Per-Tone Equalization for OFDM over Doubly-Selective Channels*

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***Abstract*— In this paper, we propose a per-tone frequency-domain equalization approach for OFDM over doubly-selective channels. We consider the most general case, where the doubly-selective channel delay spread is larger than the cyclic prefix (CP), which results into inter-block interference (IBI). IBI in conjunction with the Doppler effect destroys the orthogonality between subcarriers and hence, results into severe intercarrier interference (ICI). In this paper, we propose a novel per-tone frequency-domain equalizer (PTFEQ) that is obtained through transferring a time-varying time-domain equalizer (TV-TEQ) to the frequency-domain. The purpose of the TV-TEQ is to restore orthogonality between subcarriers and eliminate ICI. We use the mean-square error criterion to design the PTFEQ. An efficient implementation of the proposed PTFEQ is also discussed. Finally, we show some simulation results of the proposed equalization technique.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has attracted a lot of attention, as it provides a simple means to deal with frequency-selective channels. However, if the channel varies over an OFDM block, this destroys the orthogonality between the subcarriers, i.e. results into intercarrier interference (ICI). In addition, inter-block interference (IBI) arises when the channel delay spread is larger than the cyclic prefix (CP). IBI in conjunction with the Doppler effect results into severe ICI. In this paper, we focus on a per-tone frequency-domain IBI/ICI mitigation technique. We first apply a time-varying (TV) finite impulse response (FIR) time-domain equalizer (TEQ). The purpose of the TEQ is to convert the doubly-selective channel into a purely frequency-selective channel whose delay spread fits within the CP. Our per-tone frequency-domain equalizer (PTFEQ) is then obtained by transferring the TEQ operation to the frequency-domain.

Different approaches for reducing ICI in OFDM over doubly-selective channels have been proposed, including frequency-domain equalization and/or time-domain windowing. In [1], [2] the authors propose matched-filter, least-squares (LS) and minimum mean-square error (MMSE) receivers incorporating all subcarriers. Receivers considering the dominant adjacent subcarriers have been presented in [3]. For multiple-input multiple-output (MIMO) OFDM over doubly-selective channels, a frequency-domain ICI mitigation technique is proposed in [4]. A time-domain windowing (linear pre-processing) approach to restrict ICI support in conjunction with iterative MMSE estimation is presented in [5]. ICI self-cancellation schemes are proposed in [6], [7]. There, redundancy is added to enable self-cancellation, which implies a substantial reduction in bandwidth efficiency. To avoid this rate loss, partial response encoding in conjunction with maximum-likelihood sequence detection to mitigate ICI in OFDM systems is studied in [8]. All of the above mentioned works, assume the channel delay spread fits within the CP, and hence, no IBI is present.

Previously, a time-invariant (TIV) FIR TEQ [9] is used to shorten a purely frequency-selective channel when its delay spread is larger than the CP. A per-tone equalizer is then obtained by transferring the TEQ operation to the frequency-domain [10]. In this paper, we assume the doubly-selective channel to have a delay spread larger than the CP. A TV FIR TEQ is applied to convert the doubly-selective channel into a purely frequency-selective channel whose delay spread fits within the CP. The proposed PTFEQ is then obtained by transferring the TV FIR TEQ operation to the frequency-domain.

This paper is organized as follows. In Section II, we present the system model. In Section III, we discuss the basis expansion model (BEM) channel. The proposed PTFEQ is presented in Section IV. An efficient implementation of the proposed PTFEQ is discussed in Section V. In Section VI, we show through computer simulations the performance of the proposed equalizer. Finally, our conclusions are drawn in Section VII.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote the Kronecker delta as $\delta[n]$ and $\mathcal{E}\{\cdot\}$ denotes expectation. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M \times N}$. Finally, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with $\mathbf{x}$ on the diagonal, and $\mathrm{diag}\{\mathbf{A}_0, \ldots, \mathbf{A}_{M-1}\}$ denotes the

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block diagonal matrix with the submatrices $\mathbf{A}_0, \ldots, \mathbf{A}_{M-1}$ are on the diagonal.

II. System Model

We assume a single-input multiple-output (SIMO) OFDM system with N_r receive antennas. At the transmitter, the conventional OFDM modulation is applied, i.e., the incoming bit sequence is parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed into a time-domain block using an N-point IFFT. A cyclic prefix (CP) of length ν is inserted at the head of each block. The time-domain blocks are then serially transmitted over a multipath fading channel. The channel is assumed to be linear time-varying (LTV). Focusing only on the baseband-equivalent description, the received signal at the rth receive antenna, $y^{(r)}(t)$, is given by:

$$y^{(r)}(t) = \sum_{n=-\infty}^{\infty} x[n] g^{(r)}(t; t-nT) + \eta^{(r)}(t),$$

where $g^{(r)}(t;\tau)$ is the baseband-equivalent of the doubly-selective channel from the transmitter to the rth receive antenna, which constitutes the physical channel as well as the transmit and receive filters, $\eta^{(r)}(t)$ is the baseband-equivalent filtered additive noise at the rth receive antenna, and $x[n]$ is the discrete time-domain sequence transmitted at rate $1/T$, the symbol rate. Suppose $S_k[i]$ is the QAM symbol transmitted on the kth subcarrier of the ith OFDM block ($k \in \{0, \ldots, N-1\}$, with N the total number of subcarriers in the OFDM block). Then $x[n]$ can be written as:

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k[i] e^{j2\pi(m-\nu)k/N},$$

where $i = \lfloor n/(N+\nu) \rfloor$ and $m = n - i(N+\nu)$. Note that this description includes the transmission of a CP of length ν.

Sampling each receive antenna at the symbol rate $1/T$, the received sample sequence at the rth receive antenna, $y^{(r)}[n] = y^{(r)}(nT)$, can be written as:

$$y^{(r)}[n] = \sum_{\theta=-\infty}^{\infty} g^{(r)}[n;\theta] x[n-\theta] + \eta^{(r)}[n], \qquad (1)$$

where $\eta^{(r)}[n] = \eta^{(r)}(nT)$ and $g^{(r)}[n;\theta] = g^{(r)}(nT;\theta T)$.

III. BEM Channel

In this paper, we use the basis expansion model (BEM) [11], [12] to approximate the discrete-time baseband-equivalent doubly-selective channel. In this BEM, the doubly-selective channel $g^{(r)}[n;\nu]$ is modeled as an FIR filter where the taps are expressed as a superposition of complex exponential basis functions with frequencies on a discrete grid. To detect the ith OFDM block, we model each channel $g^{(r)}[n;\theta]$ for $n \in \{i(N+\nu)+\nu+d-L', \ldots, (i+1)(N+\nu)+d-1\}$ as:

$$h^{(r)}[n;\theta] = \sum_{l=0}^{L} \delta[\theta - l] \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)}[i] e^{j2\pi qn/K}, \qquad (2)$$

where L should be selected such that $LT \geq \tau_{\max}$, where $\tau_{\max}$ is the maximum delay spread of all channels, and Q and K should be selected such that $Q/(KT) \geq 2f_{\max}$, with $f_{\max}$ the maximum Doppler spread of all channels. Note that the parameters d and L' represent the delay and the order of the equalizer that we will apply to this channel later on.

In this expansion model, L represents the delay-spread (expressed in multiples of T, the delay resolution of the model), and $Q/2$ represents the Doppler-spread (expressed in multiples of $1/(KT)$, the Doppler resolution of the model). Note that the coefficients $h_{q,l}^{(r)}[i]$ remain invariant over a period of length $(N+L')T$, and may change from block to block.

Substituting (2) in (1), the received sample sequence at the rth receive antenna for $n \in \{i(N+\nu)+\nu+d-L', \ldots, (i+1)(N+\nu)+d-1\}$, can be written as:

$$y^{(r)}[n] = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/K} h_{q,l}^{(r)}[i] x[n-l] + \eta^{(r)}[n]. \qquad (3)$$

IV. Equalization of OFDM

In this section, we propose a novel per-tone frequency-domain equalizer (PTFEQ). We assume the most general case, where the TV channel order is larger than the CP ($L > \nu$). We obtain the proposed PTFEQ through transferring a time-domain equalizer (TEQ) to the frequency-domain. The purpose of this TEQ filter is to shorten the channel in the delay spread dimension as well as in the Doppler spread dimension. We assume a TV FIR TEQ, i.e., we apply at the rth receive antenna the TV FIR TEQ $w^{(r)}[n;\theta]$ to convert the doubly-selective channel of order $L > \nu$ and $f_{\max} \neq 0$ into a target impulse response (TIR) that is purely frequency-selective with order $L \leq \nu$ and $f_{\max} = 0$. Hence, subject to some decision delay d, the output of the TV FIR TEQ at the rth receive antenna after removing the CP, can be written as:

$$z^{(r)}[n-d] = \sum_{\theta=-\infty}^{\infty} w^{(r)}[n;\theta] y^{(r)}[n-\theta], \qquad (4)$$

for $n \in \{i(N+\nu)+\nu+d, \ldots, (i+1)(N+\nu)+d-1\}$.

Since we approximate the doubly-selective channel using the BEM, it is convenient also to model the TV FIR TEQ using the BEM. In other words, we design the TV FIR TEQ $w^{(r)}[n;\theta]$ to have $L'+1$ taps, where the time variation of each tap is modeled by $Q'+1$ time-varying complex exponentials basis functions. Hence, we can write the TV FIR TEQ $w^{(r)}[n;\theta]$ for $n \in \{i(N+\nu)+\nu+d, \ldots, (i+1)(N+\nu)+d-1\}$ as:

$$w^{(r)}[n;\theta] = \sum_{l'=0}^{L'} \delta[\theta - l'] \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)}[i] e^{j2\pi q'n/K}, \qquad (5)$$

Define $\hat{S}_k[i]$ as the estimate of the transmitted QAM symbol on the kth subcarrier in the ith OFDM block. This estimate is obtained by applying a 1-tap FEQ on the TEQ output after the FFT-demodulation:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \mathcal{F}^{(k)} \mathbf{D}_{q'}[i] \mathbf{Y}^{(r)}[i] \mathbf{w}_{q'}^{(r)}[i]/d_k[i] \tag{6a}$$

$$= \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \mathcal{F}^{(k)} \underbrace{\mathbf{D}_{q'}[i]\mathbf{Y}^{(r)}[i]\hat{\mathbf{D}}_{q'}^*}_{\tilde{\mathbf{Y}}_{q'}^{(r)}[i]} \underbrace{\hat{\mathbf{D}}_{q'}\mathbf{w}_{q'}^{(r)}[i]/d_k[i]}_{\tilde{\mathbf{w}}_{q'}^{(r,k)}[i]} \tag{6b}$$

where $\mathcal{F}^{(k)}$ is the $(k+1)$st row of the FFT matrix $\boldsymbol{\mathcal{F}}$, $\mathbf{D}_{q'}[i] = \text{diag}\{[e^{j2\pi q'(i(N+\nu)+\nu+d)}, \ldots, e^{j2\pi q'((i+1)(N+\nu)+d-1)/K}]^T\}$, $\mathbf{Y}^{(r)}[i]$ is an $N \times (L'+1)$ Toeplitz matrix, with the first column $[y^{(r)}[i(N+\nu)+\nu+d], \ldots, y^{(r)}[(i+1)(N+\nu)+d-1]]^T$ and first row $[y^{(r)}[i(N+\nu)+\nu+d], \ldots, y^{(r)}[i(N+\nu)+\nu+d-L']]$, $\hat{\mathbf{D}}_{q'} = \text{diag}\{[1, \ldots, e^{j2\pi q' L'/K}]^T\}$, and $d_k[i]$ is the frequency response of the TIR on the kth subcarrier in the ith OFDM block ($1/d_k[i]$ represents the 1-tap FEQ). Note that the right multiplication of $\mathbf{Y}^{(r)}[i]$ with the diagonal matrix $\hat{\mathbf{D}}_{q'}$ in (6b) is done here to restore the Toeplitz structure in $\mathbf{Y}_{q'}^{(r)}[i] = \mathbf{D}_{q'}\mathbf{Y}^{(r)}[i]$, which will simplify the analysis and implementation as will be clear later. From (6b), we can see that each subcarrier has its own $(L'+1)$-tap FEQ. This allows us to optimize the equalizer coefficients $\tilde{\mathbf{w}}_{q'}^{(r,k)}[i]$ for each subcarrier k separately, without taking into account the specific relation between $\tilde{\mathbf{w}}_{q'}^{(r,k)}[i]$, $\mathbf{w}_{q'}^{(r)}[i]$, and $d_k[i]$.

Defining $\tilde{\mathbf{Y}}^{(r)}[i] = \left[\tilde{\mathbf{Y}}_{-Q'/2}^{(r)}[i], \ldots, \tilde{\mathbf{Y}}_{Q'/2}^{(r)}[i]\right]$ and $\tilde{\mathbf{w}}^{(r,k)}[i] = \left[\tilde{\mathbf{w}}_{-Q'/2}^{(r,k)T}[i], \ldots, \tilde{\mathbf{w}}_{Q'/2}^{(r,k)T}[i]\right]^T$, (6b) reduces to:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \mathcal{F}^{(k)} \tilde{\mathbf{Y}}^{(r)}[i] \tilde{\mathbf{w}}^{(r,k)}[i], \tag{7}$$

Transferring the TEQ operation to the frequency-domain by interchanging the TEQ with the FFT in (7), we obtain:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \tilde{\mathbf{w}}^{(r,k)T}[i] \mathbf{F}^{(k)}[i] \mathbf{y}^{(r)}[i], \tag{8}$$

where $\mathbf{F}^{(k)}[i] = \left(\mathbf{I}_{Q'+1} \otimes \tilde{\boldsymbol{\mathcal{F}}}^{(k)}\right) \left[\tilde{\mathbf{D}}_{-Q'/2}^T[i], \ldots, \tilde{\mathbf{D}}_{Q'/2}^T[i]\right]^T$, with $\tilde{\mathbf{D}}_{q'}[i] = \text{diag}\{[e^{j2\pi q'(i(N+\nu)+\nu+d-L')/K}, \ldots, e^{j2\pi q'((i+1)(N+\nu)+d-1)/K}]^T\}$, $\tilde{\boldsymbol{\mathcal{F}}}^{(k)}$ is given by:

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)} = \begin{bmatrix} 0 & \cdots & 0 & \boxed{\mathcal{F}^{(k)}} & \\ \vdots & 0 & \boxed{\mathcal{F}^{(k)}} & & 0 \\ 0 & \iddots & & \iddots & 0 & \vdots \\ \boxed{\mathcal{F}^{(k)}} & & 0 & \cdots & 0 \end{bmatrix}.$$

and $\mathbf{y}^{(r)}[i] = [y^{(r)}[i(N+\nu)+\nu+d-L'], \ldots, y^{(r)}[(i+1)(N+\nu)+d-1]]^T$.

To implement (8), we require $(Q'+1)$ sliding FFTs per receive antenna. Each sliding FFT is applied to a modulated version of the received sequence on that receive antenna. The q'th sliding FFT on the rth receive antenna is shown in Figure 1. To estimate the transmitted QAM symbol on the kth subcarrier we then have to combine the outputs of all PTFEQs corresponding to the kth subcarrier of all sliding FFTs on all receive antennas. This results in a complexity of $(Q'+1)(L'+1)$ multiply-add (MA) operations per receive antenna per subcarrier, i.e., $N_r N(Q'+1)(L'+1)$ MA operations for a block of N symbols.

Defining $\tilde{\mathbf{w}}^{(k)}[i] = [\tilde{\mathbf{w}}^{(1,k)T}[i], \ldots, \tilde{\mathbf{w}}^{(N_r,k)T}[i]]^T$ and $\mathbf{y}[i] = [\mathbf{y}^{(1)T}[i], \ldots, \mathbf{y}^{(N_r)T}[i]]^T$, (8) can be written as:

$$\hat{S}_k[i] = \tilde{\mathbf{w}}_k^T[i] \left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right) \mathbf{y}[i]. \tag{9}$$

At this point we may introduce a model for the received sequence on the rth receive antenna $\mathbf{y}^{(r)}[i]$ as:

$$\mathbf{y}^{(r)}[i] = \underbrace{\sum_{q=-Q/2}^{Q/2} \tilde{\mathbf{D}}_q[i] \left[\boldsymbol{O}_1, \mathbf{H}_q^{(r)}[i], \boldsymbol{O}_2\right] (\mathbf{I}_3 \otimes \mathbf{P})(\mathbf{I}_3 \otimes \boldsymbol{\mathcal{F}}^H)}_{\mathbf{G}^{(r)}[i]} \times \underbrace{\begin{bmatrix} \mathbf{s}[i-1] \\ \mathbf{s}[i] \\ \mathbf{s}[i+1] \end{bmatrix}}_{\tilde{\mathbf{s}}} + \boldsymbol{\eta}^{(r)}[i], \tag{10}$$

where $\boldsymbol{O}_1 = \mathbf{0}_{(N+L')\times(N+2\nu+d-L-L')}$, $\boldsymbol{O}_2 = \mathbf{0}_{(N+L')\times(N+\nu-d)}$, $\mathbf{H}_q^{(r)}[i]$ is an $(N+L') \times (N+L'+L)$ Toeplitz matrix with first column $[h_{q,L}^{(r)}[i], \mathbf{0}_{1\times(N+L'-1)}]^T$ and first row $[h_{q,L}^{(r)}[i], \ldots, h_{q,0}^{(r)}[i], \mathbf{0}_{1\times(N+L'-L-1)}]$, and $\mathbf{P}$ is the CP insertion matrix given by:

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{\nu\times(N-\nu)} & \mathbf{I}_\nu \\ \mathbf{I}_N & \end{bmatrix}.$$

To obtain the PTFEQ coefficients for the kth subcarrier, we define the following mean-square error (MSE) cost function:

$$\mathcal{J} = \mathcal{E}\left\{\left\| S_k[i] - \tilde{\mathbf{w}}^{(k)T}[i] \left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right) \mathbf{y}[i] \right\|^2\right\}$$

Hence, the PTFEQ coefficients for the kth subcarrier are given by:

$$\tilde{\mathbf{w}}_{MMSE}^{(k)}[i] = \arg \min_{\tilde{\mathbf{w}}^{(k)}[i]} \mathcal{J} \tag{11}$$

The solution of (11) is obtained by solving $\partial\mathcal{J}/\partial\tilde{\mathbf{w}}^{(k)}[i] = \mathbf{0}$, which reduces to:

$$\tilde{\mathbf{w}}_{MMSE}^{(k)T}[i] = \left(\mathbf{B}[i]\left(\mathbf{G}[i]\mathbf{G}^H[i] + \mathbf{R}_\eta\right)\mathbf{B}^H[i]\right)^{-1} \mathbf{B}[i]\mathbf{G}[i]\mathbf{e}^{(k)}, \tag{12}$$

where $\mathbf{G}[i] = [\mathbf{G}^{(1)T}[i], \ldots, \mathbf{G}^{(N_r)T}[i]]^T$, $\mathbf{B}[i] = \left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right)$ and $\mathbf{e}^{(k)}$ is the unit vector with 1 in the position $(N+k)$. Note that we assume white input sequence ($\mathbf{R}_{\tilde{s}} = \sigma_s^2\mathbf{I}$).

In the next section, we show how we can further reduce the complexity of the proposed PTFEQ by replacing the $Q'+1$ sliding FFTs by only a few sliding FFTs, the number of which is entirely independent of Q' but rather depends on the BEM frequency resolution K. As will be clear later, the removed sliding FFTs are compensated for by combining the outputs of neighboring subcarriers on the remaining sliding FFTs.

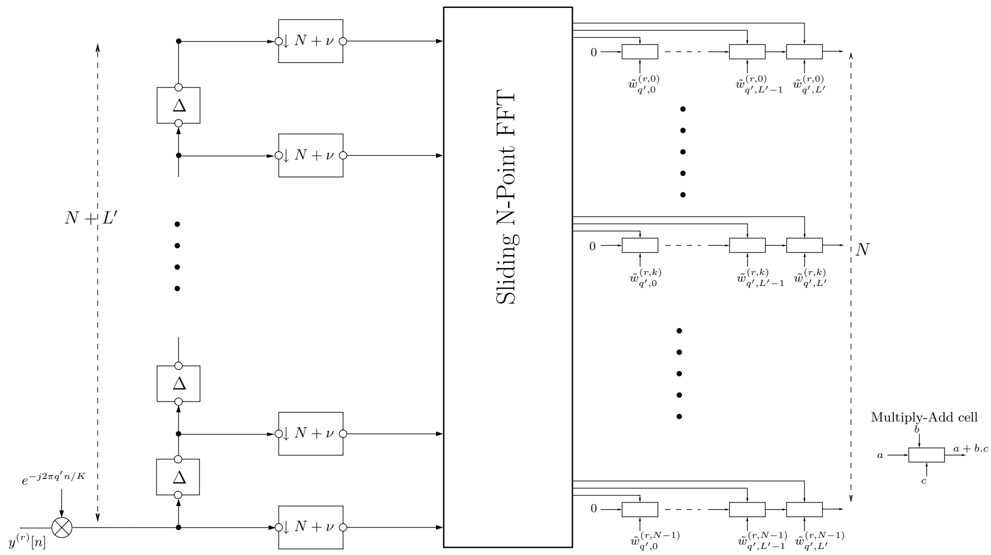

Fig. 1. Sliding FFT of the q'th phase shifted version of the received sequence $\mathbf{y}^{(r)}[i]$

V. Efficient Implementation

In Section IV, we have shown that to implement the proposed PTFEQ we basically require $(Q'+1)$ sliding FFTs. In this section, we show how we can further lower the complexity of the proposed PTFEQ by exploiting the special structure of $\mathbf{Y}^{(r)}[i]$.

In general the BEM frequency resolution K is greater than or equal to the FFT size N. In this paper, we will assume that K is an integer multiple of the FFT size i.e., $K = PN$, where P is an integer greater than or equal to 1 ($P \geq 1$). We start by defining $\mathbb{Q} = \{-Q'/2, \ldots, Q'/2\}$, and $\mathbb{Q}_p = \{q \in \mathbb{Q} \mid q \mod P = p\}$. Based on these definitions, (6b) and (7) can be written as:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \mathcal{F}^{(k-l_p)} \underbrace{\mathbf{D}_p[i]\mathbf{Y}^{(r)}\hat{\mathbf{D}}_p}_{\mathbf{Y}_p^{(r)}[i]} \underbrace{\hat{\mathbf{D}}_p^* \mathbf{w}_{p,l_p}^{(r,k)}[i]/d_k[i]}_{\bar{\mathbf{w}}_{p,l_p}^{(r,k)}[i]} \quad (13a)$$

$$= \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \mathcal{F}^{(k-l_p)} \mathbf{Y}_p^{(r)}[i] \bar{\mathbf{w}}_{p,l_p}^{(r,k)}[i], \quad (13b)$$

where $l_p = \frac{q_p - p}{P}$, and $\mathbf{w}_{p,l_p}^{(r,k)}[i] = \mathbf{w}_{q_p}^{(r,k)}[i]$. Note that (13b) splits the $Q'+1$ different terms of (7) into P different groups, with the pth group containing $|\mathbb{Q}_p|$ terms, where $|\mathbb{Q}_p|$ denotes the cardinality of the set $\mathbb{Q}_p$ for $p = 0, \ldots, P-1$. This splitting will allow us to significantly reduce the complexity. Transferring the TEQ operation to the frequency-domain, we obtain:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \bar{\mathbf{w}}_{p,l_p}^{(r,k)T}[i] \tilde{\boldsymbol{\mathcal{F}}}^{(k-l_p)} \underbrace{\tilde{\mathbf{D}}_p[i]\mathbf{y}^{(r)}[i]}_{\tilde{\mathbf{y}}_p^{(r)}[i]} \quad (14)$$

Defining $\bar{\mathbf{w}}_p^{(r,k)}[i] = [\ldots, \bar{\mathbf{w}}_{p,-1}^{(r,k)T}[i], \bar{\mathbf{w}}_{p,0}^{(r,k)T}[i], \bar{\mathbf{w}}_{p,1}^{(r,k)T}[i], \ldots]^T$, (14) can now be written as:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \bar{\mathbf{w}}_p^{(r,k)T}[i] \tilde{\mathbf{F}}_p^{(k)} \tilde{\mathbf{y}}_p^{(r)}[i] \quad (15)$$

where $\tilde{\mathbf{F}}_p^{(k)} = [\ldots, \tilde{\boldsymbol{\mathcal{F}}}^{(k-1)T}, \tilde{\boldsymbol{\mathcal{F}}}^{(k)T}, \tilde{\boldsymbol{\mathcal{F}}}^{(k+1)T}, \ldots]^T$. Let us now define $\tilde{\mathbf{F}}^{(k)} = \operatorname{diag}\left\{\tilde{\mathbf{F}}_0^{(k)T}, \ldots, \tilde{\mathbf{F}}_{P-1}^{(k)T}\right\}$, $\tilde{\mathbf{y}}_p^{(r)T}[i] = [\tilde{y}_p^{(r)T}[i(N+\nu)+d-L'], \ldots, \tilde{y}_p^{(r)T}[(i+1)(N+\nu)+d-1]]^T$ with $\tilde{y}_p^{(r)T}[n] = e^{j2\pi n/K} y^{(r)}[n]$, $\tilde{\mathbf{y}}^{(r)}[i] = [\tilde{\mathbf{y}}_0^{(r)T}[i], \ldots, \tilde{\mathbf{y}}_{P-1}^{(r)T}[i]]^T$ and $\tilde{\mathbf{y}}[i] = [\tilde{\mathbf{y}}^{(1)T}[i], \ldots, \tilde{\mathbf{y}}^{(N_r)T}[i]]^T$. Further defining $\bar{\mathbf{w}}^{(r,k)}[i] = [\bar{\mathbf{w}}_0^{(r,k)T}[i], \ldots, \bar{\mathbf{w}}_{P-1}^{(r,k)T}[i]]^T$ and $\bar{\mathbf{w}}^{(k)}[i] = [\bar{\mathbf{w}}^{(1,k)T}[i], \ldots, \bar{\mathbf{w}}^{(N_r,k)T}[i]]^T$, (15) can finally be written as:

$$\hat{S}_k[i] = \bar{\mathbf{w}}^{(k)T}[i](\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)})\tilde{\mathbf{y}}[i] \quad (16)$$

To implement (15), we require P sliding FFTs per receive antenna rather than $Q'+1$ sliding FFTs per receive antenna as in Section IV (in practice and in our simulations $P \ll Q'+1$). Each sliding FFT is applied to a modulated version of the received sequence. This reduction in the number of sliding FFTs per receive antenna is compensated for by combining $|\mathbb{Q}_p|$ neighboring subcarriers on the pth sliding FFT. Notice here, that apart from the reduction in the number of sliding FFTs, the implementation complexity remains the same as in Section IV, i.e., $N_r N(Q'+1)(L'+1)$ MA operations for a block of N symbols.

Similar to (11), we can construct the MSE cost function as:

$$\mathcal{J} = \mathcal{E}\left\{\left\| S_k[i] - \bar{\mathbf{w}}^{(k)T}[i]\left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}\right)\tilde{\mathbf{y}}[i]\right\|^2\right\} \quad (17)$$

Note the solution of the minimization of (17) is given by a formula similar to (and equivalent to) the one obtained in (12).

We can further simplify the computational complexity associated with the proposed PTFEQ by replacing each sliding FFT by only one full FFT and L' difference terms that are common to all subcarriers [13]. To explain this, we will consider only one sliding FFT. Let us consider the kth subcarrier of the pth sliding FFT, i.e., $\tilde{\boldsymbol{\mathcal{F}}}^{(k)}\tilde{\mathbf{y}}_p^{(r)}[i]$. Define $\tilde{Y}_p^{(r,k)} = \mathcal{F}^{(k)}[\tilde{y}_p^{(r)}[i(N+\nu)+\nu+d], \ldots, \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-1]]^T$ as the frequency response of the pth phase shifted version of the received sequence on the rth receive antenna on the kth subcarrier. It can then easily be shown that:

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)}\tilde{\mathbf{y}}_p^{(r)}[i] = \mathbf{T}^{(k)} \begin{bmatrix} \tilde{Y}_p^{(r,k)} \\ \Delta\tilde{\mathbf{y}}_p^{(r)}[i] \end{bmatrix} \updownarrow L' \times 1 \tag{18}$$

where $\mathbf{T}^{(k)}$ is an $(L'+1)\times(L'+1)$ lower triangular Toeplitz matrix given by:

$$\mathbf{T}^{(k)} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \beta^{(k-1)L'} & \cdots & \beta & 1 \end{bmatrix} \tag{19}$$

with $\beta = e^{-j2\pi/N}$. The difference terms $\Delta\tilde{\mathbf{y}}_p^{(r)}[i]$ are given by $\Delta\tilde{\mathbf{y}}_p^{(r)}[i] = [\tilde{y}_p^{(r)}[i(N+\nu)+\nu+d-1] - \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-1], \ldots, \tilde{y}_p^{(r)}[i(N+\nu)+\nu+d-L'] - \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-L'-1]]^T$. In a similar fashion, we can obtain an expression for the neighboring subcarriers on the same sliding FFT by replacing the subcarrier index. The symbol estimate (16) can then be written as follows. We first define $\mathbf{u}_{p,l_p}^{(r,k)T}[i] = \bar{\mathbf{w}}_{p,l_p}^{(r,k)T}[i]\mathbf{T}^{(k+l_p)}$ and also define the following $(|\mathbb{Q}_p|(L'+1)) \times (|\mathbb{Q}_p| + L')$ selection matrix:

$$\mathbf{S}_p = \begin{bmatrix} 1 & \mathbf{0}_{1\times(|\mathbb{Q}_p|+L'-1)} & \\ \mathbf{0}_{L'\times|\mathbb{Q}_p|} & & \mathbf{I}_{L'} \\ 0 \quad 1 & \mathbf{0}_{1\times(|\mathbb{Q}_p|+L'-2)} & \\ \mathbf{0}_{L'\times|\mathbb{Q}_p|} & & \mathbf{I}_{L'} \\ & \vdots & \end{bmatrix}.$$

Introducing $\mathbf{u}_p^{(r,k)T}[i] = [\ldots, \mathbf{u}_{p,-1}^{(r,k)T}[i], \mathbf{u}_{p,0}^{(r,k)T}[i], \mathbf{u}_{p,1}^{(r,k)T}[i], \ldots]^T$ and $\mathbf{v}_p^{(r,k)T}[i] = \mathbf{u}_p^{(r,k)T}[i]\mathbf{S}_p$, (16) can then be written as:

$$\hat{S}_k[i] = \sum_{r=1}^{N_r}\sum_{p=0}^{P-1} \mathbf{v}_p^{(r,k)T}[i] \left[\begin{array}{c} \vdots \\ \tilde{Y}_p^{(r,k-1)}[i] \\ \tilde{Y}_p^{(r,k)}[i] \\ \tilde{Y}_p^{(r,k+1)}[i] \\ \vdots \\ \hline \Delta\tilde{\mathbf{y}}_p^{(r)}[i] \end{array}\right] \begin{array}{l} \updownarrow |\mathbb{Q}_p| \times 1 \\ \\ \updownarrow L' \times 1 \end{array}$$

$$= \sum_{r=1}^{N_r}\sum_{p=0}^{P-1} \mathbf{v}_p^{(r,k)T}[i] \underbrace{\left[\begin{array}{ccc} \vdots & & \vdots \\ \mathbf{0}_{1\times L'} & & \mathcal{F}^{(k-1)} \\ \mathbf{0}_{1\times L'} & & \mathcal{F}^{(k)} \\ \mathbf{0}_{1\times L'} & & \mathcal{F}^{(k+1)} \\ \vdots & & \vdots \\ \hline \bar{\mathbf{I}}_{L'} & \mathbf{0}_{L'\times(N-L')} & -\bar{\mathbf{I}}_{L'} \end{array}\right]}_{\tilde{\mathbf{F}}_p^{(k)}} \tilde{\mathbf{y}}_p^{(r)}[i] \tag{20}$$

where $\bar{\mathbf{I}}_{L'}$ is the anti-diagonal identity matrix of size $L' \times L'$. Defining $\mathbf{v}^{(r,k)}[i] = [\mathbf{v}_0^{(r,k)T}[i], \ldots, \mathbf{v}_{P-1}^{(r,k)T}[i]]^T$, $\mathbf{v}^{(k)}[i] = [\mathbf{v}^{(1,k)T}[i], \ldots, \mathbf{v}^{(N_r,k)T}[i]]^T$ and $\tilde{\mathbf{F}} = \text{diag}\left\{\tilde{\mathbf{F}}_0^T, \ldots, \tilde{\mathbf{F}}_P^T\right\}$, (20) can finally be written as:

$$\hat{S}_k = \mathbf{v}^{(k)T}[i](\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)})\tilde{\mathbf{y}}[i] \tag{21}$$

Note that, due to the fact that the difference terms are common to all subcarriers in a particular sliding FFT, the implementation complexity is $P(L'+1) + Q' + 1$ MA operations per receive antenna per subcarrier, compared to $(Q'+1)(L'+1)$ per receive antenna per subcarrier in Section IV. In Figure 2, we show how (21) can be realized for the pth sliding FFT on the rth receive antenna. Note that replacing the sliding FFT with one full FFT and L' difference terms in Section IV, will not reduce the implementation complexity. This is due to the fact that we only consider a single PTFEQ output for each sliding FFT to estimate a particular symbol.

We can also show that our approach unifies and extends many existing frequency-domain approaches. For the case of a TIV channel ($Q = 0$) with delay spread larger than the CP, the proposed PTFEQ with $Q' = 0$ comes down to the per-tone equalizer of [10]. On the other hand, for a TIV channel with delay spread smaller than or equal to the CP, the proposed PTFEQ with $Q' = 0$ comes down to the well-known MMSE equalizer [14]. For the case of a TV channel with delay spread smaller than or equal to the CP, the proposed PTFEQ comes down to the MMSE FEQ proposed in [3] for $P = 1$ and to the MMSE FEQ proposed in [15] for an arbitrary P.

VI. Simulation Results

In this section, we show some simulation results for the proposed ICI mitigation technique. We consider a SISO system as well as a SIMO system with $N_r = 2$ receive antennas. The channel is assumed to be doubly-selective of order $L = 6$ with a maximum Doppler frequency of $f_{\max} = 100Hz$. The channel taps are simulated as i.i.d., correlated in time with a correlation function according to Jakes' model $\mathcal{E}\{h^{(r)}[n_1; l_1]h^{(r')*}[n_2; l_2]\} = \sigma_h^2 J_0(2\pi f_{\max}T(n_1-n_2))\delta[l_1-l_2]\delta[r-r']$, where J_0 is the zeroth-order Bessel function of the first kind and σ_h^2 denotes the variance of the channel. We consider $N = 128$ subcarriers, and a cyclic prefix of length $\nu = 3$. The sampling time is $T = 50\mu sec$, the total OFDM symbol duration is 6.6 $msec$. QPSK signaling is assumed. We define the SNR as $SNR = \sigma_h^2(L+1)E_s/\sigma_\eta^2$, where E_s is the QPSK symbol power.

We use the BEM to approximate the channel. The BEM coefficients of the approximated channel are used to design the MMSE equalizer. The BEM resolution is determined by $K = PN$ where $P = 1, 2$. The number of TV basis functions of the channel is chosen such that $Q/(2KT) \geq f_{\max}$, which results into $Q = 2$ for $P = 1$, and $Q = 4$ for $P = 2$. The TV FIR TEQ is modeled using the BEM with $Q' = 14$ and $L' = 14$ for $N_r = 1$ receive antenna, and $Q' = 8$ and $L' = 8$ for $N_r = 2$ receive antennas. The decision delay d is always chosen as $d = \lfloor(L + L')/2\rfloor + 1$. The proposed equalizer is used to equalize the true Jakes' channel.

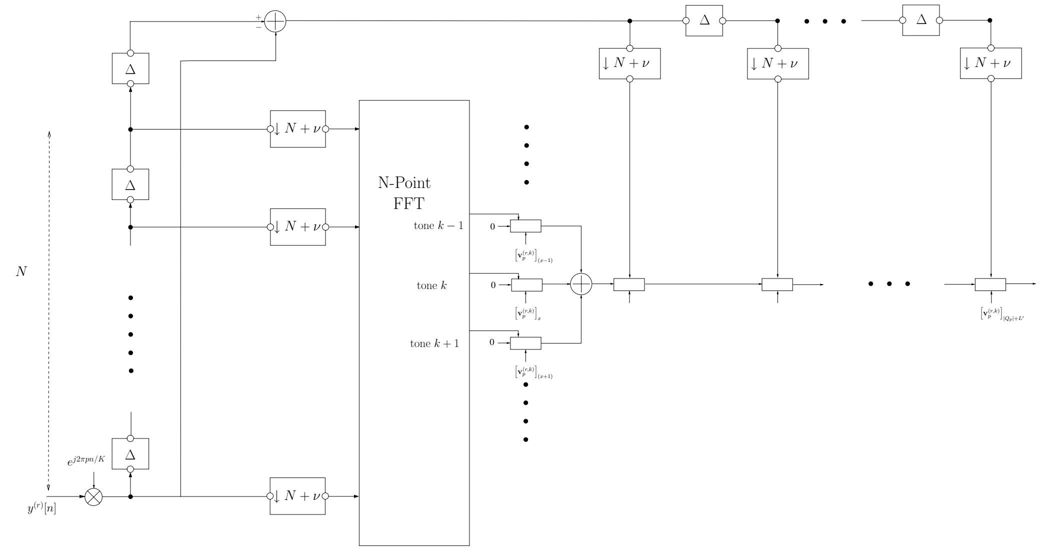

Fig. 2. Low complexity PTFEQ

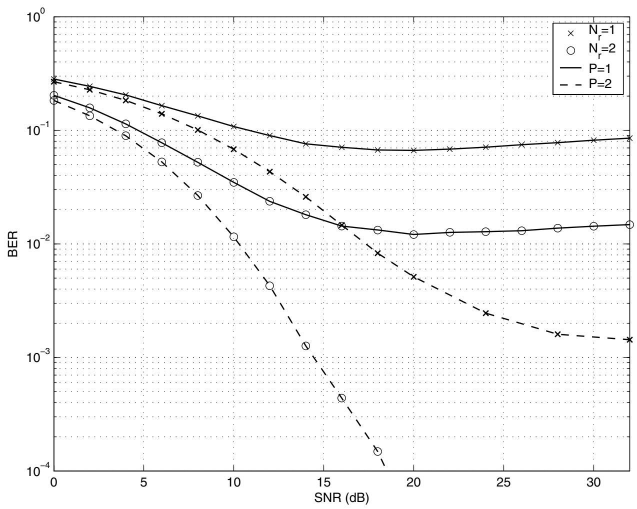

Fig. 3. BER vs. SNR for $N_r = 1, 2$ and $P = 1, 2$

As shown in Figure 3, the performance of the proposed equalizer is significantly improved using $P = 2$ over $P = 1$, where the latter suffers from an early error floor (4×10^{-1} for $N_r = 1$ and 10^{-2} for $N_r = 2$).

VII. Conclusion

A frequency-domain equalizer for OFDM over doubly-selective channels has been proposed, where the channel delay spread is larger than the CP. The devised PTFEQ is obtained by transferring a TV FIR TEQ operation to the frequency-domain. We also show how we can efficiently implement the proposed PTFEQ. A key role in the performance of the proposed PTFEQ is the BEM frequency resolution. We show that by choosing the BEM frequency resolution equals twice the FFT resolution (FFT size), the performance is significantly improved.

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