Waves in Complex Media

and Efficient Ways to Compute Them

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Waves come in all forms and shapes. Some are localized like the waves that make a drum vibrate. Others travel for long distances like the electromagnetic waves that transmit your latest snapchat message to your friend on the other side of this planet. Waves are often used to transport energy or information from one point to another, but can also be used to gather information about the location and constitution of some body or object of interest. With acoustic waves, for example, we can image a fetus inside a mother's pregnant belly, while seismic and electromagnetic waves may be used to image the subsurface of the Earth.

What acoustic, electromagnetic, and elastodynamic waves have in common is that in each case the waves are characterized by two fundamental wave field quantities. For acoustic waves these quantities are the acoustic pressure and the particle velocity, electromagnetic waves are characterized by the electric and magnetic field strength, and for elastodynamic waves the fundamental wave field quantities are the stress and the particle velocity again. In all three cases, these quantities are coupled through a set of first-order partial differential equations, which give a precise description of how first-order spatial variations of one field quantity are coupled to first-order temporal variations of the other quantity. In electromagnetics, for example, the curl or rotation of the magnetic field (a first-order spatial variation) is coupled to first-order temporal variations of the electric field (Maxwell-Ampere law), while the curl of the electric field is coupled to a first-order temporal variation of the magnetic field (Faraday's law).

To compute an acoustic or electromagnetic wave field in a complex medium, the governing wave equations have to be discretized in space resulting in large-scale systems of equations that can only be solved numerically on a computer. What constitutes a large-scale system is actually not precisely defined, but typically we think of systems with at least one million unknowns. In addition, these unknowns are time or frequency dependent and so we are actually dealing with millions of unknowns for each fixed time instant or frequency of interest. It is not difficult to imagine that computing an acoustic or electromagnetic wave field throughout a three-dimensional volume and on large time intervals or wide frequency ranges of interest is a formidable task.

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Fortunately, in practice we can exploit the symmetry of the first-order wave equations in our computations and particular properties of the configuration or device of interest can be exploited as well. For example, energy conservation is related to a particular symmetry property of the wave equations and wave field reciprocity is linked to another symmetry property. Both of these properties can be used to develop very efficient solution strategies for large-scale wave field computations. When modeling wave propagation in complex media (or any other physical phenomenon) all relevant physical laws must be satisfied, of course, and symmetry that follows from these laws can be exploited to efficiently compute the wave field quantities of interest.

As an illustration of how we can exploit certain properties of a given configuration to efficiently compute required field responses, consider the configuration shown in Figure 1. The cylindrical bar in this figure is a golden nanobar (note the scale) and the small arrow located just above the bar represents an electric dipole source. This simple dipole model can be used to compute the spontaneous decay rate of a two-level quantum

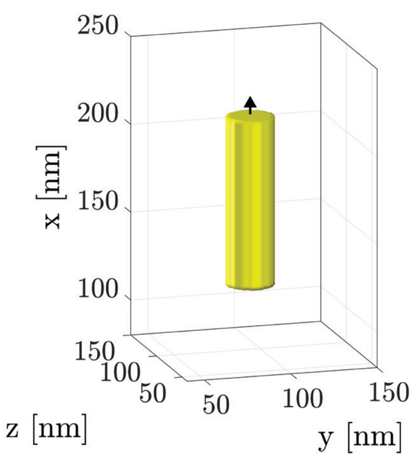

Figure 1. *Quantum emitter (arrow) located above a golden cylindrical nanorod (diameter 30nm, length 100nm). The emitter is located 10nm above the rod.*

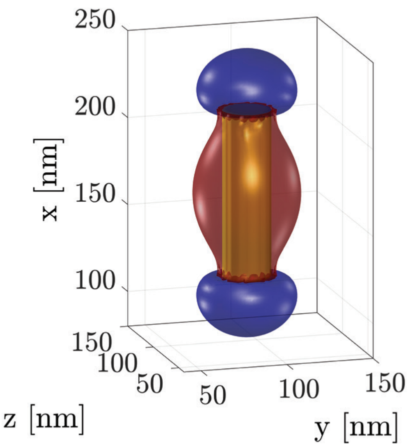

Figure 2. *Magnitude of the x-component of the electric field of the dominant quasi-normal mode.*

emitter (as represented by the dipole source). This decay rate depends on the surroundings of the quantum emitter and can be enhanced by placing the emitter in the neighborhood of the golden bar as illustrated in Figure 1. Enhancing the decay rate of a quantum emitter is exploited in many different areas in nano-optics and is utilized in light emitting diodes (LEDs), for example.

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To determine this possible enhancement using classical electromagnetic field theory, it can be shown that the imaginary part of the projection of the electric field strength onto the electric dipole moment at the location of the dipole is required over a frequency interval for which enhancement is expected [1]. Now gold is a dispersive material meaning that its reaction to the presence of an electromagnetic field varies with frequency. Therefore, a straightforward approach would be to select a frequency from the interval of interest, compute the electric field strength at this frequency by solving the large-scale discretized Maxwell system for this configuration and repeat this procedure for all frequencies in the frequency interval. By following this approach, N large-scale Maxwell systems need to be solved for N frequencies of interest leading to prohibitively long computation times.

Fortunately, the spontaneous decay rate is essentially determined by a single so-called quasi-normal mode over the complete frequency interval of interest. This mode is a characteristic mode or natural vibration of the golden nanobar that is excited by the dipole source. The magnitude of the x-component of the electric field of this mode is illustrated in Figure 2. Therefore, a much more efficient approach is to compute this mode since then we are essentially done in one step. By exploiting the symmetry properties of Maxwell's equations this dominant field mode (and additional modes as well) can be efficiently computed. In other words, by exploiting physics-based symmetry the same spontaneous decay rate results can be obtained as with the above-mentioned brute-force approach, but at significantly reduced costs. The spontaneous decay rate of the emitter in case the nanobar is present normalized to the decay rate in case the emitter is absent is shown in Figure 3. The presence of the nanobar clearly enhances the decay rate of the quantum emitter on the frequency (wavelength) interval of interest.

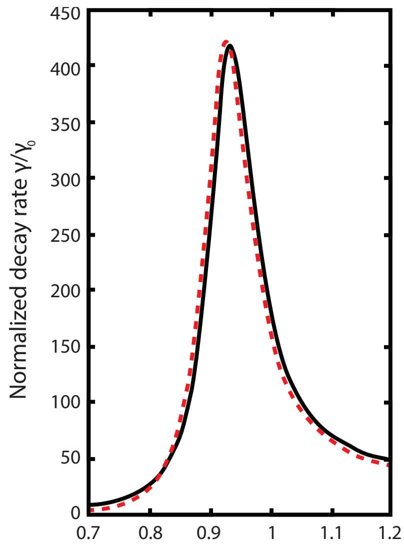

Figure 3. *Normalized spontaneous decay rate on wavelength interval of interest.*

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[1] L. Novotny, B. Hecht, Principles of Nano-Optics, Second Edition, Cambridge University Press, 2012.

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