SOURCE SEPARATION OF ASYNCHRONOUS OFDM SIGNALS USING SUPERIMPOSED TRAINING

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Abstract- In multi-user orthogonal frequency division multiplexing (OFDM) systems, an asynchronous interferer is defined as a signal whose cyclic prefix (CP) does not align with the desired user CP. Such *ad-hoc* multi-user OFDM systems are considered and an algorithm to perform blind space-time source separation is presented. In this paper we consider a scheme where a periodic sequence is added to the transmitted OFDM signal to be able to identify it. The periodic sequence has a cyclostationary property and can be used to estimate the channel and to separate the interfering users from the desired user signals.

Index Terms— Ad hoc networks, asynchronous OFDM, blind source separation, superimposed training.

1. INTRODUCTION

Ubiquitous communication of orthogonal frequency division multiplexing (OFDM) signals among multiple users, that require neither slot synchronization nor coordination, is a very attractive option to achieve maximum spectral efficiency and data rate. In such asynchronous OFDM systems [1-2], the CP of the desired user is not synchronized to the CP of the interfering user. This leads to poor error rate performance when frequency domain weights are used to separate the sources, since all the components of the interfering users in each of the subcarriers must be suppressed [1]. Consider an asynchronous multiuser-OFDM system as shown in Fig. 1 where several users occupy a common wireless channel. Each user transmits data in the form of packets, containing a sequence of OFDM symbols with arbitrary start time and packet lengths. The transmitted signal of the desired user (user 1) is received using an antenna array at the receiver, corrupted by interference from other users and noise. Our objective is to use the signals from the antenna array to separate the interfering signals at the receiver and estimate desired user.

Superimposed training has been recently considered by several authors [3-5], where the training (a periodic non random) sequence is arithmetically added to the transmitted sequence. In this way no bandwidth is lost when sending the training data. The received data has periodically time-varying mean and this property can be exploited to separate the required user and to estimate the channel. We consider a scenario where users can transmit packages at random moments and where the receiver knows the super imposed training sequence and the packet length of the user of interest. The transmission offset of the desired user can be determined using a multiple signal classification (MUSIC) type search by transforming the time varying mean by a DFT followed by deconvolution with the superimposed sequence, which maps the delays into phase shifts in the frequency domain [6]. In [7] the desired user is synchronized for a convolutive multi-user channel and a beamformer is designed based on correlation between received antenna array and superimposed sequence. In some ways this work is an improvement over [7] for multi-user scenario where the source separation algorithm exploits the time varying mean property to cancel the interfering users as well as compute the offset of the desired user for a CP based system.

Notation: $(.)^T$, $(.)^H$ and $(.)^\dagger$ represent transpose, hermitian and pseudo inverse. diag$(.)$ maps a vector into a diagonal matrix. $\circledast$ and $\odot$ represent circular convolution and Schur-Hadamard product between matrices and $\|.\|$ represents Frobenius norm.

2. DATA MODEL

In a cyclic prefix (CP) based OFDM communication system, a redundant CP of length L_{cp} is inserted between successive data blocks $s_{b,0:N-1}$ of length N, where $s_{b,0:N-1}$ is the information sequence corresponding to block number b and time instants 0 to $N-1$. The transmitted signal consisting of CP and data block is of length $N + L_{cp}$ as shown for desired user in Fig. 1.

From now on we assume that data is transmitted in blocks and omit the b term in $s_{b,k}$. We also assume for the time being that the desired user is synchronized to the receiver as shown in Fig. 1. In an OFDM transmission system using superimposed training, a known pilot sequence $c_n^{(1)}$, with period P is added to the time domain OFDM signal $x_k^{(1)} = c_k^{(1)} + s_k^{(1)}$ (superscript 1 denotes user 1) and padded with CP. These superimposed training schemes can be used in acquiring channel state infor-

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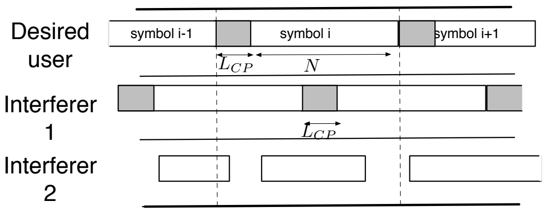

Fig. 1: *packet transmission with non alignment of cyclic prefix*

mation (CSI) for any block transmission systems such as [3-5]. In the well known pilot tone assisted modulation (PTAM) scheme for OFDM systems, a number of dedicated training pilots are placed in specific frequency bins to acquire the CSI, resulting in reduced bandwidth. The PTAM scheme can be seen as a special case of superimposed training where a pilot sequence is inserted into the frequency bins which is equivalent to being added onto the time domain OFDM signal.

Let N_t be the number of users occupying the common wireless channel with user 1 being the desired user and the other users being interferers. We make the following assumptions about the transmitted signals from desired and interfering users:

A1: The information sequence of the desired user $s_k^{(1)}$ is zero mean and white with $E\{|s_k^{(1)}|^2\} = \sigma_s^2$.

A2: The superimposed sequence of the desired user $c_k^{(1)}$ is a non-random periodic sequence with period P such that $c_k^{(1)} = c_{k+mP}^{(1)}$ for all m, k.

A3: The information sequence of the asynchronous interfering users is zero mean. The interfering users may or may not use superimposed training. For interfering users with superimposed training, the period of training sequence is $P' \neq P$ [1].

Each user is associated with one antenna and the receiver is supplied with N_r receive antennas. Consider a $N_r \times N_t$ frequency selective fading channel modeled by a time invariant FIR filter H_l of order L ($L \leq L_{CP}$ cyclic prefix length). The received $N_r \times 1$ signal is given by

$$\mathbf{y}_k = \begin{bmatrix} H_0 H_1 \cdots H_L \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_{k-1} \\ \vdots \\ \mathbf{x}_{k-L} \end{bmatrix} + \mathbf{w}_k \tag{1}$$

where $\mathbf{x}_k = [x_k^{(1)} \cdots x_k^{(N_t)}]^T$ is a $N_t \times 1$ vector of the transmitted signals from the desired and interfering users and $\mathbf{w}_k$ is the $N_r \times 1$ noise vector at the receive antennas at time instant k. For simplicity of expression, we will assume that all $N_r \times N_t$ MIMO channels have same length.

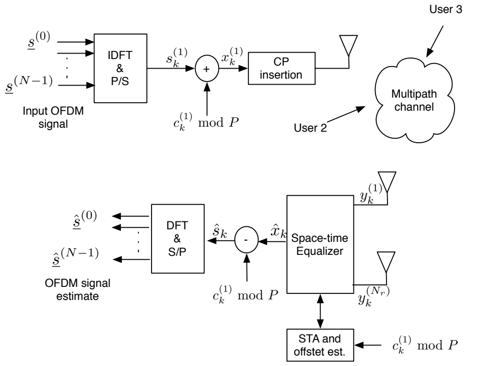

Fig. 2: *superimposed training algorithm (STA) used to recognize user 1*

3. EQUALIZER DESIGN

Fig. 2 shows a multi-user discrete-time baseband schematic of an OFDM transmission system using the superimposed training algorithm (STA) that will be proposed here. The information sequence from the user of interest is added to a non-periodic sequence (known at the receiver) and transmitted over a wireless channel corrupted by interference from other users. It has been shown in [1] that if OFDM demodulation is performed directly after the receive antenna array, equalizer coefficients of degree L are required for each subcarrier to cancel the asynchronous interfering users. To avoid this situation, the setup in Fig. 2 will cancel the asynchronous interference using a time domain space time (ST) equalizer, independent of the transmission scheme such as OFDM. This superimposed sequence allows the space time equalizer at the receiver to detect and filter out the desired user among other interfering users.

3.1. Equalizer design for asynchronous OFDM systems

The objective is to estimate the desired user data using a space-time equalizer ($mN_r \times 1$ vector) $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T \cdots \boldsymbol{\theta}_m^T]$, where m is the length of the equalizer which combines $N_r \times 1$ receive antenna array signals at successive time instants to obtain an estimate of the transmitted user data. For time instants $k, k \in \{0, 1, \cdots N-1\}$, stacking the successive antenna array signals from time k to $k-m+1$ gives

$$\begin{aligned} \mathbf{y}_{k:k-m+1} &= [\mathbf{y}_k^T \ \cdots \ \mathbf{y}_{k-m+1}^T]^T \\ &= \mathcal{H}\mathbf{x}_{k:k-m-L+1} + \mathbf{w}_{k:k-m+1} \end{aligned} \tag{2}$$

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[1] This is a strong assumption that will be lifted in future work

$$\mathcal{H} = \begin{bmatrix} H_0\, H_1 \dots H_L & & \\ & \ddots \qquad \ddots & \\ & & H_0\; H_1 \dots H_L \end{bmatrix}$$

and $\mathbf{x}_{k:k-m-L+1}$ and $\mathbf{w}_{k:k-m+1}$ are defined in a similar way as $\mathbf{y}_{k:k-m+1}$ and $\mathcal{H}$ is a $mN_r \times (m+L+1)N_t$ block toeplitz matrix. The condition for equalization relies on the existence of a filtering matrix $\boldsymbol{\theta}$ such that $\mathbf{y}^T_{k:k-m+1}\boldsymbol{\theta}$ is an estimate of the transmitted superimposed signal $\hat{x}^{(1)}_k$. A necessary condition for space-time equalization is $\mathcal{H}$ should be a tall matrix [8]. The instantaneous error of ST equalizer $\boldsymbol{\theta}$ is

$$\begin{aligned} \varepsilon_k &= \mathbf{y}^T_{k:k-m+1}\boldsymbol{\theta} - x^{(1)}_k \\ &= \left[\mathbf{y}^T_k \cdots \mathbf{y}^T_{k-m+1}\right] \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_m \end{bmatrix} - (s^{(1)}_k + c^{(1)}_k) \end{aligned}$$

Extending the cost function ε_k for P successive time instants, to a $P \times 1$ vector by left shifting and stacking P times and dropping $\arg\min$, we obtain for $k, k \in \{0, 1, \cdots N-1\}$ the least squares (LS) problem

$$\begin{bmatrix} \varepsilon_k \\ \vdots \\ \varepsilon_{k+P-1} \end{bmatrix} = \begin{bmatrix} \mathbf{y}^T_k & \mathbf{y}^T_{k-1} & \cdots \mathbf{y}^T_{k-m+1} \\ \vdots & \ddots & \vdots \\ \mathbf{y}^T_{k+P-1} & \mathbf{y}^T_{k+P-2} & \cdots \mathbf{y}^T_{k+P-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_m \end{bmatrix} - \begin{bmatrix} s^{(1)}_k + c^{(1)}_k \\ \vdots \\ s^{(1)}_{k+P-1} + c^{(1)}_{k+P-1} \end{bmatrix} \quad (3)$$

To remove the effect of unknown symbols $s^{(1)}_k$, we propose to use the assumptions A1, A2 and A3 and average them out over M superimposed sequence periods $M = \frac{N}{P}$, where M is an integer and N is the number of subcarriers. When M is sufficiently large (obtained by including successive OFDM blocks), the cost function is can be averaged for $\bar{\varepsilon}_j = \sum_{i=0}^{M-1} \varepsilon_{iP+j}$ for all $j = 0 : P-1$. Exploiting the time varying mean property A1 and A2, the resulting block average cost function is given by

$$\begin{bmatrix} \bar{\varepsilon}_0 \\ \vdots \\ \bar{\varepsilon}_{P-1} \end{bmatrix} = \begin{bmatrix} \mathbf{z}^T_0 & \cdots \mathbf{z}^T_{-m+1} \\ \vdots \quad \ddots & \vdots \\ \mathbf{z}^T_{P-1} & \cdots \; \mathbf{z}^T_{P-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_m \end{bmatrix} - \begin{bmatrix} c^{(1)}_0 \\ \vdots \\ c^{(1)}_{P-1} \end{bmatrix}$$
$$\Leftrightarrow \bar{\boldsymbol{\varepsilon}} = \mathcal{Z}\boldsymbol{\theta} - \mathbf{c}^{(1)} \quad (4)$$

where $\mathbf{z}_j = \frac{1}{M}\sum_{i=0}^{M-1} \mathbf{y}_{iP+j}$. (The negative indices correspond to receive antenna array averages of the cyclic prefix). The LS estimate for $\boldsymbol{\theta}$ is given by

$$\hat{\boldsymbol{\theta}} = \mathcal{Z}^{\dagger}(\bar{\varepsilon} + \mathbf{c}^{(1)}) \quad (5)$$

The $\hat{\boldsymbol{\theta}}$ is used as an initial point and the estimate $\hat{x}^{(1)}_k$ is alternatingly projected onto the row span of $\mathbf{y}_{0:N-1}$ [6].

$$\begin{gathered} \hat{\mathbf{x}}^{(1)} = \mathbf{Y}^T\boldsymbol{\theta}(i) \\ \boldsymbol{\theta}(i+1) = [\hat{\mathbf{x}}^{(1)T}\mathbf{Y}^{\dagger}]^T \end{gathered}$$

where $\mathbf{Y} = [\mathbf{y}_{0:-m+1} \;\cdots\; \mathbf{y}_{N-1:N-m}]$, $\hat{\mathbf{x}}^{(1)} = \hat{x}^{(1)}_{0:N-1}$ and i is the iteration count. This process is iterated until the algorithm converges (say 5 times).

3.2. Joint offset and equalizer estimation

In the previous section we assumed that the OFDM receiver was synchronized to the user of interest. Now we consider a situation where the receiver is not synchronized to the user of interest. Without loss of generality assume that the maximum offset of the desired user is less than P and use a MUSIC type search to estimate the offset. Our aim is to compute

$$\{\hat{\boldsymbol{\theta}},\, \hat{\tau}\} = \arg \min_{\boldsymbol{\theta},\, \tau} \begin{bmatrix} \mathbf{z}^T_0 & \cdots \mathbf{z}^T_{-m+1} \\ \vdots \quad \ddots & \vdots \\ \mathbf{z}^T_{P-1} & \cdots \; \mathbf{z}^T_{P-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_m \end{bmatrix} - \begin{bmatrix} c^{(1)}_{\tau} \\ \vdots \\ c^{(1)}_{(\tau+P-1)_N} \end{bmatrix}$$
$$\Leftrightarrow \bar{\varepsilon} = \mathcal{Z}\boldsymbol{\theta} - \mathbf{c}^{(1)}_{\tau} \quad (6)$$

Here $\mathbf{c}^{(1)}_{\tau}$ is a circular shift of the superimposed sequence $\mathbf{c}^{(1)}$ by integer τ. We exploit the shift invariance property that a delay in time domain corresponds to a phase progression in frequency domain. Similar to [6]

$$\mathcal{F}\mathbf{c}^{(1)}_{\tau} = \mathcal{F}\mathbf{c}^{(1)} \odot \boldsymbol{\phi}_{\tau}$$

where $\boldsymbol{\phi}_{\tau} = [1 \;\; \phi \;\; \cdots \;\; \phi^{P-1}]^T$ and $\mathcal{F}$ is a $P \times P$ DFT matrix, $\phi = \exp\frac{-j2\pi\tau}{P}$. Let $\hat{\mathcal{Z}} = [\text{diag}(\mathcal{F}\mathbf{c}^{(1)})]^{-1}\mathcal{F}\mathcal{Z}$. Equation (6) indicates that the column span of $\hat{\mathcal{Z}}$ has shift invariance structure. Applying the DFT matrix $\mathcal{F}$ to (6) gives the LS problem

$$\mathcal{F}\mathcal{Z}\boldsymbol{\theta} = \mathcal{F}\mathbf{c}^1 \odot \boldsymbol{\phi}_{\tau} \;\; \Leftrightarrow \;\; \hat{\mathcal{Z}}\boldsymbol{\theta} = \boldsymbol{\phi}_{\tau} \quad (7)$$

Let $\hat{\mathcal{Z}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$: $\mathbf{U} \in \mathbb{C}^{N_r mp \times N_r mp}$, $\boldsymbol{\Sigma} \in \mathbb{R}^{N_r mp \times P}$, $\mathbf{V} \in \mathbb{C}^{P \times P}$ be a singular value decomposition of $\hat{\mathcal{Z}}$. $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices containing the singular vectors and $\boldsymbol{\Sigma}$ is a real diagonal matrix with non-negative elements. The delay can be estimated by a MUSIC type search: $\hat{\tau} = \arg\max_{\tau} \|\boldsymbol{\phi}^H_{\tau}\mathbf{U}_s\|$ where $\mathbf{U}_s$ consists of the dominant $N_t(P+m-1)$ columns of $\mathbf{U}$.

3.3. MMSE using known transmitted signal

The single user minimum mean square equalizer (MMSE) receiver is designed using transmitted signal values of user 1 known to the receiver and is used as a reference to compare to the performance of STA:

$$\hat{\boldsymbol{\theta}}_1 = \arg \min_{\boldsymbol{\theta}} \parallel \mathbf{Y}^T\boldsymbol{\theta} - \mathbf{x}^{(1)} \parallel^2_F = [\frac{1}{N}\mathbf{Y}\mathbf{Y}^T]^{-1}\frac{1}{N}\mathbf{Y}\mathbf{x}^{(1)}$$

where $\mathbf{x}^{(1)} = x^{(1)}_{0:N-1}$.

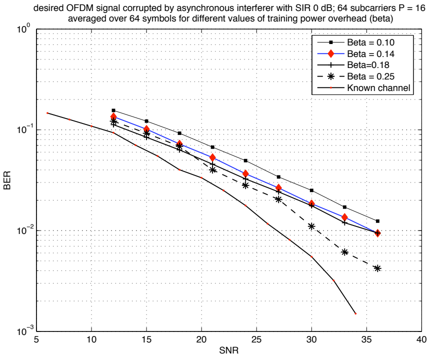

Fig. 3: BER performance for asynchronous interfering users having random offsets, $L = 4$ and β

4. SIMULATION RESULTS

The performance of STA is tested for a multi-user OFDM system. All users transmit QPSK modulated information sequences superimposed with periodic sequences and the receiver knows the superimposed sequence of the desired user. The training power overhead introduced by superimposed sequences is β is given by $\beta = \frac{\sigma_c^2}{\sigma_c^2+\sigma_s^2}$. We consider a frequency selective Raleigh fading channel for $N_t = 2$ users, $N_r = 2$ receive antennas and signal to interference ratio (SIR) = 0 dB. An oversampling ratio of 2 is used to increase the dimensions of $\mathcal{H}$.The interfering users are superimposed with gold sequences. We choose $N = 64$ as number of subcarriers and $P = 16$. The time varying mean is averaged over 128 OFDM symbols, which is approximately equal to packet length of wireless LAN systems.

Case 1- receiver synchronized to desired user: Fig. 3 shows the BER performance for different values of β. In these simulations, the desired user is synchronized to the receiver. Each OFDM symbol of the interfering user is transmitted with random offset and the receiver does not have any information regarding the superimposed sequence and offset of the interfering user.

Case 2 - receiver not synchronized to desired user: Fig. 4 shows the failure rate when the receiver is not synchronized to the desired user. An offset estimate is labeled as a failure if the rounded value of the offset obtained by a MUSIC type search is not equal to the true (integer) offset of the desired user.

5. CONCLUSION

In this paper, an algorithm to perform blind source separation for asynchronous WLAN is proposed. From theoretical analysis and simulation results, it has been observed that this technique results in performance independent of channel characteristics and modulation formats of desired and interfering users. The STA is combined with a MUSIC type search, to jointly estimate the equalizer coefficients and delay of the desired user signal.

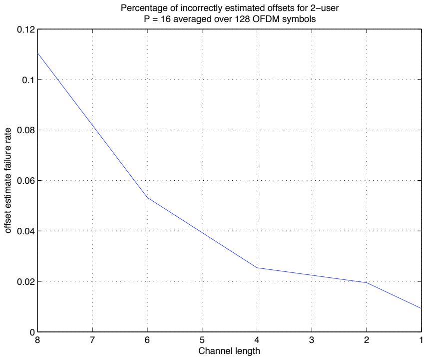

Fig. 4: incorrectly estimated offsets of desired user at $SNR = 36dB$ and $\beta = 0.5$

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