Computation of Field Approximations in Dispersive Media Using Quasinormal Mode Expansions

J. T. Zimmerling[1] and R. F. Remis[2]

[1]Department of Mathematics, University of Michigan, Ann Arbor, USA
[2]Circuits and Systems Group, Delft University of Technology, Delft, The Netherlands

Abstract— In the proximity of resonators, the electromagnetic field can sometimes be expanded in a few resonances, so called quasinormal modes. During the design of integrated photonics or optical sensors, resonators from dispersive materials, such as noble metals, are often encountered. The efficient computation of electromagnetic fields in such structures is crucial for design optimization.

In this contribution, we propose a computational method that consists of two steps. In order to reduce the computational complexity, the discretized Maxwell equations are first projected onto a Krylov subspace and thereby approximated by a so-called reduced-order model. From the reduced-order model, field approximations and quasinormal modes can be drawn. In the second step, the reduced-order model responses are approximated by low-order quasinormal mode expansion. The two steps are necessary, in order to control the error introduced by the mode expansion and check if the medium of interest allows for a low-order mode expansion.

Straightforward discretization of the Maxwell equations with dispersive materials on an open domain leads to a system that depends nonlinearly on the frequency. The truncation of the domain by a PML and the frequency-dependent materials both introduce such non-linearity. A linear system is desirable in order to leverage methods from computational linear algebra.

If the dispersion of the medium can be approximated by a finite sum of Drude and Lorentz media the material dependence on the frequency can be factored out at the cost of introducing new, auxiliary field variables. Now, a linear system can be obtained if the PML is linearized in a frequency range. However, the linearization of the PML leads to a discrete Maxwell operator with a spectrum that is stable for the frequencies of the same sign as the linearization frequencies and unstable for frequencies of opposite sign. Essentially, due to linearization of the PML, one obtains an outgoing radiation condition for frequencies of one sign and an incoming wave condition for the opposite sign.

We will show how stable field approximations and mode expansions can nonetheless be drawn from such a system and how basic properties of the Maxwell equations can be preserved. Furthermore, by preserving the symmetry of the Maxwell equations throughout the discretization process we show that quasinormal mode expansions can be computed efficiently using the memory efficient Lanczos algorithm.