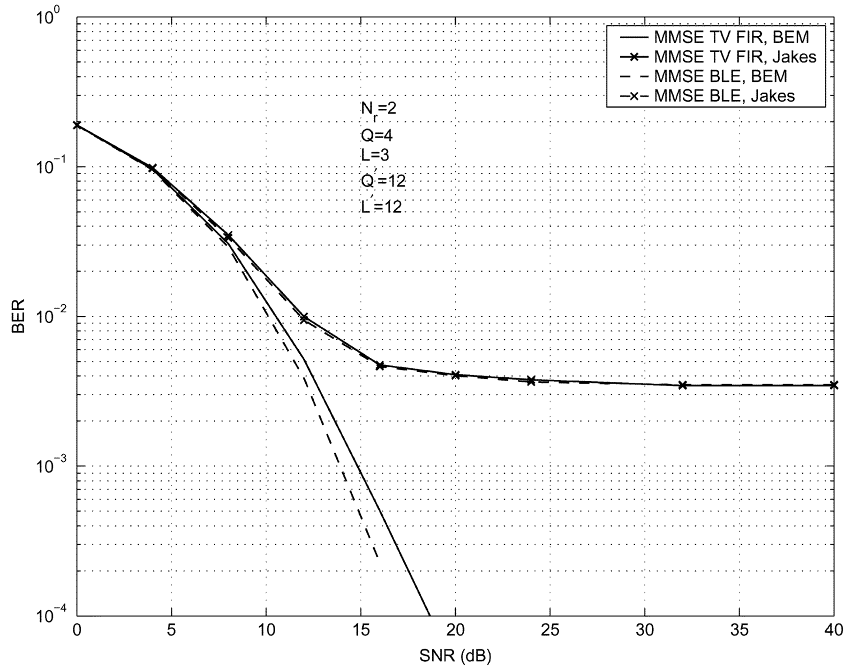

Fig. 11. BER versus SNR for MMSE receivers ($N_r = 2$ receive antennas).

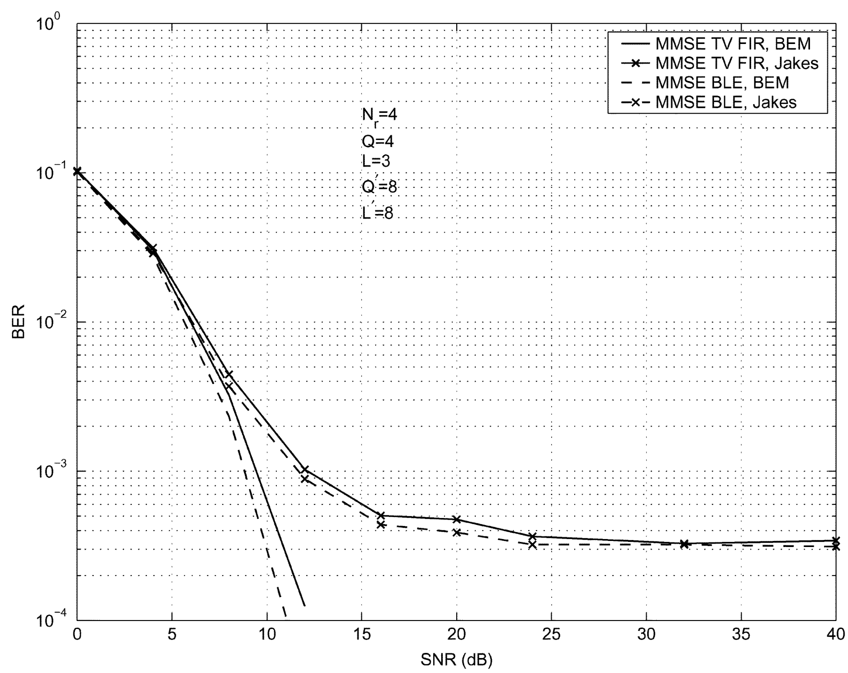

Fig. 12. BER versus SNR for MMSE receivers ($N_r = 4$ receive antennas).

TABLE II
TV FIR EQUALIZER COMPLEXITY TABLE

	ZF TV FIR				MMSE TV FIR			
N_r	Design*	% red	Run-Time†	% red	Design*	% red	Run-Time†	% red
1	NA	NA	NA	NA	216,000,000	57.81	352,800	44.87
2	216,000,000	57.81	352,800	44.87	20,123,648	96.07	135,200	78.88
4	3796416	99.26	64,800	89.87	3,796,416	99.26	64,800	89.87

* in flops (order of magnitude)

† in MA operations

X. CONCLUSION

In this paper, we have proposed a TV FIR equalizer to compensate for doubly selective channels. In our equalizer design, we consider the MMSE as well as the ZF solutions. We have used the BEM to approximate the doubly selective channel and to design the TV FIR equalizer. This allows us to turn a complicated TV 1-D deconvolution problem into an equivalent simpler TIV 2-D deconvolution problem, containing only the BEM coefficients of both the doubly selective channel and the TV FIR equalizer. It is shown that a TV FIR equalizer generally requires at least two receive antennas for the ZF solution to exist. In contrast to the BLE, the proposed TV FIR equalizer allows for a flexible tradeoff between complexity and performance. Moreover, through computer simulations we have shown that the performance of the proposed MMSE TV FIR equalizer comes close to the performance of the MMSE BLE, at a point where the design as well as the implementation complexity are much lower.

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