Space-time block codes for MIMO systems

1. Array and diversity gain

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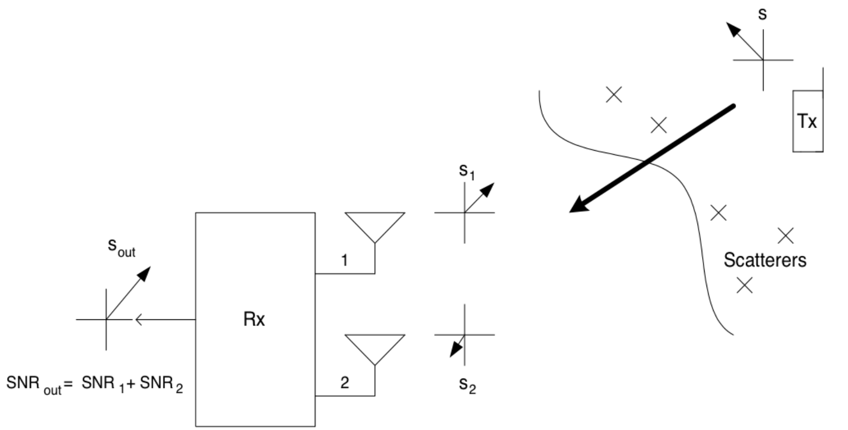
Space-time processing - array gain

Consider a SIMO system with M_r receive antennas and 1 transmit antenna



Received signal $\mathbf{x} = \mathbf{h}s + \mathbf{n}$ processed by a linear filter

$$\mathbf{w}^H\mathbf{x} = \mathbf{w}^H\mathbf{h}s + \mathbf{w}^H\mathbf{n}.$$

$\mathbf{h} = [h_1, \ldots, h_m]^T$ is the channel between the transmitter and receiver array.

Matched filter, $\mathbf{w} = \mathbf{h}$, maximizes the output SNR (*maximum ratio combining*):

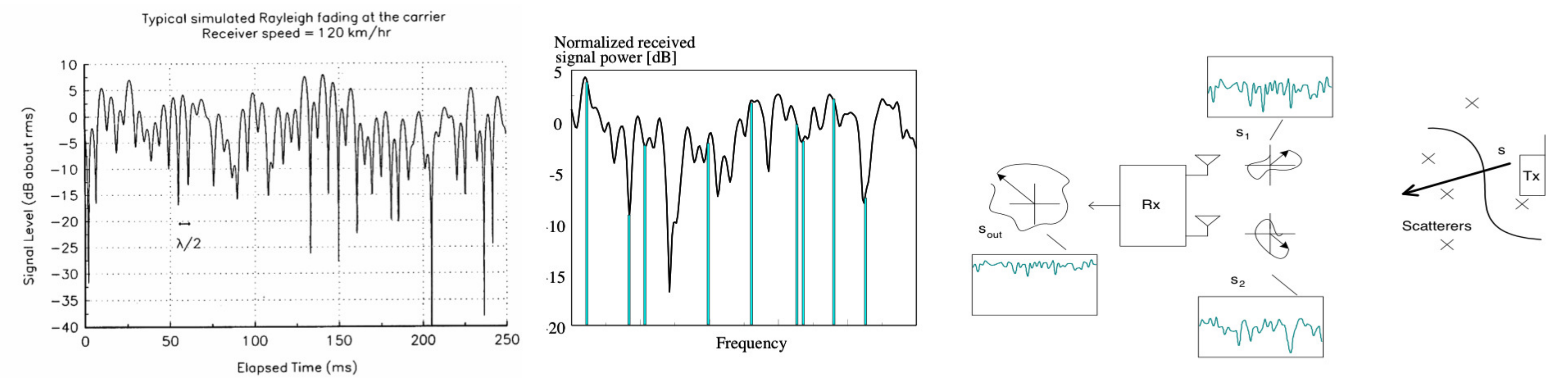
$$\mathrm{SNR}_{out} = \frac{\|\mathbf{h}\|_2^2}{\sigma^2} = \sum_{m=1}^{M_r} \frac{|h_m|^2}{\sigma^2}.$$

The output SNR increases with the number of antennas, and is called *array gain*.

Diversity gain

Signal power in a wireless channel fluctuates (or “fades") with time/frequency/space.

Diversity is used to combat fading - "independent" fading links are combined.



- **Time diversity**: successive transmission of the same symbol (reduces symbol rate, more energy per symbol)

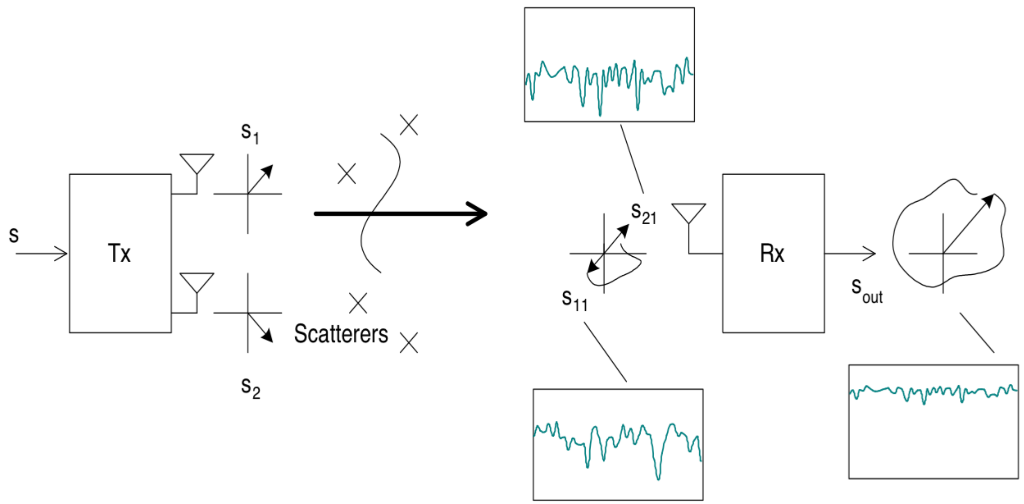
- **Frequency diversity**: transmission of the same narrowband signal on different frequencies (requires more bandwidth and power)

- **Spatial (receiver) diversity**: requires more hardware, but no extra bandwidth or time required

Diversity gain is related to the number of independent fading branches.

Transmit diversity

Consider a MISO system with 1 receive antenna and M_t transmit antennas



Diversity is created by transmit symbols over multiple antennas.

Assuming the “channel state information" is known, transmit symbol at antenna m is precoded as $s_m = \bar{h}_m s$ / || h ||

$$\mathbf{s} = \bar{\mathbf{h}} s \text{ / || h ||}$$

h_m is the channel between antenna m and the receiver with $\mathbf{h} = [h_1, \ldots, h_{Mt}]^T$.

TUDelft

Transmit diversity

The received signal

$$x = \sum_{m=1}^{M_t} h_m s_m + n = \mathbf{h}^T \mathbf{s} + n$$

has an output SNR

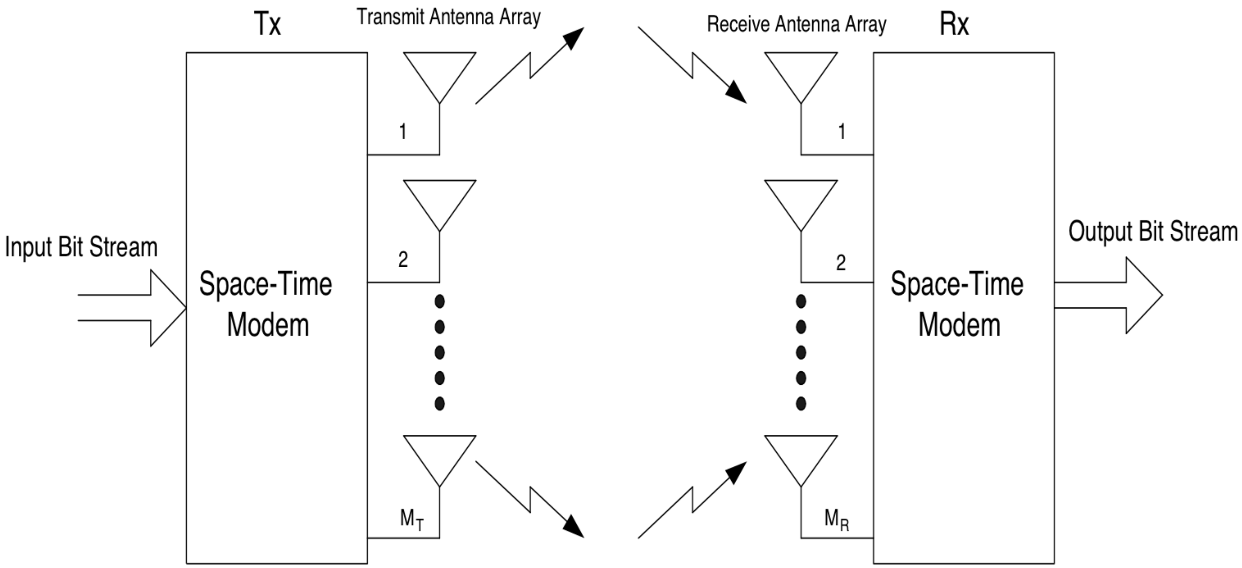
$$\mathrm{SNR}_{out} = \sum_{m=1}^{M_t} \frac{|h_m|^2}{\sigma^2}$$

This results in the same performance as the *maximum ratio combiner*.

- Knowing the channel state information (CSI) at the transmitter is not easy (feedback link required)
- If channel state information is not known at the transmitter, and the powers are randomly allocated, no diversity gain is achieved.

MIMO system

Consider a MIMO system with M_t transmit antennas and M_r receive antennas



Signal received $\mathbf{x} \in \mathbb{C}^{M_r \times 1}$ at the receive antenna array

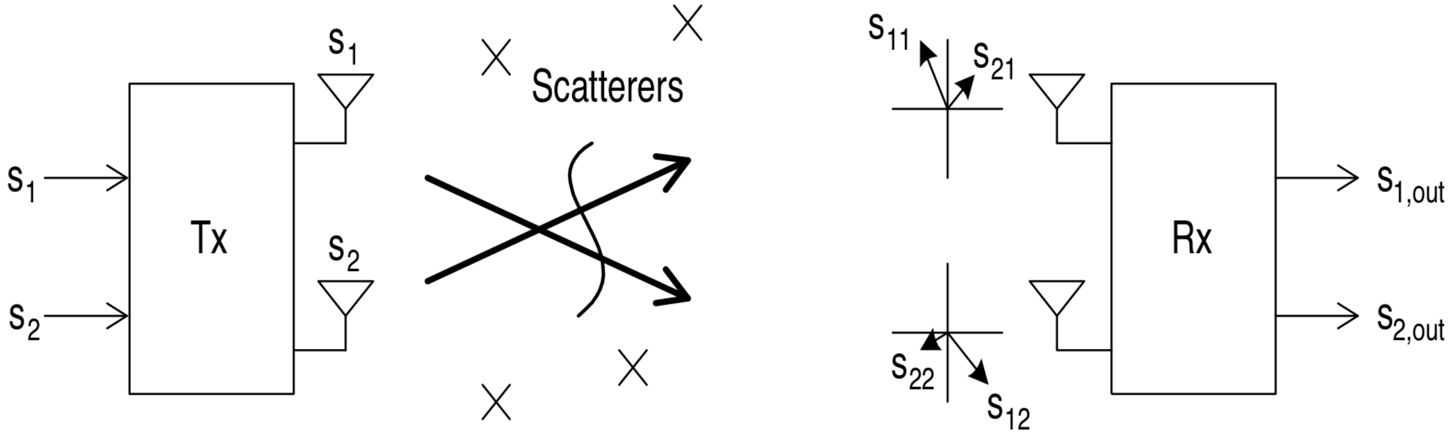
$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

$\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the MIMO channel matrix with (i, j)th entry $h_{i,j}$

$\mathbf{s} \in \mathbb{C}^{M_t \times 1}$ contains the data symbols

Multiplexing gain

Obtain independent channels in MIMO system



- Suppose the channel state information is known both at the transmitter and reciver
- Let $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ be the singular value decomposition of $\mathbf{H}$.
- Precode symbols as $\mathbf{s} = \mathbf{V}\tilde{\mathbf{s}}$ and on the receiver side reshape the signal as $\tilde{\mathbf{x}} = \mathbf{U}^H\mathbf{x}$.

Multiplexing gain

After transmitter precoding and receiver shaping

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad \Rightarrow \quad \tilde{\mathbf{x}} = \mathbf{U}^H \mathbf{x} = \mathbf{U}^H (\mathbf{H}\mathbf{s} + \mathbf{n}) = \mathbf{\Sigma}\tilde{\mathbf{s}} + \mathbf{U}^H \mathbf{n}$$

Suppose $\mathbf{H}$ has a rank $R \leq \min\{M_r, M_t\}$: rich scattering is good to have a well conditioned $\mathbf{H}$

$$\left[\begin{array}{c} \tilde{\mathbf{x}} \\ \hline \mathbf{0} \end{array}\right] = \left[\begin{array}{ccc|c} \sigma_1 & & & \\ & \ddots & & \mathbf{0} \\ & & \sigma_R & \\ \hline & \mathbf{0} & & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \tilde{\mathbf{s}} \\ \hline \mathbf{0} \end{array}\right] + \tilde{\mathbf{n}}$$

This is equivalent to R “parallel" SISO channels.

The noise covariance matrix of $\tilde{\mathbf{n}} = \mathbf{U}^H \mathbf{n}$ is same as that of $\mathbf{n}$ as $\mathbf{U}$ is unitary

$$E\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\} = E\{\mathbf{U}^H \mathbf{n} (\mathbf{U}^H \mathbf{n})^H\} = \sigma^2 \mathbf{I}$$

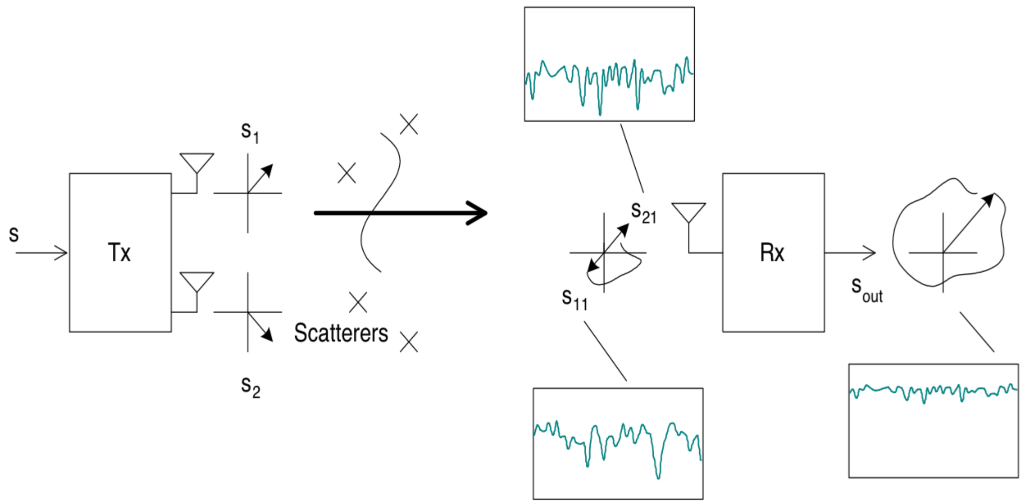
Channel gains σ_i are typically all different. Power allocation is required, e.g., via waterfilling. Bad channels are not used.

Space-time coding - with no CSI

Suitable design of transmit signal (codebook design) can lead to transmit diversity, even without knowing the channel at the transmitter.

Decoding should be using a linear filter.

Consider a MISO channel with $M_r = 1$ and $M_t = 2$.



Let $\mathbf{S} \in \mathbb{C}^{T \times M_t}$ be the codeword spanning T samples.

Space-time coding - Alamouti code

In time slot 1, transmit: $\mathbf{s} = [s_1, s_2]^T$ so that the received signal is

$$x_1 = h_1 s_1 + h_2 s_2 + n_1$$

In time slot 2, transmit: $\mathbf{s} = [-\bar{s}_2, \bar{s}_1]^T$ so that the received signal is

$$x_2 = -h_1 \bar{s}_2 + h_2 \bar{s}_1 + n_2 \Rightarrow \bar{x}_2 = -\bar{h}_1 s_2 + \bar{h}_2 s_1 + \bar{n}_2$$

The codebook is

$$\mathbf{S} = \begin{bmatrix} s_1 & -\bar{s}_2 \\ s_2 & \bar{s}_1 \end{bmatrix}$$

Since two symbols are transmitted in two time-slots, this is full rate (rate 1) code. In general, a code that encodes k symbols in T slots has a code rate k/T.

Since, $\mathbf{S}^H \mathbf{S} = \alpha \mathbf{I}$ with $\alpha = |s_1|^2 + |s_2|^2$, such codes are called orthogonal codes.

Space-time coding - Alamouti code

The received signal

$$\begin{bmatrix} x_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ \bar{h}_2 & -\bar{h}_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ \bar{n}_2 \end{bmatrix} \Leftrightarrow \mathbf{x}' = \mathbf{H}'\mathbf{s} + \mathbf{n}'$$

The channel matrix is now orthogonal and is assumed to be known at the receiver.

Therefore, the receiver simply implements

$$\hat{\mathbf{s}} = \mathbf{H}'^T \mathbf{x}'$$

The power is split equally across the transmit antennas, so SNR per symbol is given by

$$\mathrm{SNR}_i = \frac{|h_1|^2 + |h_2|^2}{2\sigma^2}$$

Space-time coding - Alamouti code

For $M_t > 2$, full rate codes are not available, in general, e.g., for $M_t = 3$

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -\bar{s}_2 & \bar{s}_1 & \frac{s_3}{\sqrt{2}} \\ \frac{\bar{s}_3}{\sqrt{2}} & \frac{\bar{s}_3}{\sqrt{2}} & \frac{-s_1-\bar{s}_1+s_2-\bar{s}_2}{2} \\ \frac{\bar{s}_3}{\sqrt{2}} & -\frac{\bar{s}_3}{\sqrt{2}} & \frac{s_2+\bar{s}_2+s_1-\bar{s}_1}{2} \end{bmatrix}$$

with $k/T = 3/4$.

It has uneven power among the symbols it transmits: the signal does not have a constant envelope and that the power each antenna must transmit has to vary, both of which are undesirable.

To avoid such issues, quasi-orthogonal codewords are available (have full rate with linear decoding).