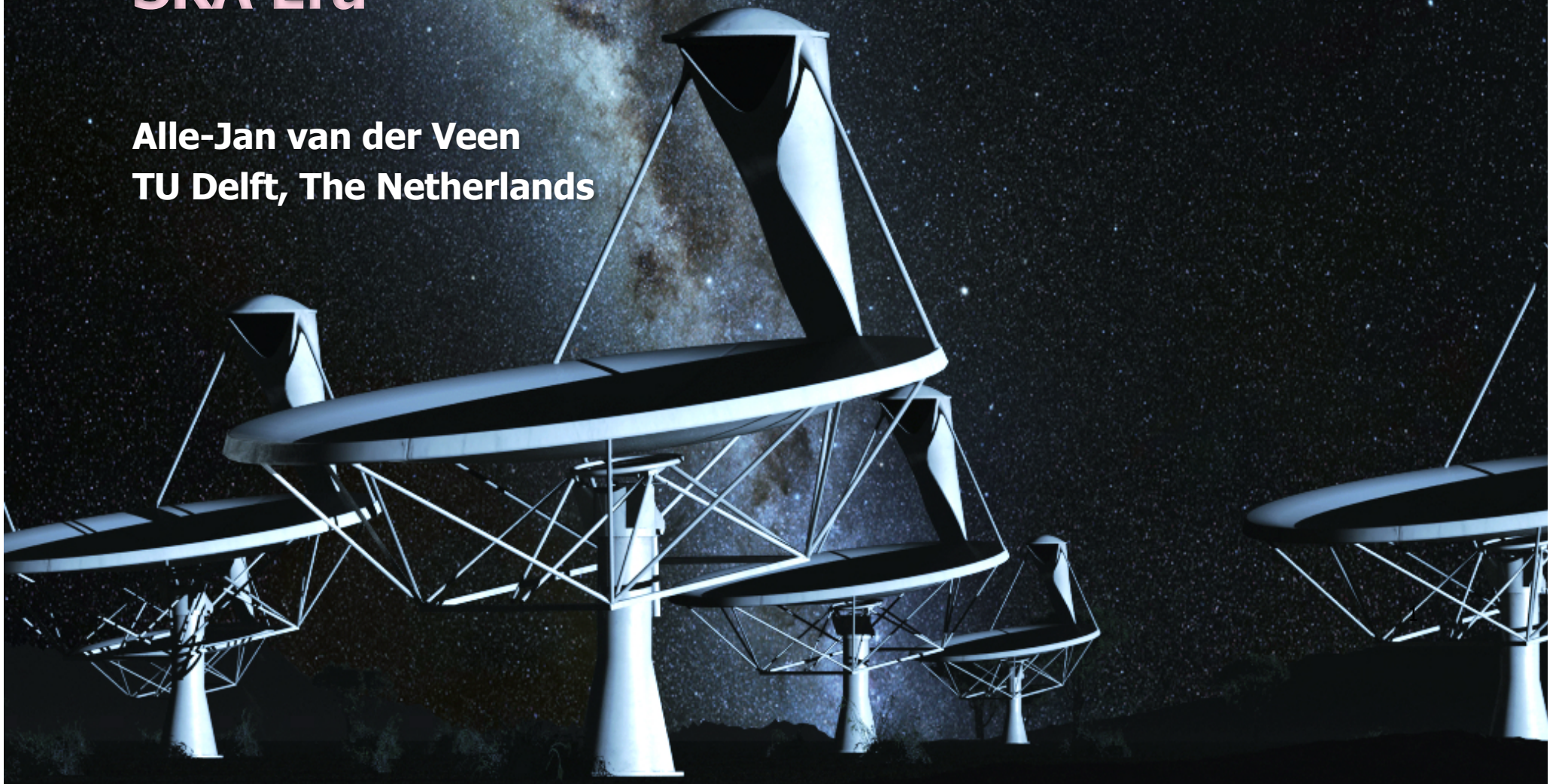


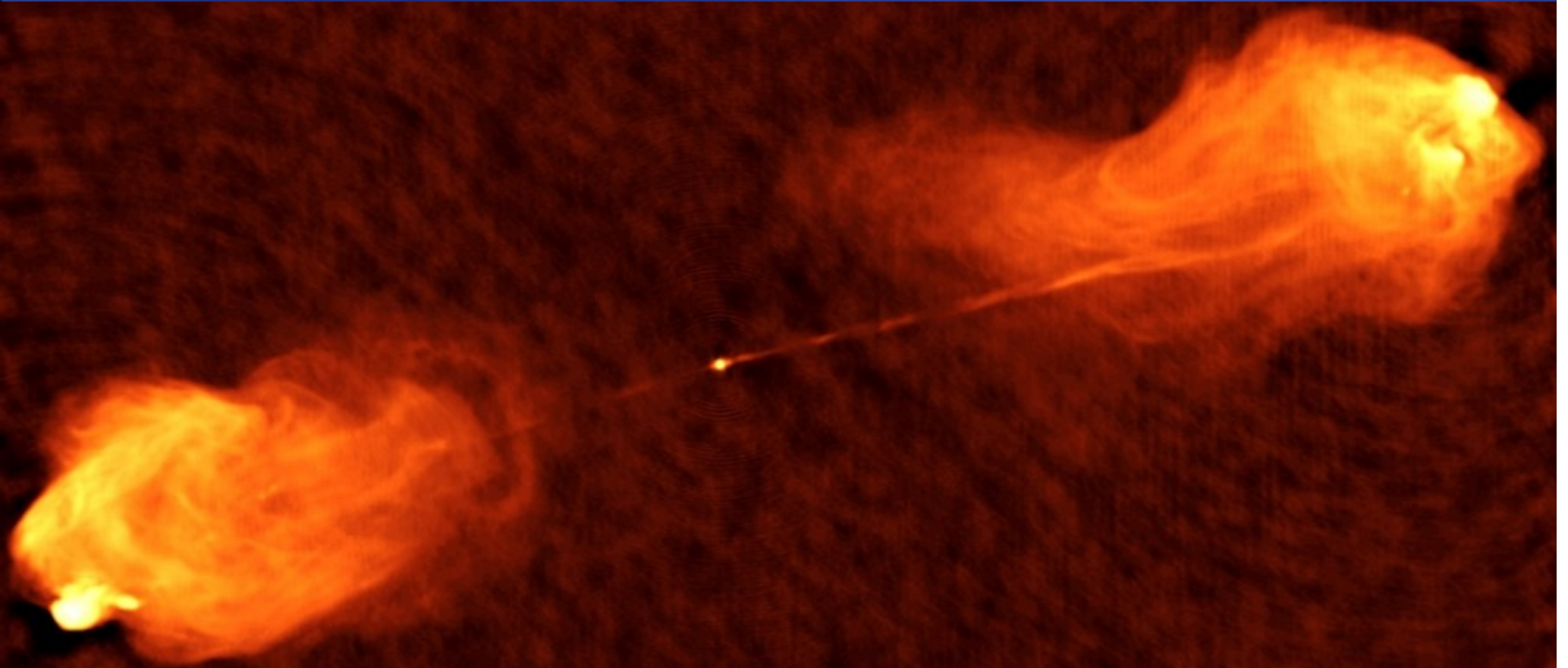
# Radio Astronomy Image Formation in the SKA Era

**Alle-Jan van der Veen**  
**TU Delft, The Netherlands**



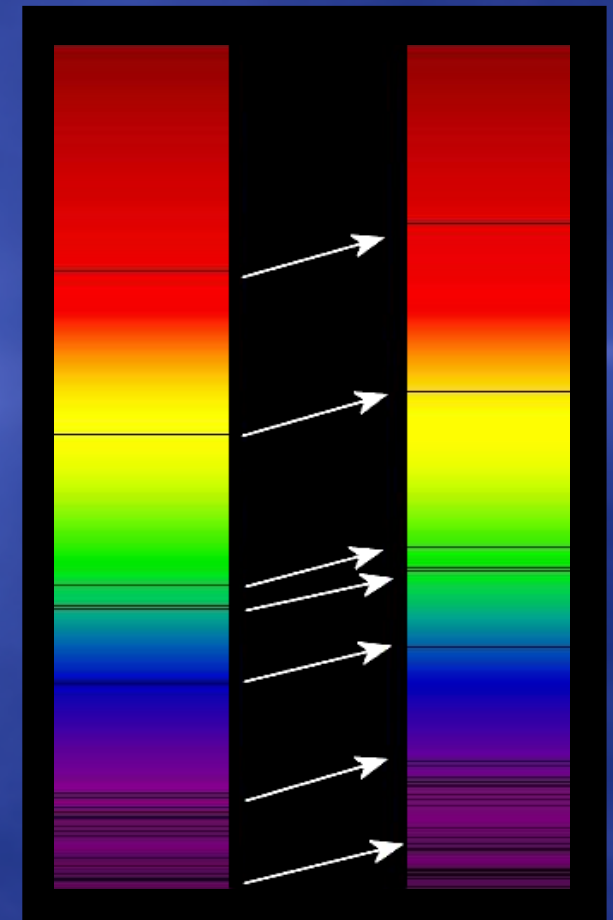
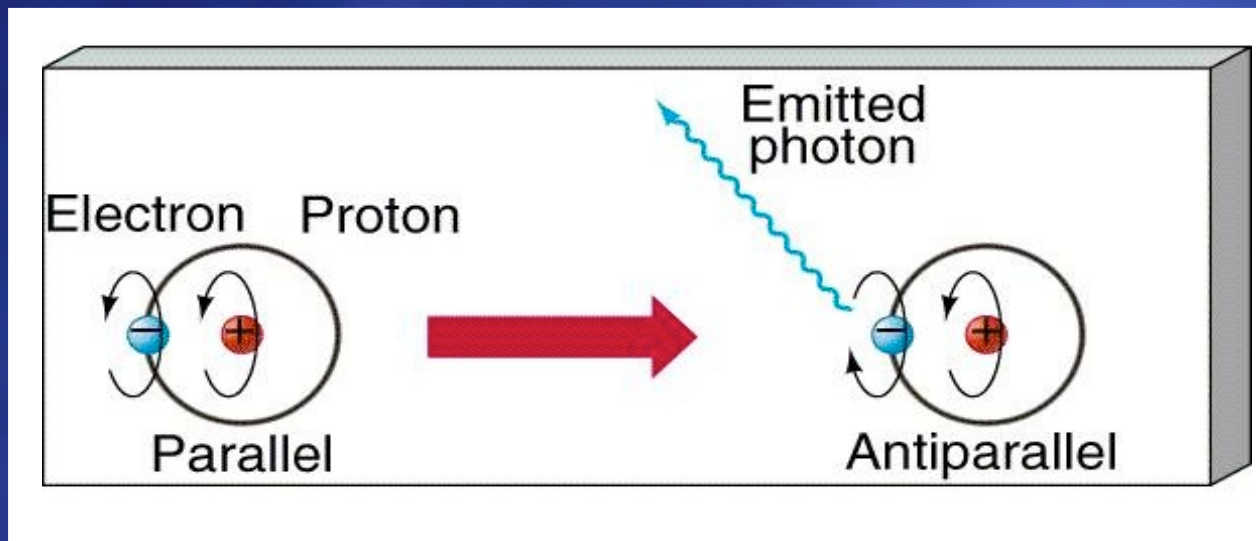
# Introduction—the science

- Cygnus A (a quasar)

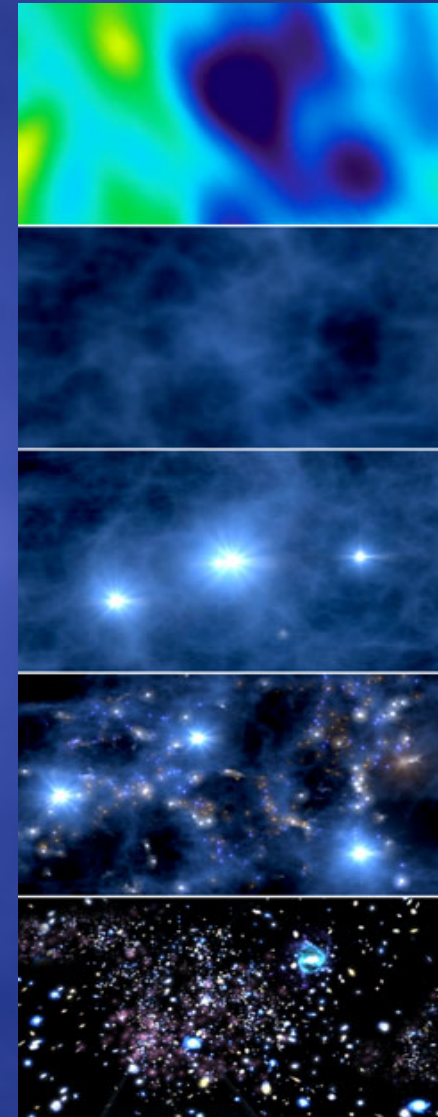
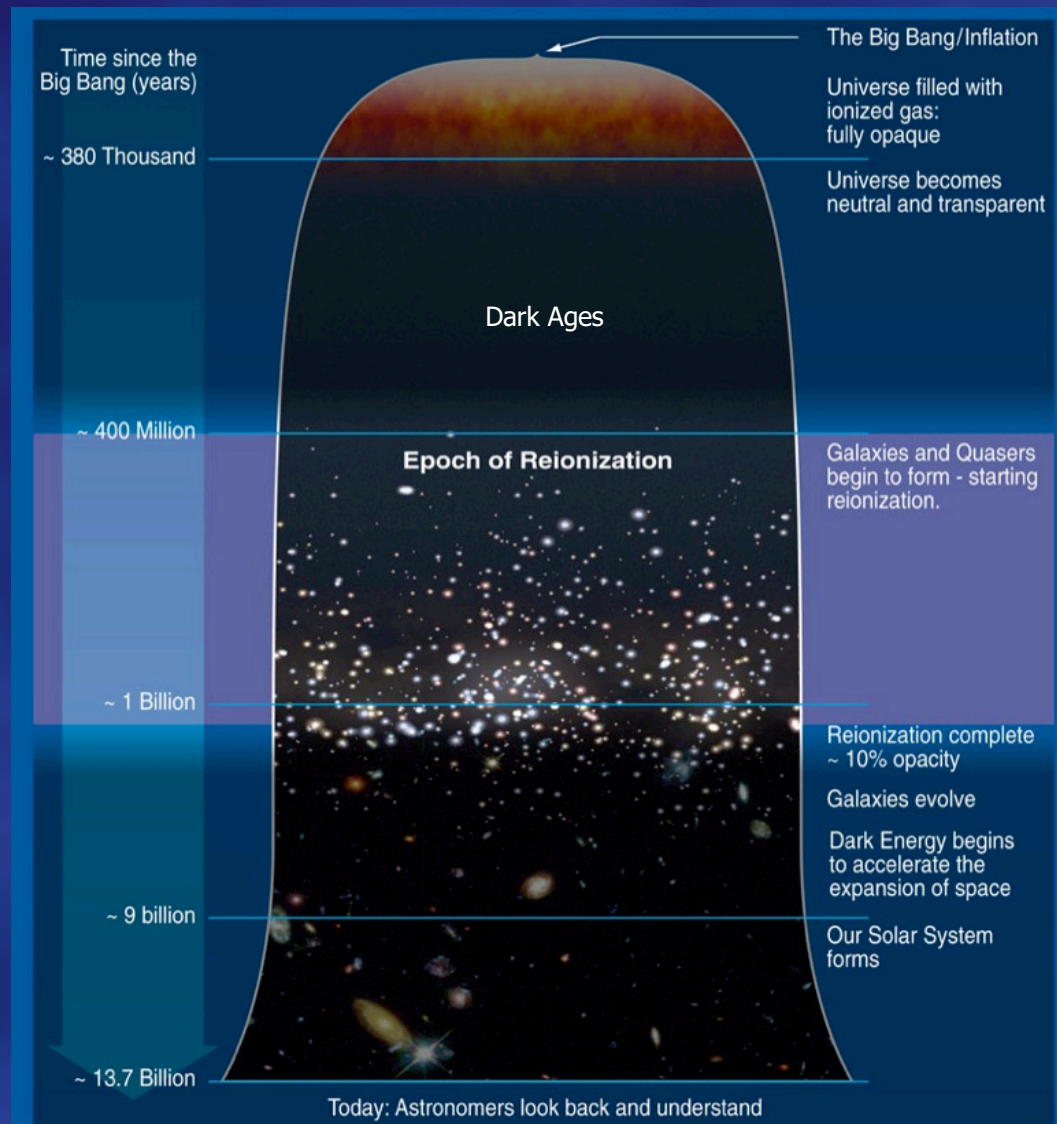


# Introduction—the science

- Blackbody radiation; synchrotron radiation
- Spectral line due to neutral hydrogen (1420 MHz)
- Redshift (“Doppler”)—up to a factor 10!

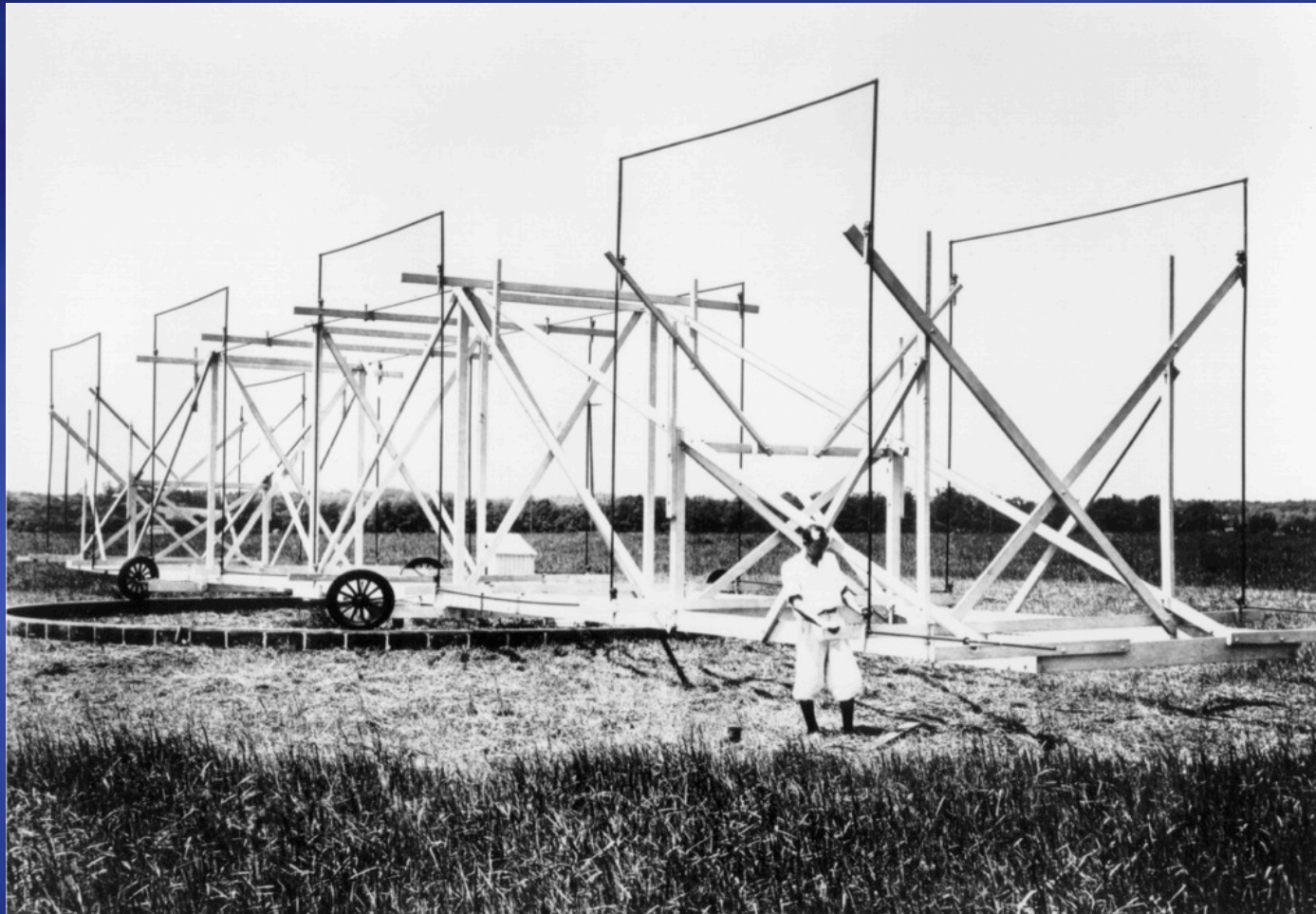


# Introduction—the science



# Introduction—the instruments

- Karl Jansky, Bell Labs (1928), 20 MHz



# Introduction—the instruments

- Oort/van der Hulst (1950s), prediction and discovery of the HI 21 cm line (1420 MHz)



- Dwingeloo (Netherlands), 1956



- Jodrell Bank (Manchester, UK), 1957  
76 m

# Introduction—the instruments

- Effelsberg, Germany (1972) 100m
- Arecibo, Puerto Rico (1960) 305m
- “FAST”, Guizhou, China (2016), 500m



# Introduction—the instruments

- Westerbork, Netherlands (1970), 14 dishes



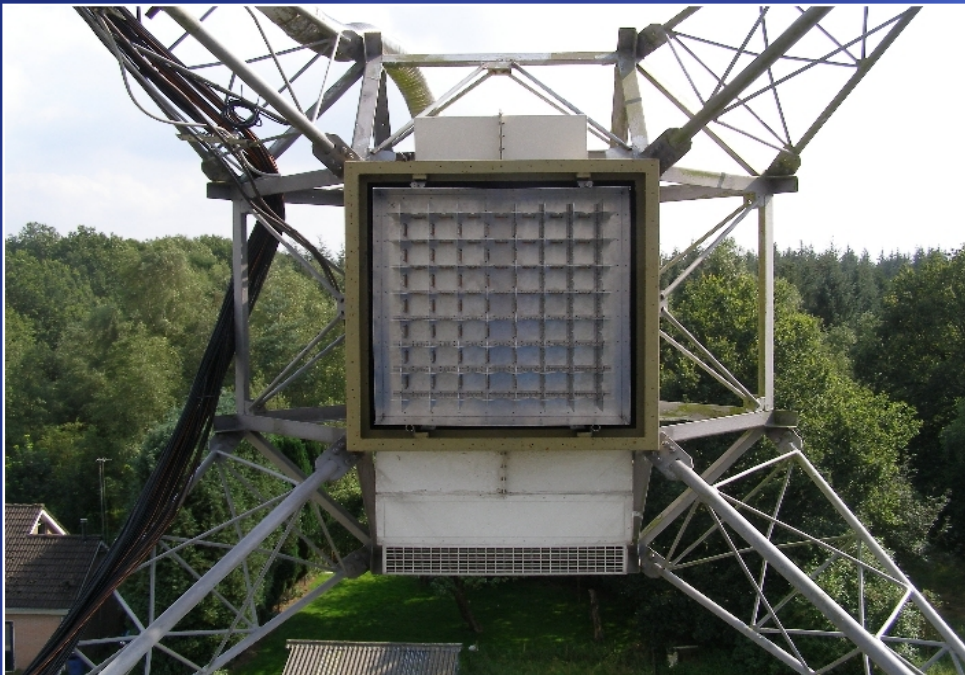
- Very Large Array, New Mexico (1980), 27 dishes



# New instruments – Focal Plane Arrays

## Westerbork upgrade (2017)

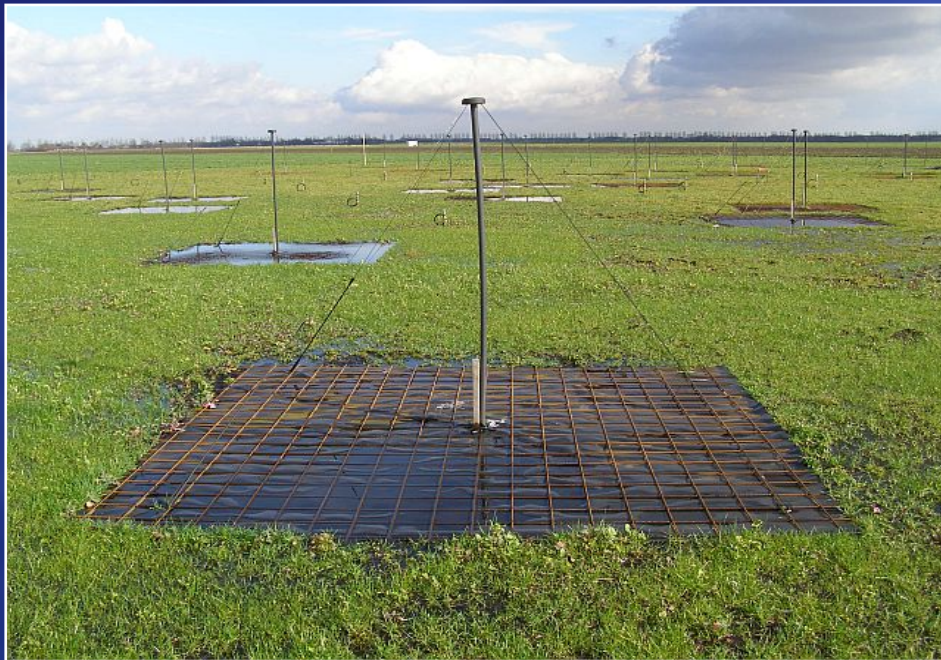
- 8 x 8 phased array
- 37 beams for faster surveys
- Beamshape control



# New instruments – Aperture Arrays

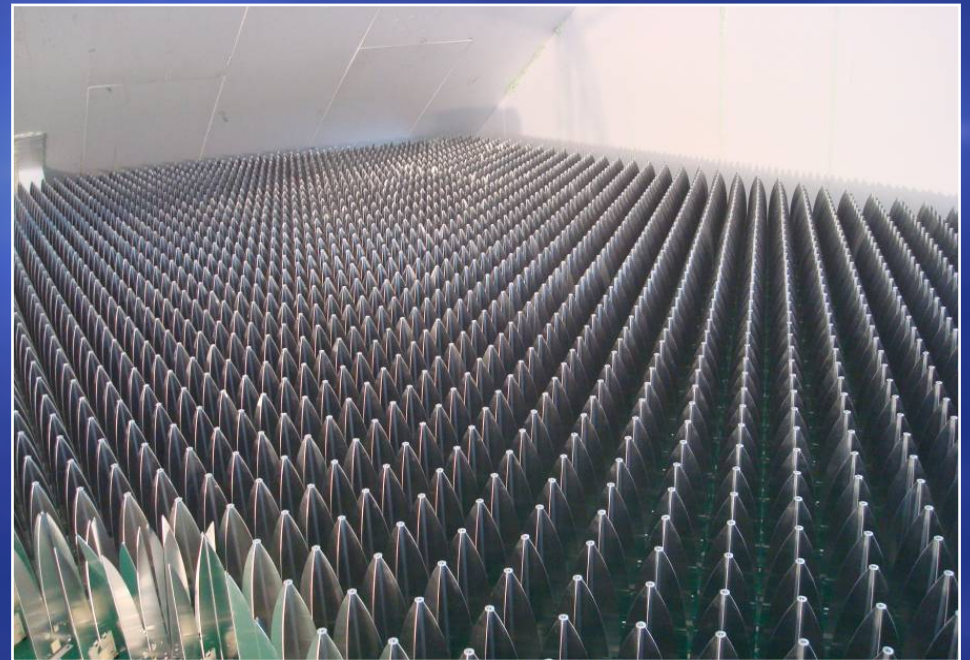
## The new era: massive phased arrays

(a) Sparse phased arrays



- LOFAR (Europe), 100,000 antennas

(b) Dense phased arrays



- EMBRACE (NL), prototype AA

# LOFAR radio telescope array (2008)

## Design parameters

- 38 stations (Netherlands)
- 13+ additional stations in Germany, UK, Sweden, France, Poland, Ireland, ...
- ~1000 antenna elements/station
- Dense core, 24 stations within 3.5 km
- Lowband array: 10-90 MHz
- Highband array: 110-240 MHz
- Resolution ~1 kHz
- 8 simultaneous beams
- Up to 48 MHz per beam

Designed and operated by ASTRON



# LOFAR radio telescope array

## Station parameters

- Each station has 2 high band array substations, and 96 low band antennas
- Beamformed output mimicks a telescope dish
- 24 core stations
- 14 NL remote stations
- 13+ international stations



# LOFAR core stations

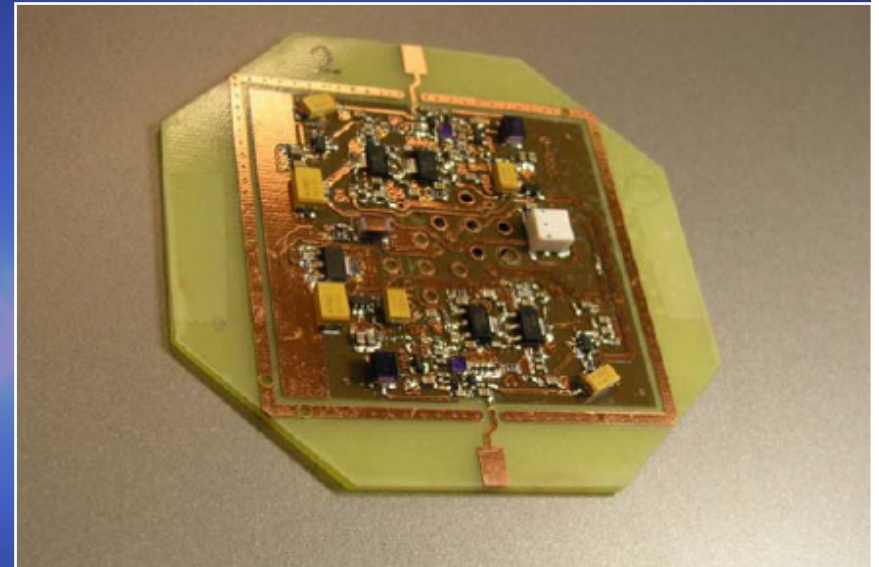
## Core station parameters

- `Superterp': 6 stations, diameter 300m
- 24 core stations within 3.5 km



# LOFAR lowband antennas

- 10-90 MHz
- Dipole antenna, 96 dipoles per station



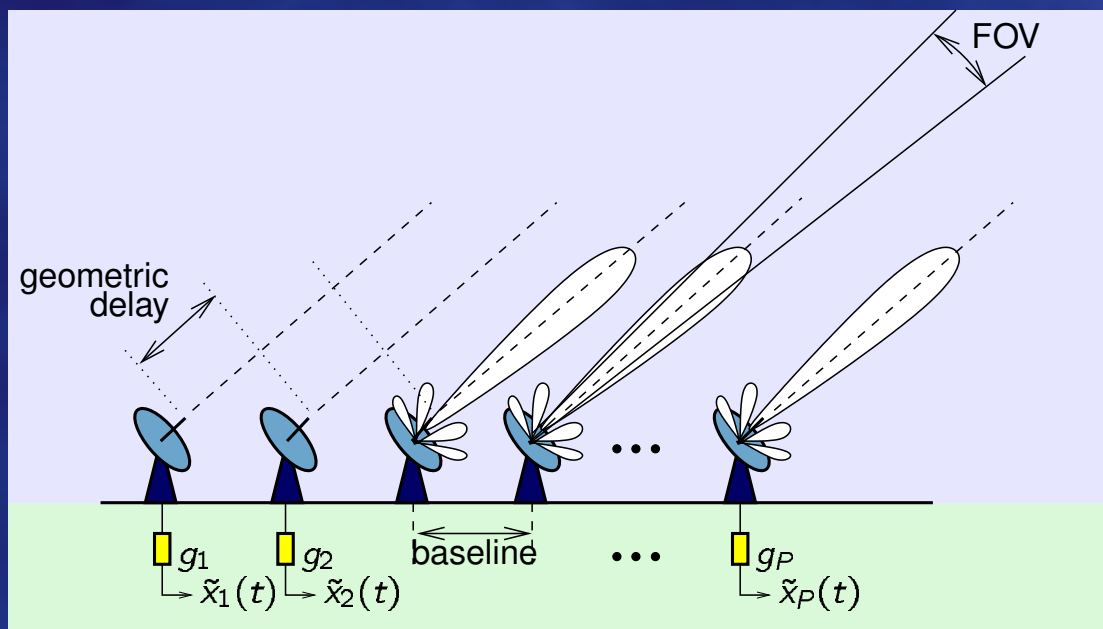
# LOFAR high-band array

- 120-240 MHz
- Each tile is 5 x 5 meter and has 4 x 4 'spider' antennas encapsulated in polystyrene
- 24 tiles form a sub-station
- 2 substations per station (total 768 antennas)



# Basic concepts of interferometry

## Observations



Consider  $P$  antennas,  $Q$  sources,  $N$  samples per short-term interval (STI), a single subband:

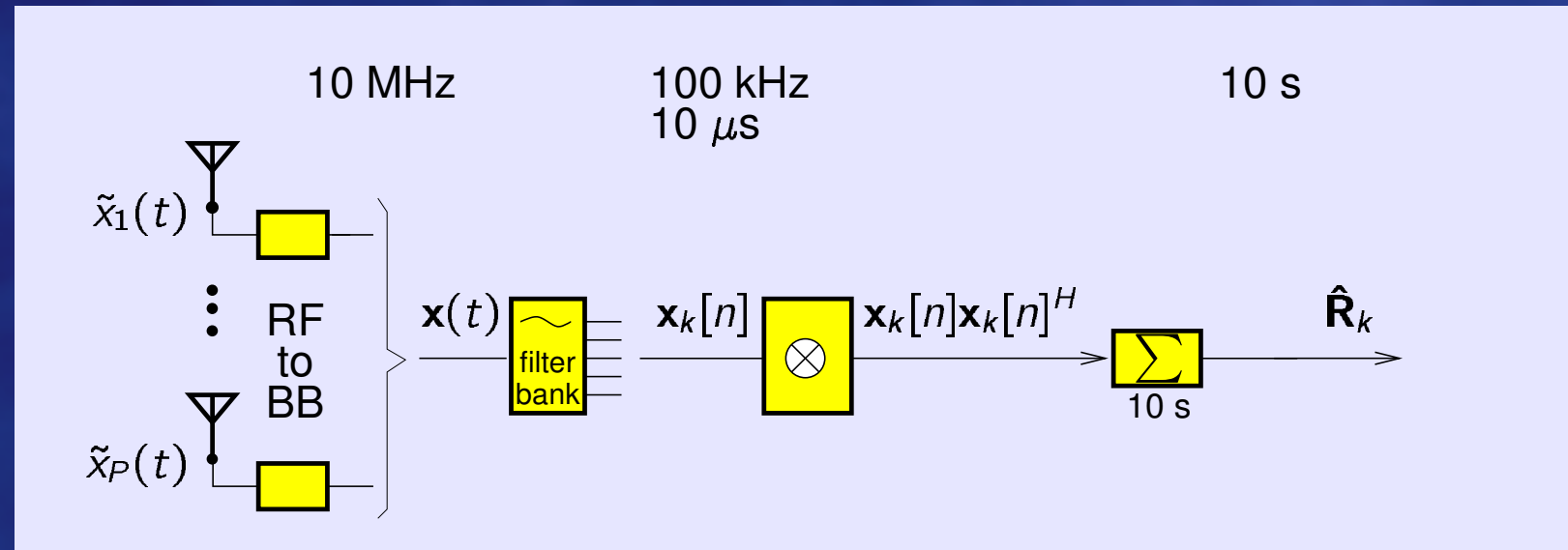
$$\mathbf{x}_k[n] = \mathbf{A}_k \mathbf{s}_k[n] + \mathbf{n}_k[n], \quad n = 1, \dots, N$$

where  $\mathbf{A}_k = [\mathbf{a}_1, \dots, \mathbf{a}_Q]$  of size  $P \times Q$  is the array response matrix.

Index  $k$  represents “slow time” and/or frequency index.

# Basic concepts of interferometry

## Correlations



From the measurements, compute

$$\hat{\mathbf{R}}_k = \frac{1}{N} \sum_{n=1}^N \mathbf{x}_k[n] \mathbf{x}_k[n]^H$$

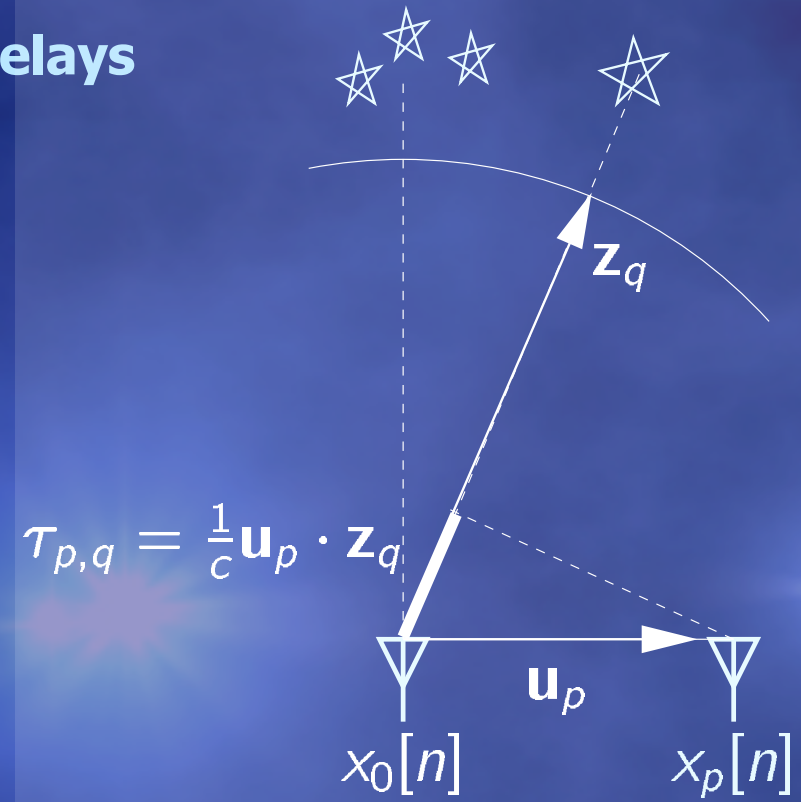
A filter bank splits the signal into subbands to satisfy narrowband conditions.

The stationarity of  $\mathbf{R}_k$  is limited due to earth rotation.

# Basic concepts of interferometry

## Array response due to geometric delays

- $\mathbf{u}_p$ : antenna position in meter (time varying as the earth rotates)
- $\mathbf{z}_q$ : source direction (unit vector with direction cosines)



Ideally, the response to a source only depends on the geometric delays, which, under narrowband conditions, can be represented by phase shifts:

$\mathbf{A}_k$  has entries  $a_{p,q} = e^{-j2\pi f_c \tau_{p,q}} = e^{-j \frac{2\pi}{\lambda} \mathbf{u}_p^T \mathbf{z}_q}$

# Basic concepts of interferometry

## Narrow-band and stationarity conditions

$$a_{p,q} = e^{-j2\pi f_c \tau_{p,q}} = e^{-j \frac{2\pi}{\lambda} \mathbf{u}_p^T \mathbf{z}_q}$$

- **Narrowband condition:**  $f_c \tau \approx \text{constant}$  for  $f_c \in (f_{\min}, f_{\max})$   
 $\Rightarrow$  Determines maximum processing bandwidth, function of array diameter  $D$
- **Stationarity condition:**  $f_c \tau \approx \text{constant}$  while  $\mathbf{u}_p$  rotates (earth rotation)  
 $\Rightarrow$  Determines maximum processing time (STI), also function of array diameter
- In practice, also **calibration parameters** are introduced to model antenna gains, amplifier gains, ionosphere,  $\dots$ .

# Basic concepts of interferometry

## Measurement equation

The model for  $\hat{\mathbf{R}}_k$  is

$$\mathbf{R}_k := E\{\mathbf{x}_k[n]\mathbf{x}_k^H[n]\} = \mathbf{A}_k \boldsymbol{\Sigma}_s \mathbf{A}_k^H + \boldsymbol{\Sigma}_n$$

$\boldsymbol{\Sigma}_s = \text{diag}\{\boldsymbol{\sigma}_s\}$ : source powers corresponding to the directions in  $\mathbf{A}_k$ ,

$\boldsymbol{\Sigma}_n = \text{diag}\{\boldsymbol{\sigma}_n\}$ : noise covariance (diagonal, known from calibration).

## Model options

- $Q$  point sources (small number), each represented by one column in  $\mathbf{A}_k(\boldsymbol{\theta})$ , where  $\boldsymbol{\theta}$  are the DOAs — "gridless";
- Alternatively, each pixel contains a source (possibly with zero power), hence  $\mathbf{A}_k$  is known and can be very wide. E.g.,  $Q = 3000 \times 3000$  pixels.  
We will consider this case.

# Basic concepts of interferometry

## Imaging problem formulation

- Data model:

$$\mathbf{R}_k = \mathbf{A}_k \boldsymbol{\Sigma}_s \mathbf{A}_k^H + \boldsymbol{\Sigma}_n$$

- Without the noise, a single entry of  $\mathbf{R}_k$  (called a *visibility*) is

$$V_{ij,k} := (\mathbf{R}_k)_{ij} = \sum_{q=1}^Q \sigma_q^2 a_{i,q} \overline{a_{j,q}} = \sum_{q=1}^Q I(\mathbf{z}_q) e^{-j \frac{2\pi}{\lambda} (\mathbf{u}_i(k) - \mathbf{u}_j(k))^T \mathbf{z}_q}.$$

This is called the *measurement equation* and has the form of a Fourier transform.

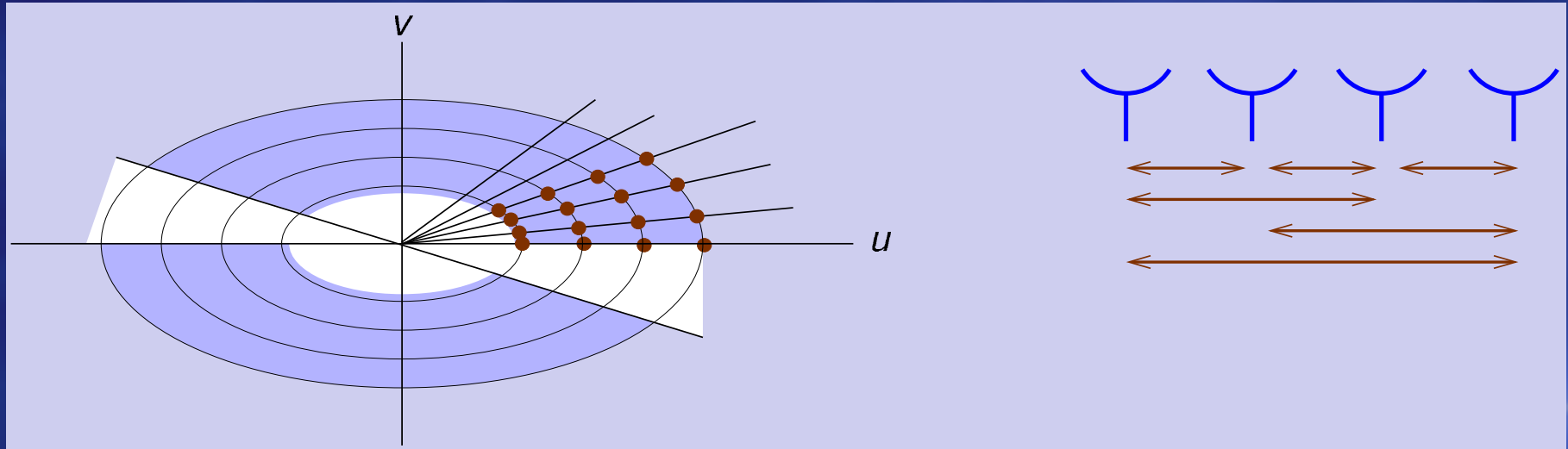
- $I(\mathbf{z}_q) = \sigma_q^2$  is the brightness (power) in direction  $\mathbf{z}_q$ .

For a discrete point-source model:  $I(\mathbf{z}) = \sum_{q=1}^Q \sigma_q^2 \delta(\mathbf{z} - \mathbf{z}_q)$ .

$I(\mathbf{z})$  is the image (called the *map*).

# Basic concepts of interferometry

## Baseline samples



- $\mathbf{u}_{ij,k} = \frac{2\pi}{\lambda}(\mathbf{u}_i(k) - \mathbf{u}_j(k))$  is called a *baseline*, with vector coordinates  $\mathbf{u} = (u, v, w)$ .
- With  $P$  antennas, we have at most  $P^2$  independent baselines (visibility samples). But we obtain more samples as the earth rotates or if we consider multiple frequencies (scaling by  $\lambda$ ).

# Basic concepts of interferometry

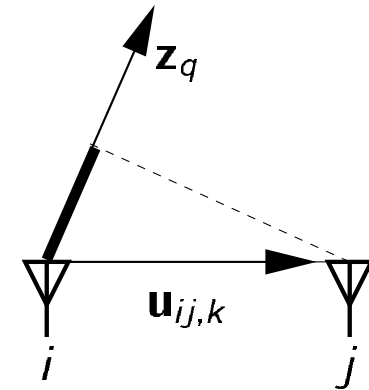
## Fourier-based image formation

- Measurement equation:

$$V_{ij,k} = \sum_{q=1}^Q I(\mathbf{z}_q) e^{-j\mathbf{u}_{ij,k}^T \mathbf{z}_q}$$

- Try inverting the Fourier transform:

$$I_D(\mathbf{z}) := \sum_{i,j,k} V_{ij,k} e^{j\mathbf{u}_{ij,k}^T \mathbf{z}}$$



This *dirty image* is not quite equal to the true image  $I(\mathbf{z}) = \sum_{q=1}^Q I(\mathbf{z}_q) \delta(\mathbf{z} - \mathbf{z}_q)$ .

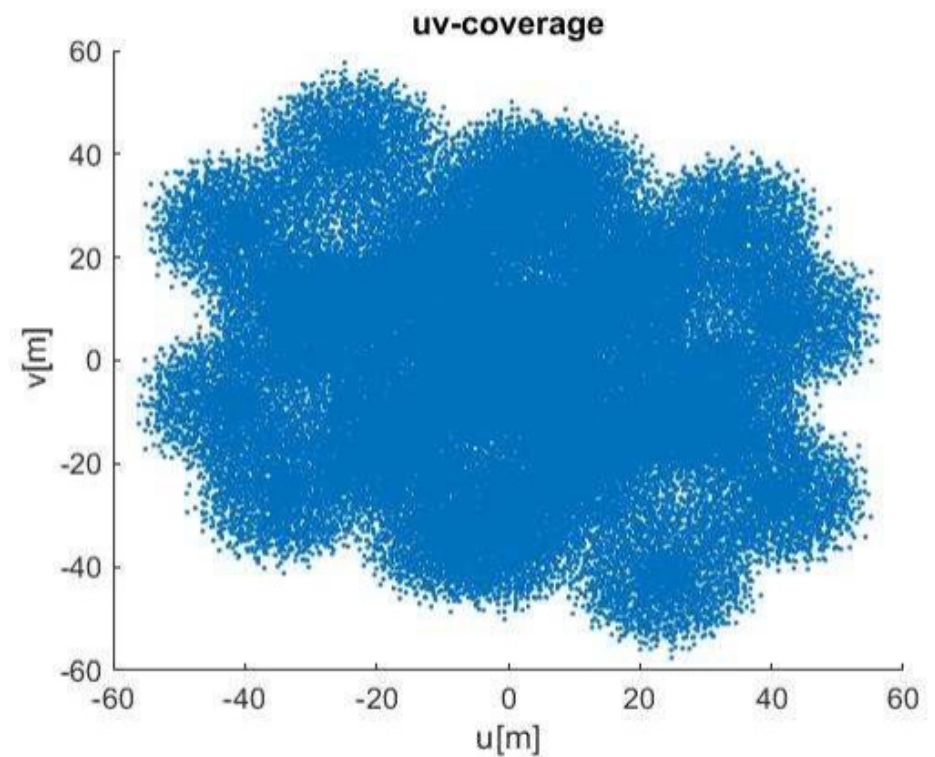
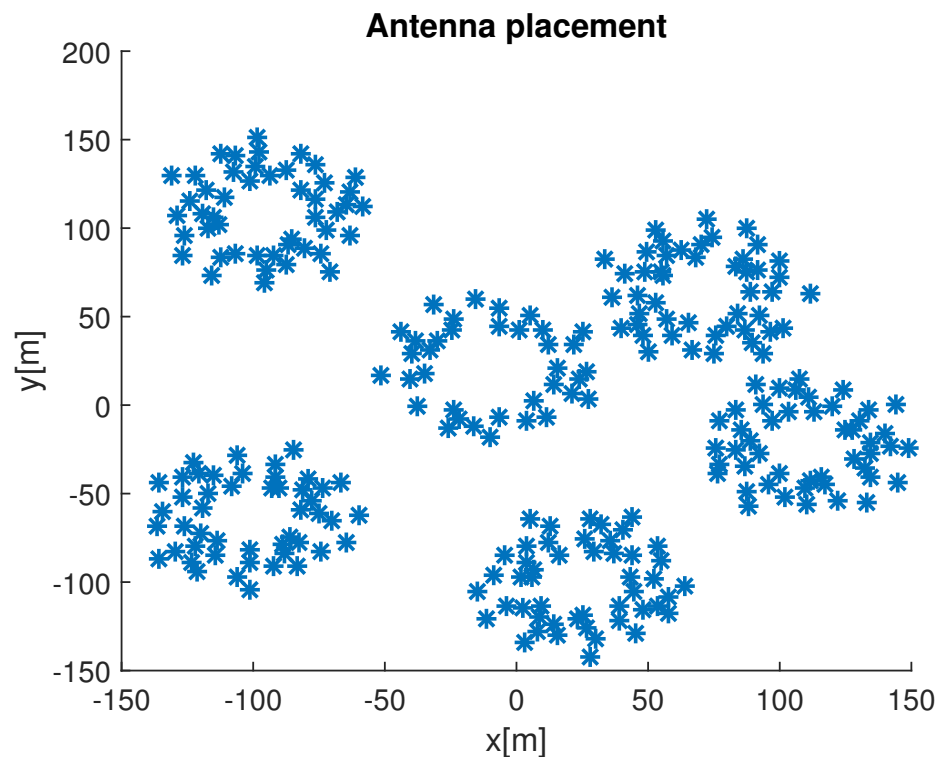
- To see this, insert the measurement equation:

$$I_D(\mathbf{z}) = \sum_{i,j,k} \sum_q I(\mathbf{z}_q) e^{j\mathbf{u}_{ij,k}^T (\mathbf{z} - \mathbf{z}_q)} = \sum_q I(\mathbf{z}_q) B(\mathbf{z} - \mathbf{z}_q) = I(\mathbf{z}) * B(\mathbf{z})$$

where the PSF  $B(\mathbf{z}) = \sum_{i,j,k} e^{j\mathbf{u}_{ij,k}^T \mathbf{z}}$  is called the *dirty beam* — a function of the available  $(u, v)$  samples.

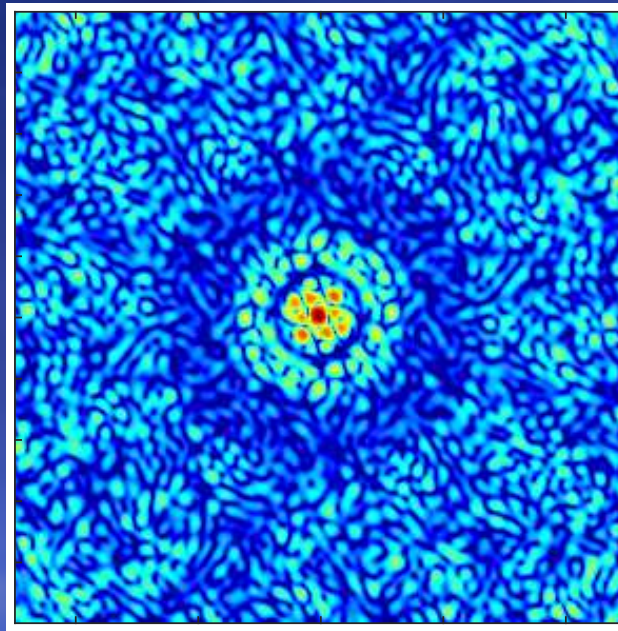
# Basic concepts of interferometry

- LOFAR “Superterp” antenna configuration  
 $P = 288$  antennas, max baseline 326 m
- $(u,v)$  samples (snapshot)



# Basic concepts of interferometry

PSF (dirty beam), dB scale, range 0..-35 dB

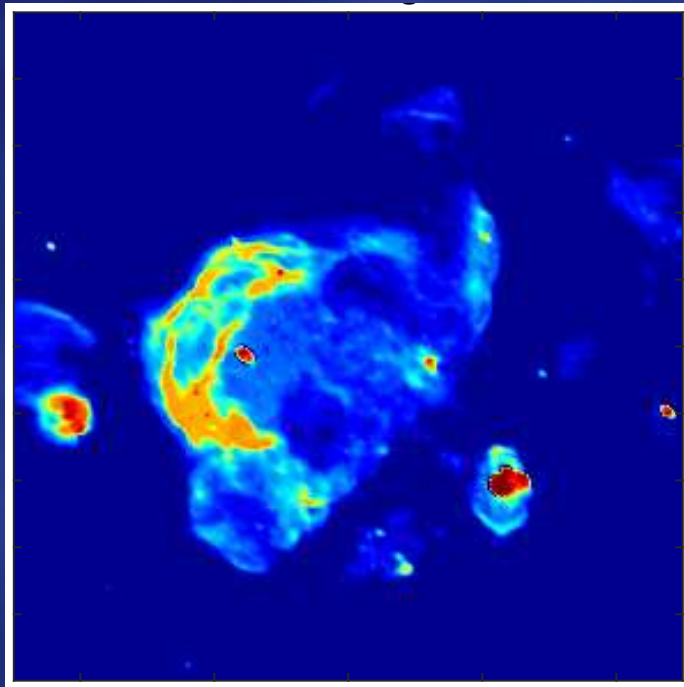


Beamsize (resolution) is inversely proportional to the array diameter; sidelobes cause *confusion*. Could use windowing (tapering).

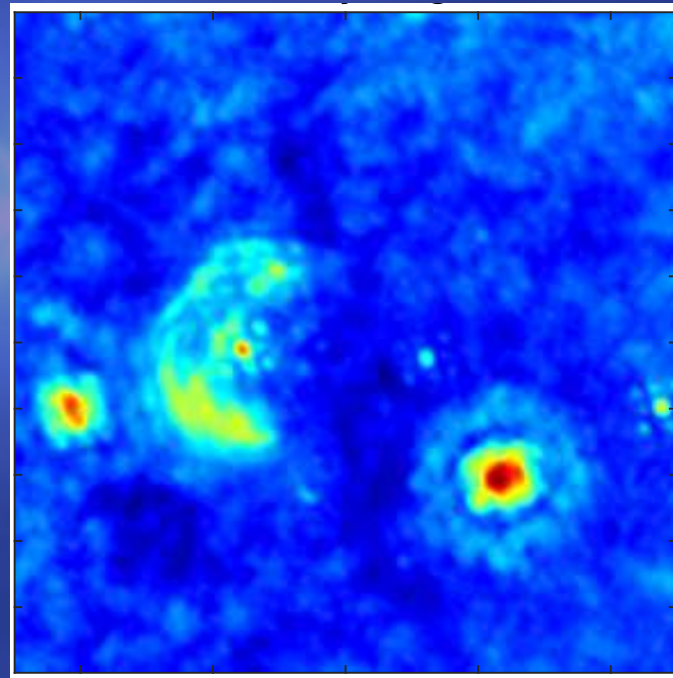
# Basic concepts of interferometry

**Test on model image:** W28 superremnant (from CASA Guides)

- From the image, correlation data is computed and corresponding measured data  $x[n]$  is generated, with added noise;  $N = 10^5$  samples.  
Operating frequency: 58:975 MHz, single time snapshot



Test image in dB (0 till -35 dB)  
 $Q=84681$  pixels



MF dirty image

# Basic concepts of interferometry

## Deconvolution: The CLEAN algorithm

A source at location  $\mathbf{z}_q$  has a response  $B(\mathbf{z} - \mathbf{z}_q)$  in the dirty image.

CLEAN tries to locate sources one by one from the peaks in the image:

for  $q = 1, 2, \dots$

$$\left[ \begin{array}{l} \mathbf{z}_q = \arg \max_{\mathbf{z}} I_D(\mathbf{z}) \\ \hat{\sigma}_q^2 = I_D(\mathbf{z}_q)/B(\mathbf{0}) \\ I_D(\mathbf{z}) := I_D(\mathbf{z}) - \gamma \hat{\sigma}_q^2 B(\mathbf{z} - \mathbf{z}_q), \quad \forall \mathbf{z} \end{array} \right.$$

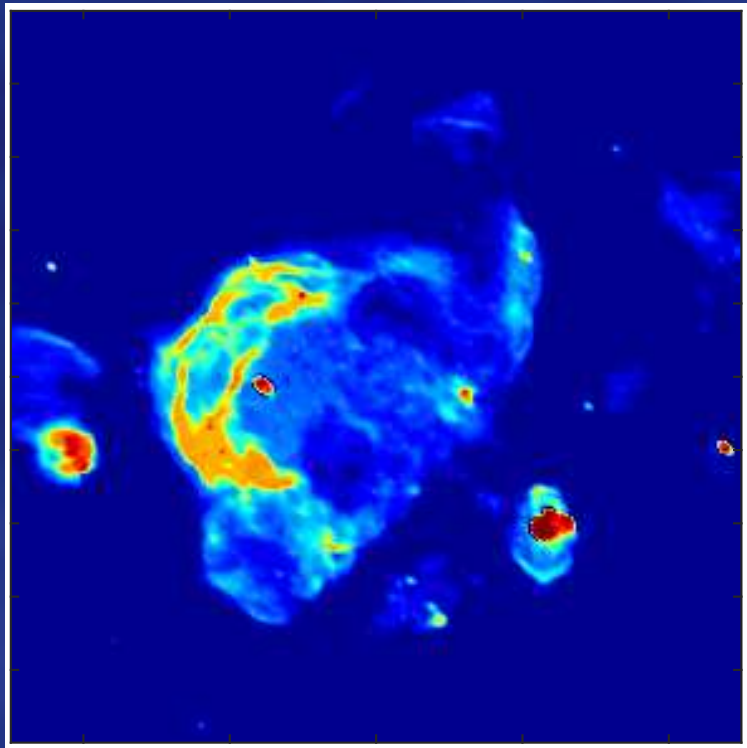
$$I_{clean}(\mathbf{z}) = I_D(\mathbf{z}) + \sum_q \gamma \hat{\sigma}_q^2 B_{synth}(\mathbf{z} - \mathbf{z}_q), \quad \forall \mathbf{z}.$$

- $B_{synth}(\mathbf{z})$  is a ‘synthetic beam”, usually a Gaussian bell-shape.
- Instead of the subtractions on the dirty image, it is considered more accurate to do the subtractions on the sample covariance matrix  $\hat{\mathbf{R}}$  instead,

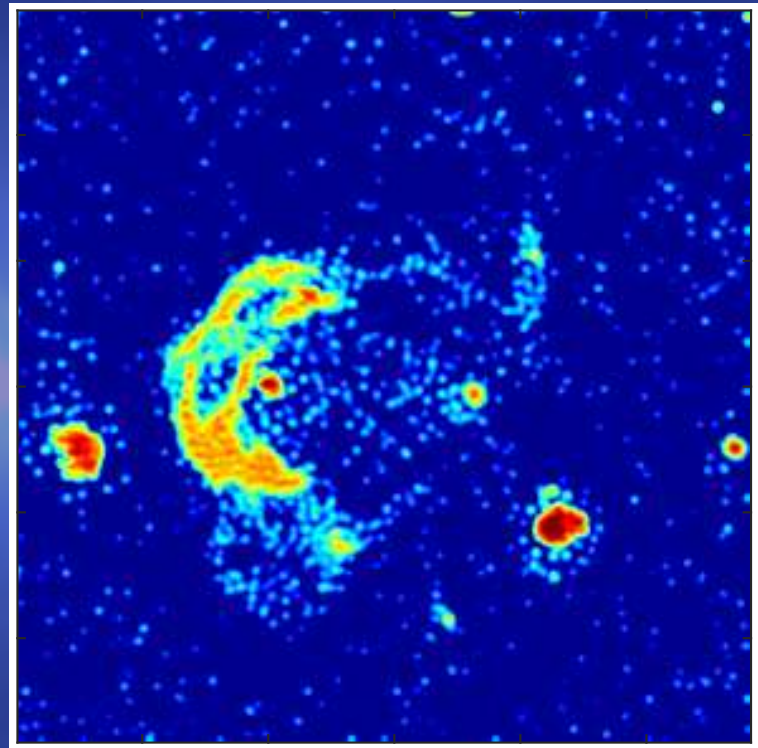
$$\hat{\mathbf{R}} := \hat{\mathbf{R}} - \gamma \hat{\sigma}_q^2 \mathbf{a}(\mathbf{z}_q) \mathbf{a}(\mathbf{z}_q)^H$$

# Basic concepts of interferometry

## The CLEAN algorithm



Original image



CLEAN + postprocessing

10 outer, 500 inner iterations ( $\sim 1$  min)

# Basic concepts of interferometry

## Rewrite in array processing language

- Computing the dirty image is equal to (dropping index  $k$  for simplicity):

$$I_D(\mathbf{z}) = \mathbf{a}^H \mathbf{R} \mathbf{a}, \quad (\mathbf{a})_i \equiv e^{-j\mathbf{u}_i^T \mathbf{z}}$$

Indeed, 
$$I_D(\mathbf{z}) = \sum_{i,j} (\mathbf{R})_{ij} \overline{(\mathbf{a})_i} (\mathbf{a})_j = \sum_{i,j} (\mathbf{R})_{ij} e^{j(\mathbf{u}_i - \mathbf{u}_j)^T \mathbf{z}}$$

## ■ More general: beamformed image

For the  $i$ th pixel, let  $\mathbf{w}_i$  be a spatial beamformer, and compute

$$\sigma_i = \mathbf{w}_i^H \mathbf{R} \mathbf{w}_i, \quad i = 1, \dots, Q$$

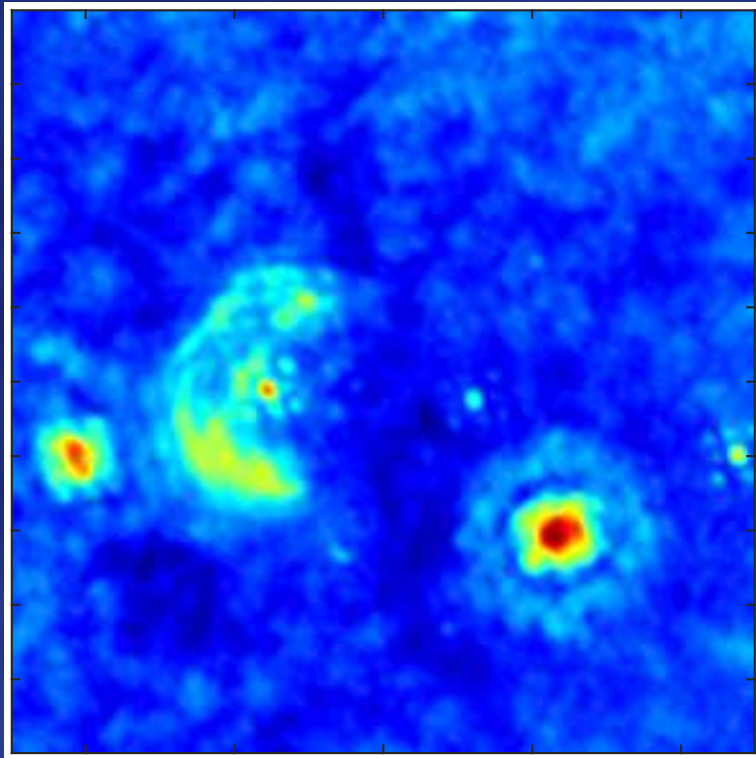
## ■ Minimum Variance Distortionless Response (MVDR) solution

Setting  $\mathbf{w}_i = \frac{\mathbf{R}^{-1} \mathbf{a}_i}{\mathbf{a}_i^H \mathbf{R}^{-1} \mathbf{a}_i}$  leads to the MVDR dirty image (where  $\mathbf{w}_i^H \mathbf{a}_i = 1$ )

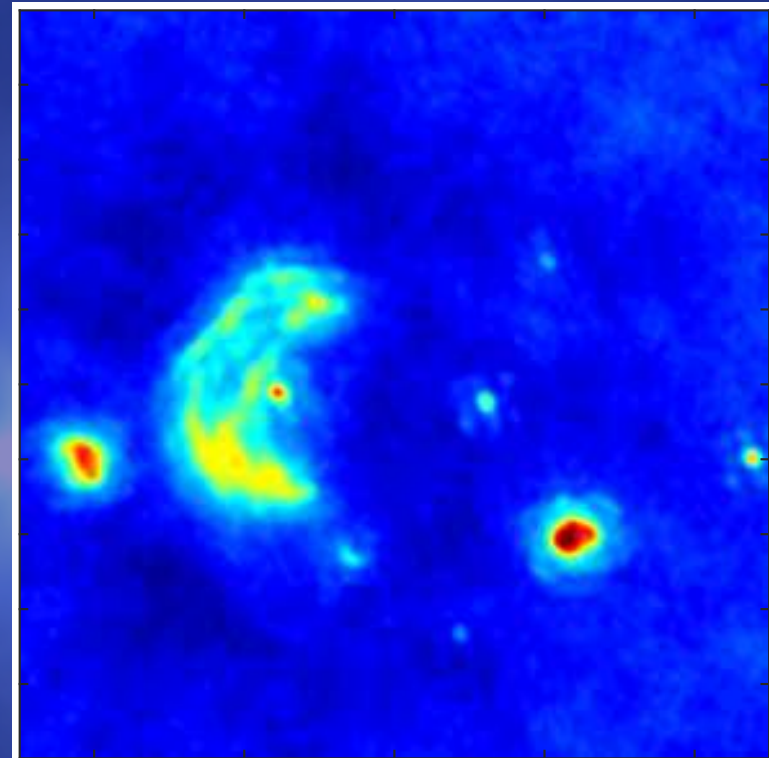
$$\sigma_{\text{MVDR},i} = \frac{1}{\mathbf{a}_i^H \mathbf{R}^{-1} \mathbf{a}_i}$$

# Basic concepts of interferometry

## Beamformed images



MF image (classical dirty image)



MVDR dirty image; dB scale

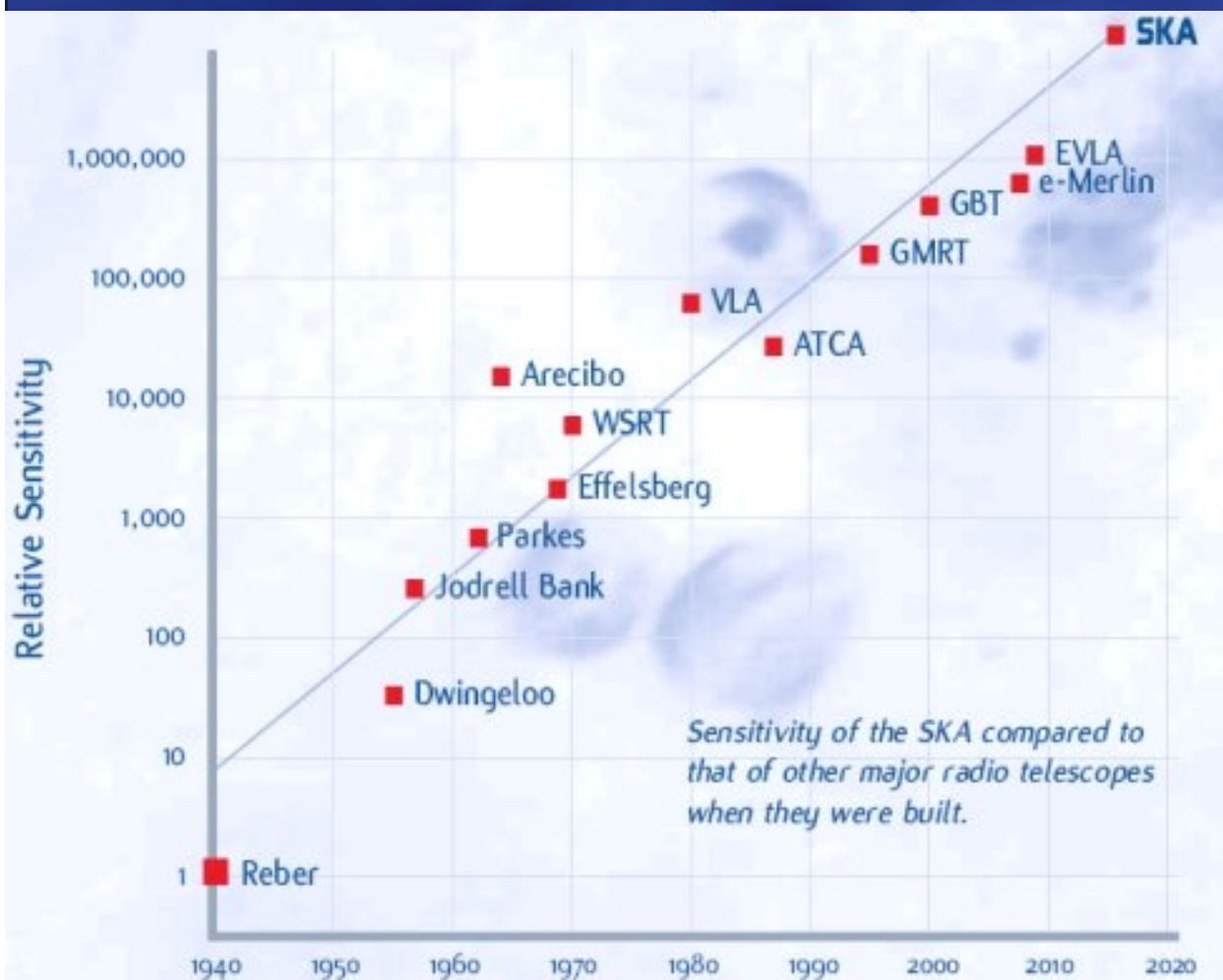
We can derive:

$$0 \leq \sigma_{\text{true}} \leq \sigma_{\text{MVDR}} \leq \sigma_{\text{MF}}$$

# New instruments

## Sensitivity evolution (1940-2010)

- Exponential: doubles every 3 years



## Other parameters

- Resolution
- Freq. range, bandwidth
- Survey speed (FoV)

## SKA science drivers

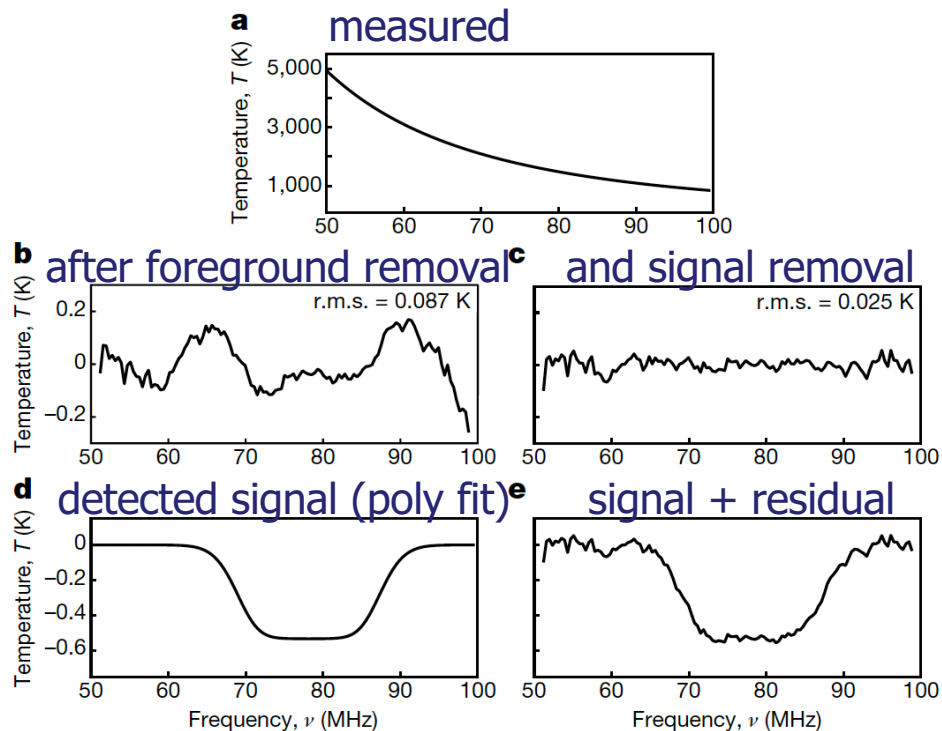
- Early universe:
  - Dark Ages and EoR: first star formation
  - Galaxy formation and evolution
- Cosmic magnetism
- Gravitation (pulsar timing)
- Origins of life

# First EOR detection (?): EDGES telescope

## ■ EDGES telescope at MRO

Single dipole blade antenna  
(2x1 m)

Published in Nature, Mar 2018  
(Judd Bowman e.a.):



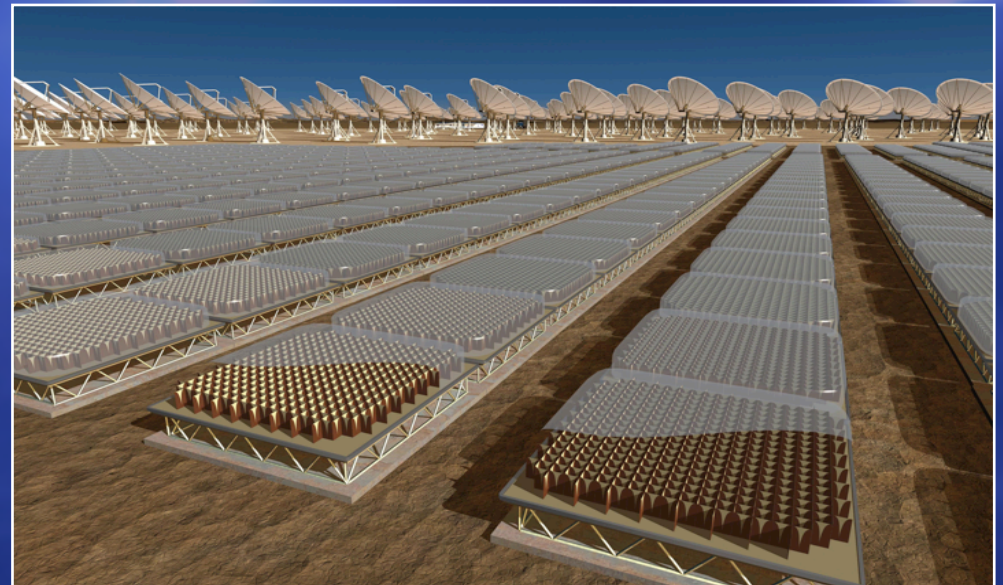
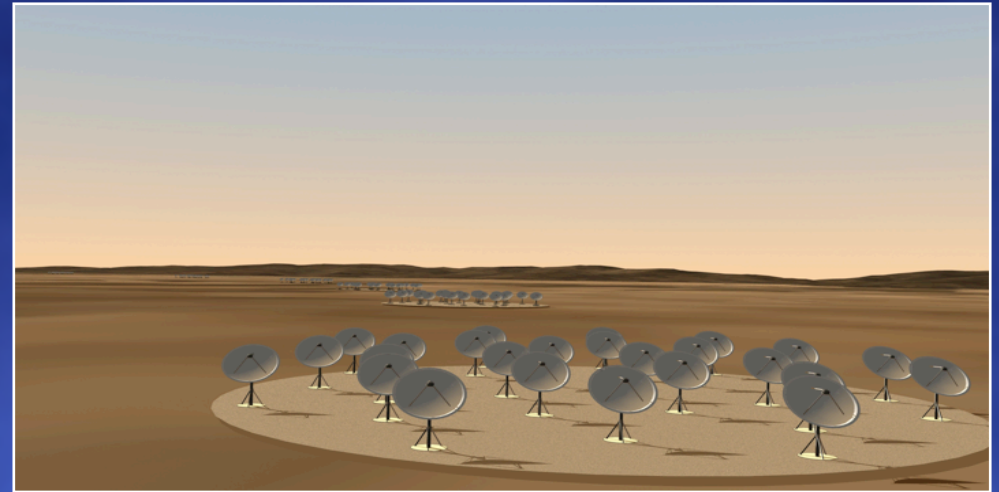
Detected EOR based on 100 h all-sky observations and parameterized (polynomial) models of the sky foreground and EOR spectrum

SKA is aimed to image this signal

# The Square Kilometer Array

## Initial design parameters (2010)

- Operational in 2019/2023
- 1.000.000 m<sup>2</sup>: sensitivity 50 x VLA
- 70 MHz--25 GHz; instantaneous bandwidth  $0.25 \times f_c$
- Core 5 km has 50% of elements); max 3000 km
- Combination of sparse phased arrays of dipoles, dense aperture arrays, dishes with focal plane arrays
- 50 instantaneous beams



# The Square Kilometer Array

## SKA1-Low (2013 param.)

- Aperture array with 250,000 log-periodic elements
- 866 stations, 90 km diameter
- 50 MHz—350 MHz
- Location: Western Australia



# The Square Kilometer Array

## Initially considered

- 400 MHz—1.4 GHz
- 250 dense aperture array stations
- Location: Southern Africa

## SKA1-Mid (2013 param.)

- 350 MHz—3 GHz (...14.5 GHz)
- 190+64 dishes (MeerKAT) with single-pixel feeds
- Location: Southern Africa

2018: 64 dishes commissioned



# SKA Precursor: MRO site

## Murchison Radio-astronomy Observatory (MRO) site

Western Australia (Boolardy station)

- **Australian SKA Pathfinder (2019)**

36 identical 12-metre wide dish antennas, 18 currently operational

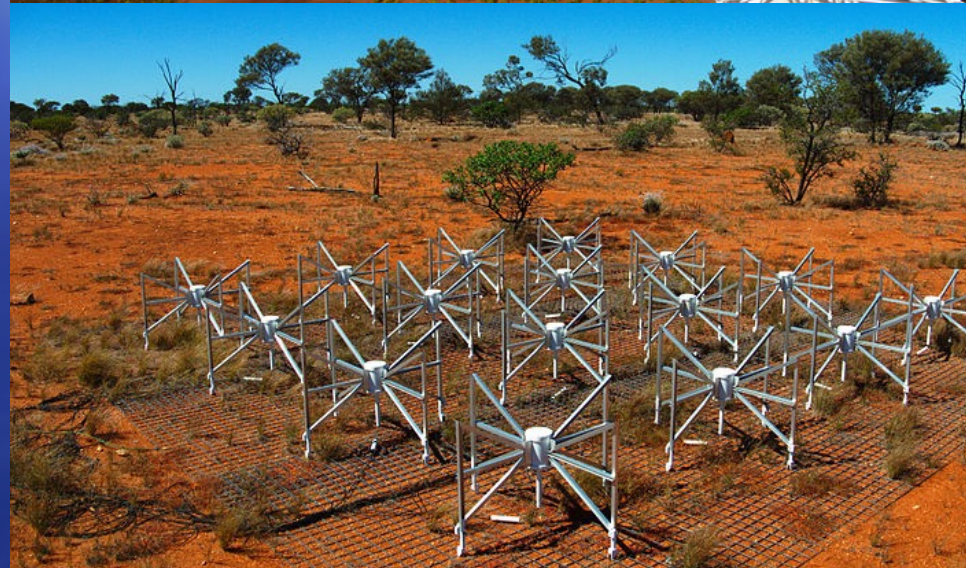
Phased Array Feed (PAF) receivers (giving survey speed)

- **Murchison Widefield Array (2013)**

80-300 MHz; 2048 dipole antennas in 128 tiles (4 x 4); max baseline ~ 3 km

Total BW 30 MHz; resolution 40 kHz x 0.5 s

2018: upgrade to 256 tiles over 6 km



# SKA Precursor: MRO site



- **SKA1-Low** prototype station  
256 low-frequency antennas  
(AAVS1)

Completed 25 May 2018



# SKA – signal processing requirements

## Quest for higher performance

- Higher resolution  $\Rightarrow$  longer baselines  $\Rightarrow$   $\left\{ \begin{array}{l} \text{shorter integration time (due to earth rotation)} \\ \text{more (narrower) subbands} \end{array} \right.$
- Higher sensitivity  $\Rightarrow$  larger number of antennas (grouped in stations), longer observing times, better calibration (direction dependent)
- Higher survey speed  $\Rightarrow$  larger total bandwidth, larger FOV, multiple beams, direction dependent calibration

Results in larger data sets, higher computational demands, and the need for better algorithms

Also: cannot store all “raw” correlation data for later processing, need new intermediate data products (snapshot images, plus  $\dots$ ).

# SKA – signal processing requirements

## Typical numbers (SKA1-Low, 2013 design parameters) [Dewdney'13]

Frequency range: 50–300 MHz, sampled using log-periodic (somewhat directional) antennas.

Max baseline 100 km (later scaled down to 65 km), station diameter 35 m.

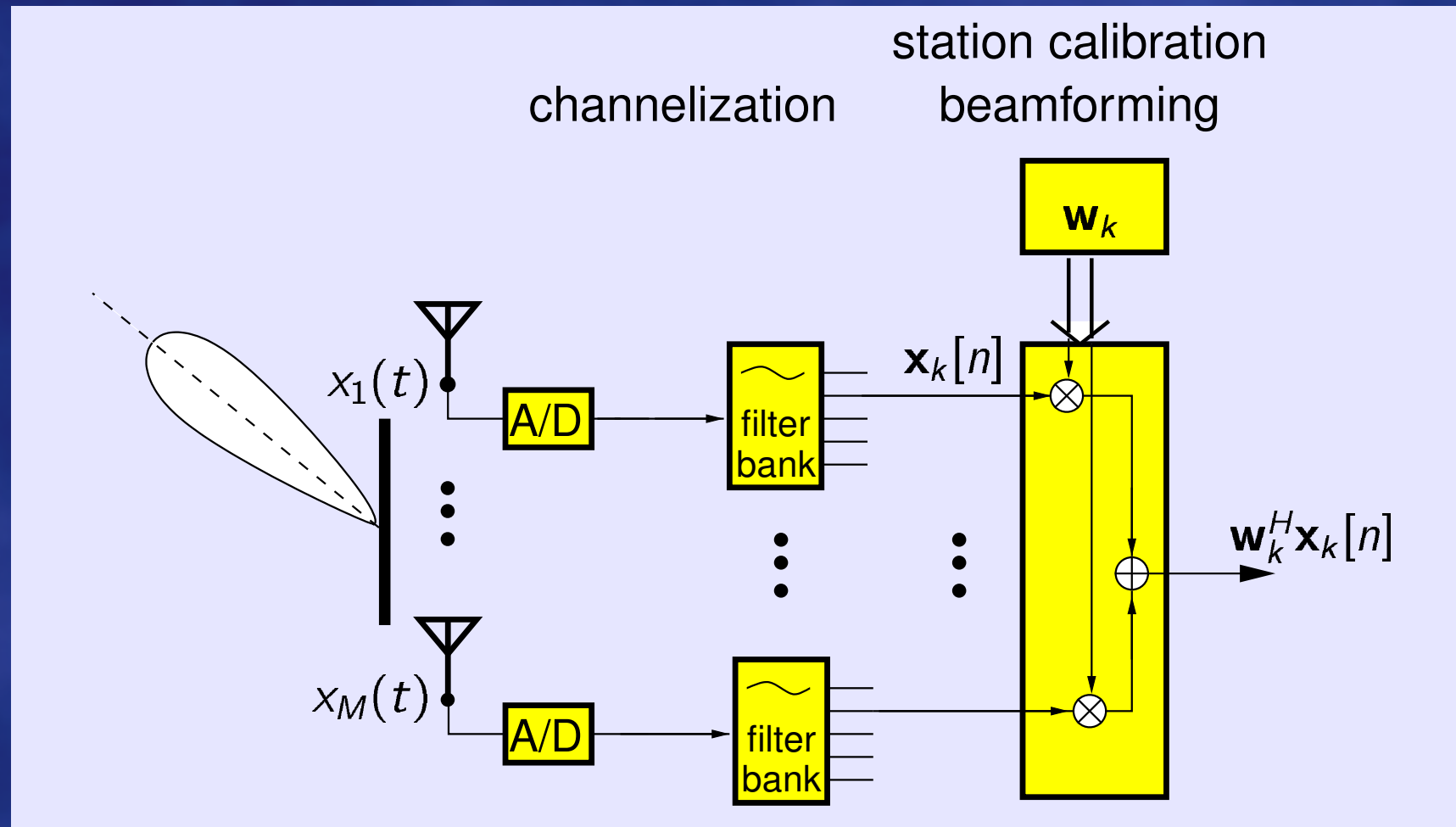
## Raw data

250 MHz bandwidth (2 polarizations),  $2 \times 4$  bit per complex sample; 286 antennas/station; 911 stations (later scaled down to 512), i.e.  $> 260,000$  antennas

**Total: 286 GBps** (one station); **260 TBps** (all stations)

# SKA – signal processing requirements

## Station processing



# SKA – signal processing requirements

## Station processing

- Station beamforming reduces raw data rate by a factor 286 to **1 GBps (1 station)**
- Number of coarse frequency channels depends on station diameter (e.g.  $D = 35$  m  $\Rightarrow$  BW  $\ll$  8.6 MHz). But narrower subbands are used e.g. for station calibration processing.

	<i>one station</i>	<i>1024 stations</i>
Channelization (polyphase filter bank)	14.1 TOps/s	
Beamforming	1.2 TOps/s	
Calibration	1.3 TOps/s	
<i>Total</i>	16.7 TOps/s	17.1 POps/s
<i>Rate</i>	1.2 GBps	1.2 TBps

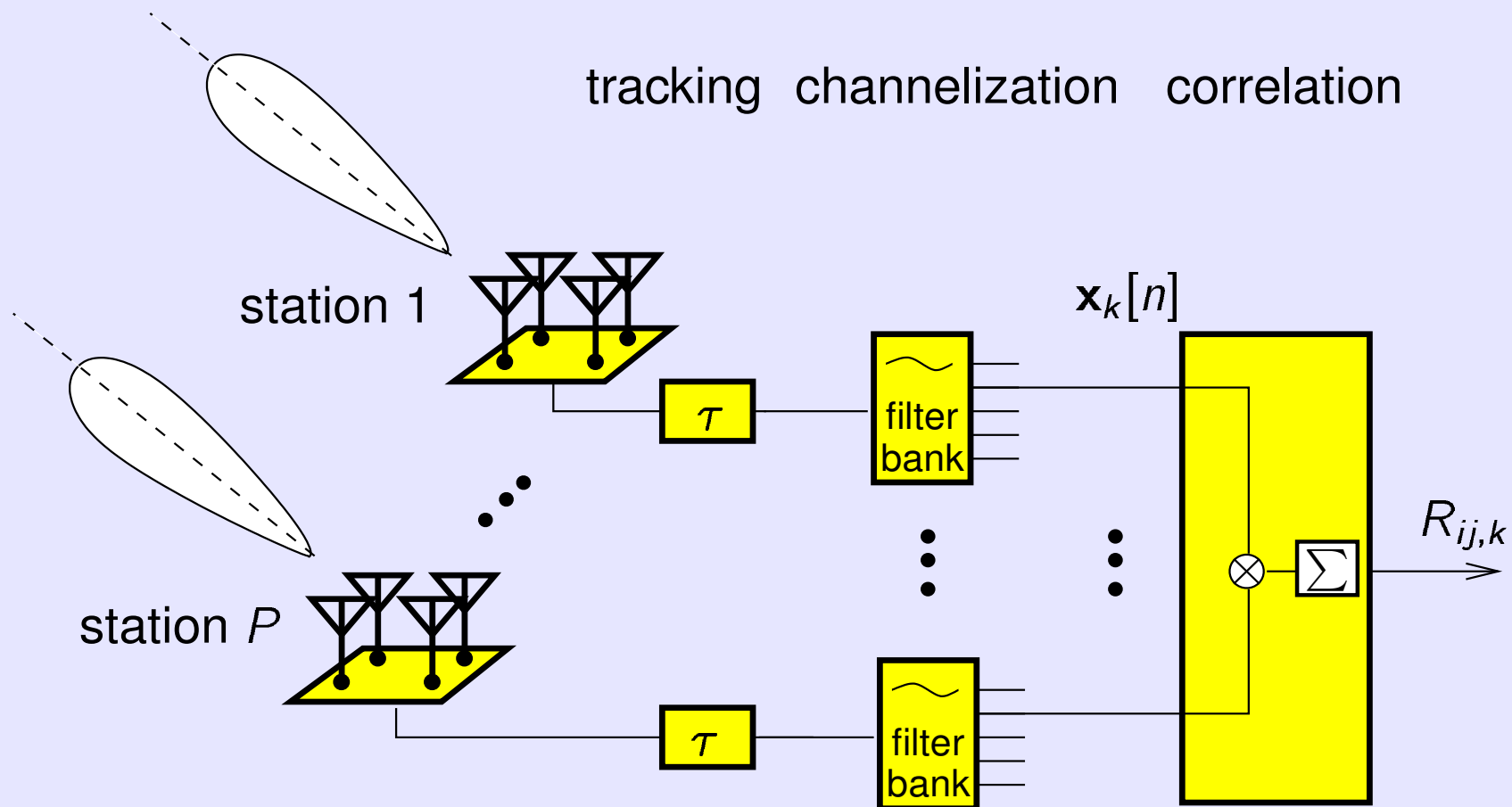
M	$10^6$
G	$10^9$
T	$10^{12}$
P	$10^{15}$

[Jongerius'16, not based on current parameters]

# SKA – signal processing requirements

## Central signal processing

At a central location, the beamformed data from stations are correlated, and averaged over short time intervals.



# SKA – signal processing requirements

## Maximum integration time

Based on earth rotation:  $T < 1200 \frac{D}{B} \Rightarrow$  Output data rate per channel :  $R = \frac{1}{1200} \cdot \frac{B}{D}$

## Number of subband channels

For the complete instrument, the longest baseline  $B$  determines the narrowband constraint (to avoid bandwidth smearing).

The maximum subband bandwidth is (taking into account the reduced FOV)

$$W_{chan} \ll \frac{D}{B} f_{\min} \Rightarrow W_{chan} = 0.1 \cdot \frac{D}{B} f_{\min}$$

For a total bandwidth  $W_{tot}$ , the number of subband channels is proportional to

$$N_{chan} = \frac{W_{tot}}{W_{chan}} = 10 \cdot \frac{B}{D} \frac{W_{tot}}{f_{\min}}$$

The main drivers for complexity are thus  $P^2$  and the ratios  $\left(\frac{B}{D}\right)^2$  and  $\frac{W_{tot}}{f_{\min}}$ .

# SKA – signal processing requirements

## Central signal processing

- With  $P$  stations, the correlation matrices have size  $P \times P$ .

SKA1-Low (2013) asked for  $P = 911$  stations.

- Number of frequency channels depends on instrument diameter.

$B = 100$  km for SKA1-Low  $\Rightarrow$  BW  $\ll 4.2$  kHz

Design: **250,000 channels** with  $W_{chan} = 1$  kHz.

- Integration time is limited due to earth rotation.

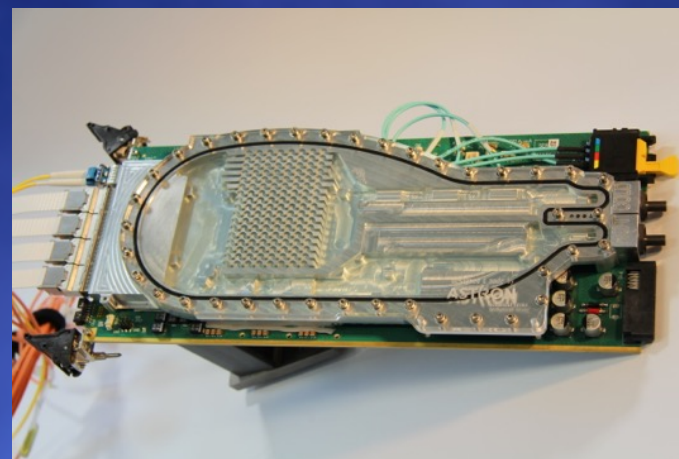
With  $B = 100$  km (longest baseline),  $D = 25$  m (station diameter), the maximal integration time is  $T \ll 0.4$  sec

**Output data:** complex correlation matrices of size  $911 \times 911$  times 250,000 channels, each 0.4 sec, times 4 polarizations

# SKA – signal processing requirements

## Central signal processing

Channelization	60.8 TOps/s
Correlation	5.0 POps/s
RFI flagging	0.3 POps/s
<i>Total</i>	5.3 POps/s
<i>Rate</i>	7.3 TBps (!)



288 "Gemini" FPGA boards (ASTRON)

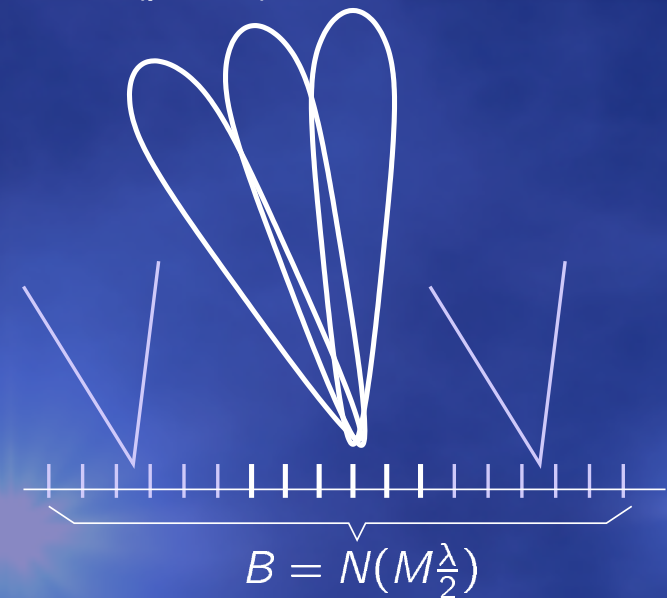
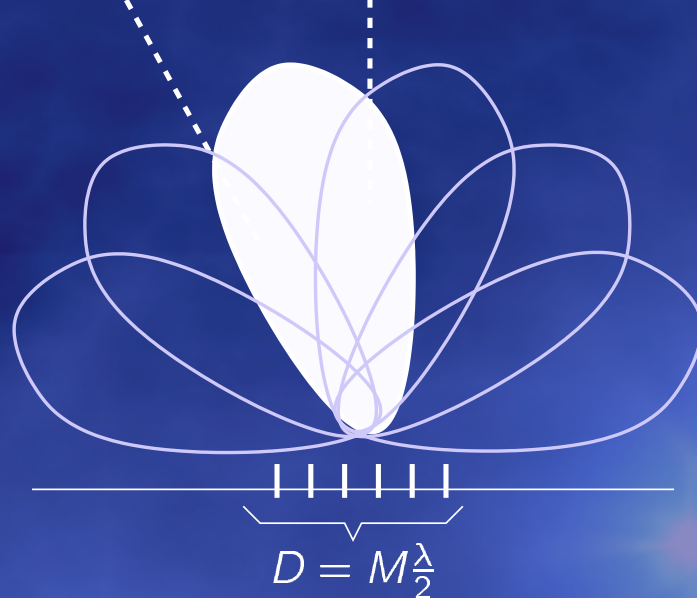
M	$10^6$
G	$10^9$
T	$10^{12}$
P	$10^{15}$

- Note that the correlator produces six times more output "data" than flows in (due to  $P \Rightarrow P^2$  and short integration length).
- "Baseline dependent averaging" [analyzed in Wijnholds'15] exploits the fact that up to 90% of the baselines are short and can be integrated over longer times and larger bandwidths. This reduces the datarate by a factor 10 (about 700 GBps).
- Hardware limitations perhaps not flops but bandwidth.

# SKA – signal processing requirements

How many pixels in an image?

$N$  beams (pixels) in the FOV



■ Uniform linear array of  $N$  stations (each  $M$  antennas), 1 dimension

$$\left. \begin{array}{l} \text{Station diameter: } D = M \frac{\lambda}{2} \\ \text{Max baseline: } B = N(M \frac{\lambda}{2}) \end{array} \right\} \Rightarrow N = \frac{B}{D} \text{ pixels}$$

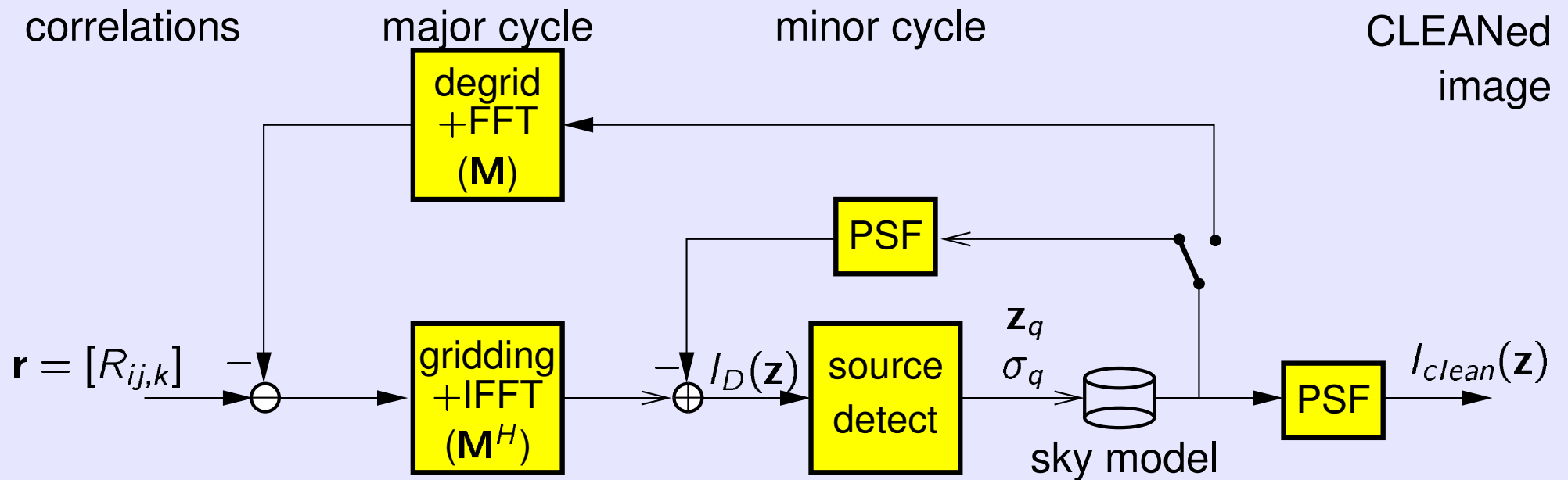
For 2 dimensions, we obtain images of size  $N \times N$ .

■ SKA1-Low (2013):  $D = 35 \text{ m}$ ,  $B = 100 \text{ km} \Rightarrow N \sim 3000$

# SKA – signal processing requirements

## Science Data Processing

Science Data Processing is the actual image formation step. Also calibration parameters are estimated and applied.



# SKA – signal processing requirements

## Science Data Processing

Gridding + IFFT (dirty image)	84 POps/s
Source subtraction (image domain)	94 TOps/s
FFT + degrid (visibility predict)	75 POps/s
<i>Total</i>	158 POps/s

M	$10^6$
G	$10^9$
T	$10^{12}$
P	$10^{15}$

Direction-dependent calibration: add 213 POps/s (?)

Resulting power requirements: at least 630 kW (SKA-1 Low) for the CPUs, and much more for the data transport from/to storage. Also consider data rearrangements . . .

**Conclusion:** Improved imaging algorithms cannot be much more complex...

# Image formation

- Data model (measurement equation):  $\mathbf{R}_k = \mathbf{A}_k \boldsymbol{\Sigma}_s \mathbf{A}_k^H + \boldsymbol{\Sigma}_n$ .

- Vectorized data model:  $\mathbf{r}_k = \text{vec}(\mathbf{R}_k)$ ; vectorized image:  $\boldsymbol{\sigma}_s = \text{vecdiag}(\boldsymbol{\Sigma}_s)$

$$\mathbf{r}_k = (\mathbf{A}_k^* \circ \mathbf{A}_k) \boldsymbol{\sigma}_s + \boldsymbol{\sigma}_{n,k} = \mathbf{M}_k \boldsymbol{\sigma}_s + \boldsymbol{\sigma}_{n,k}, \quad \text{where } \mathbf{M}_k := \mathbf{A}_k^* \circ \mathbf{A}_k$$

*Notation:*  $\circ$ : Khatri-Rao product (column-wise Kronecker product)

- Stack for all STI's (+freq bins)  $k$ :

$$\mathbf{r} = \mathbf{M} \boldsymbol{\sigma}_s + \boldsymbol{\sigma}_n$$

- Observations:  $\hat{\mathbf{r}} = \mathbf{r} + \mathbf{e}$ ,

where  $\mathbf{e}$  is finite sample noise, zero mean with covariance  $\mathbf{C}_{\mathbf{e},k} = \frac{1}{N} \mathbf{R}_k^T \otimes \mathbf{R}_k$ .

## Image formation problem

Given  $\hat{\mathbf{r}}$  and  $\mathbf{M}$ , can we recover  $\boldsymbol{\sigma}_s$ ?

*First step:* subtract or project out  $\boldsymbol{\sigma}_n$  from  $\hat{\mathbf{r}}$  (for simplicity, no change in notation; also dropping subscript  $k$ ).

# Image formation

## Initial approach: Least Squares

The LS RA imaging problem is

$$\hat{\sigma} = \arg \min_{\sigma} \|\hat{\mathbf{r}} - \mathbf{M}\sigma\|_2^2.$$

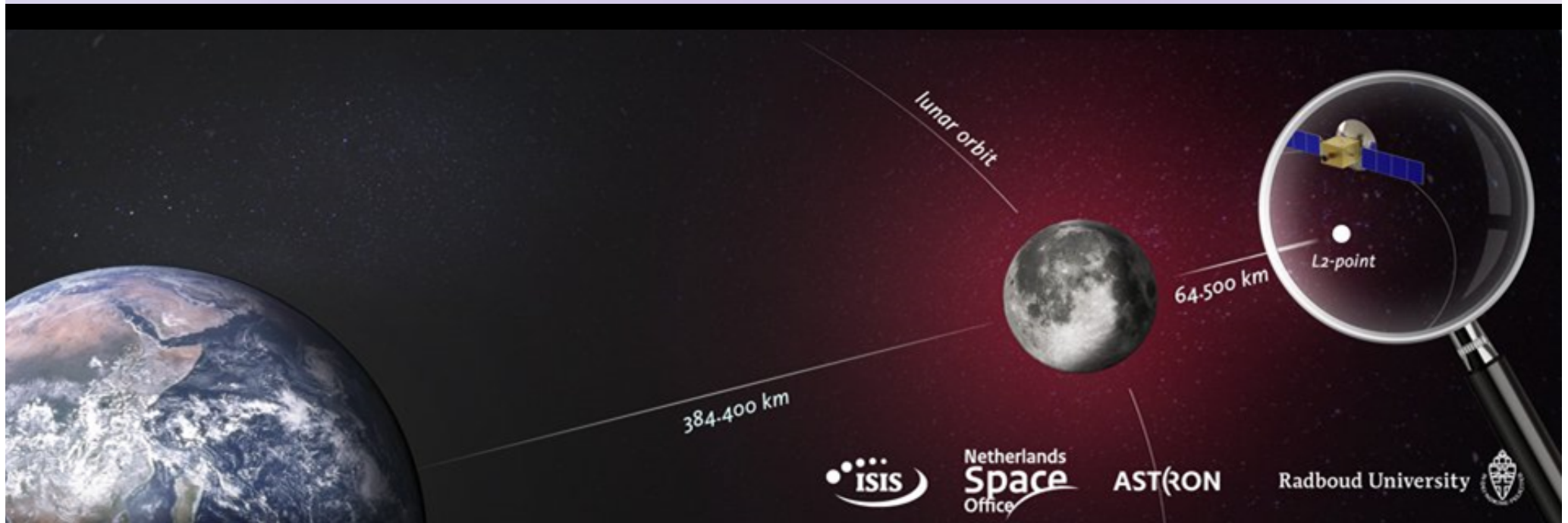
The solution satisfies the normal equations

$$\mathbf{M}^H \mathbf{M} \hat{\sigma} = \mathbf{M}^H \hat{\mathbf{r}},$$

$\hat{\sigma}_{\text{MF}} = \mathbf{M}^H \hat{\mathbf{r}}$  is the MF dirty image, and  $\mathbf{M}^H \mathbf{M}$  represents the convolution of the image pixels with the beampattern of the array.

- For large number of pixels  $Q$ ,  $\mathbf{M}^H \mathbf{M}$  is ill conditioned or not even invertible, and a solution cannot be obtained without regularizing assumptions.
- Taking a pseudo-inverse of  $\mathbf{M}^H \mathbf{M}$  using a truncated SVD suppresses high frequencies and results in a Gibbs phenomenon effect.

# Next – to the moon



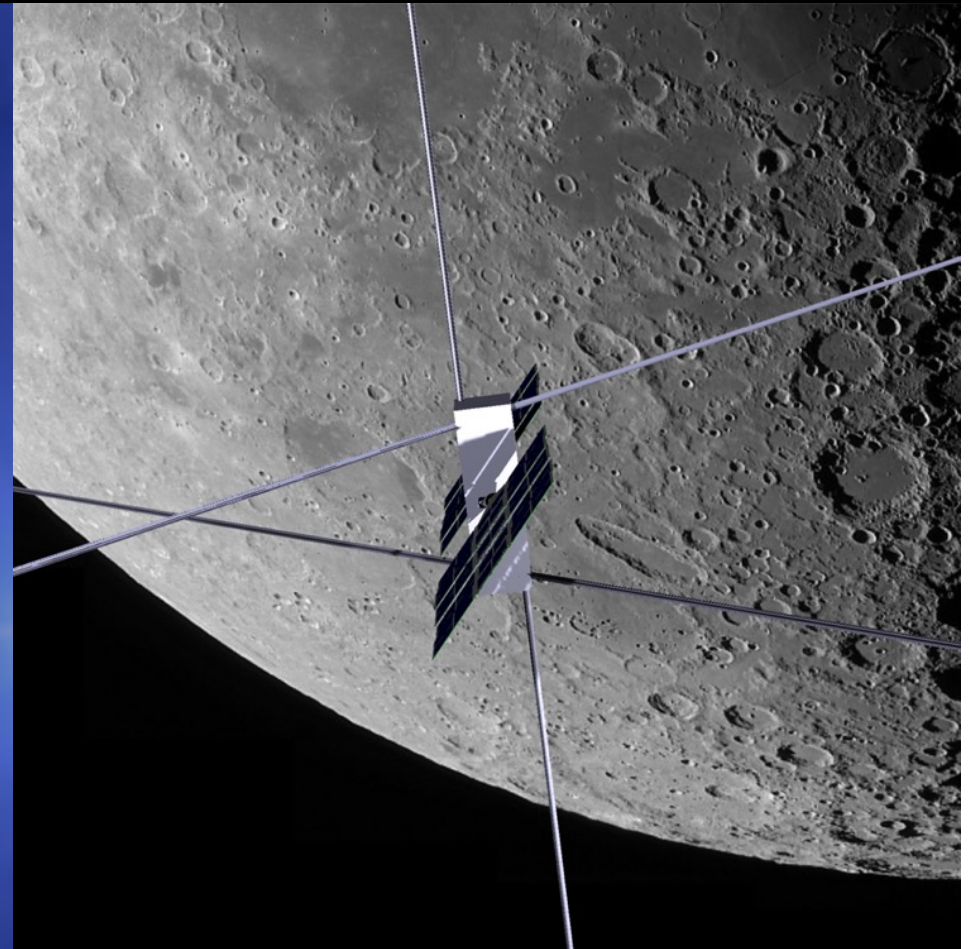
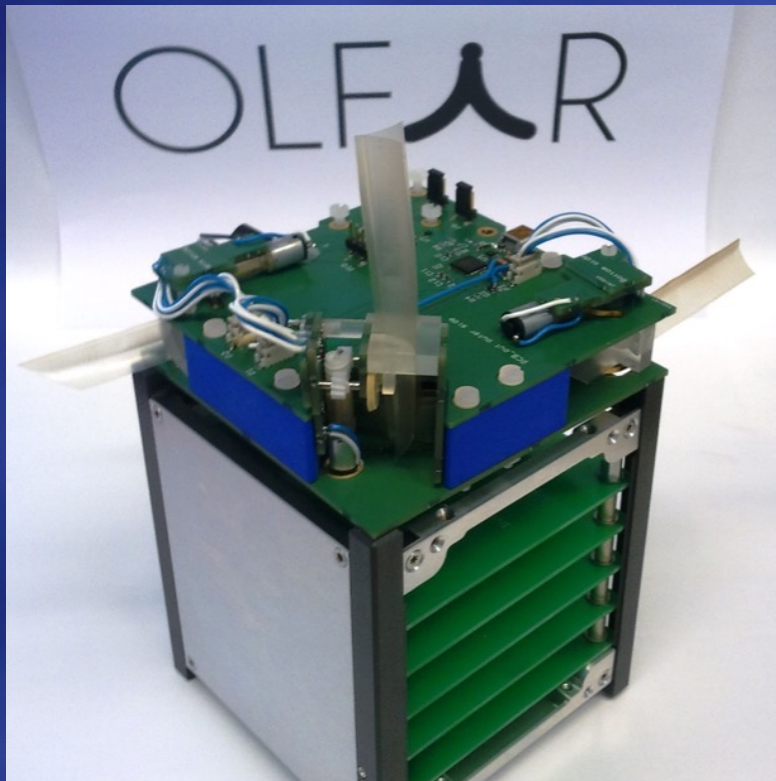
## Netherlands-China Low-Frequency Explorer (20 May 2018)

- Piggyback on a relay satellite for the Chinese lunar explorer mission (Chang'e 4)
- 3 antennas (orthogonal dipoles), (80 kHz till) 1-80 MHz
- Also: launch of two moon-orbiting microsatellites for testing interferometry at 1-30 MHz (but one failed?)

# Next – to the moon

## OLFAR – orbiting low frequency array, design concept

- Swarm of some 50 nano-satellites, orbiting the moon



# Conclusions

**Radio astronomy is an interesting application area for Array Signal Processing**

- Imaging and deconvolution
- Interference reduction using subspace techniques
- Calibration

**Thanks to collaborators:** Amir Leshem, Albert-Jan Boonstra, Stefan Wijnholds, Sebastiaan van der Tol, Millad Sardarabadi, Shahrzad Naghibzadeh, Raj Rajan, ...

A.J. van der Veen, S.J. Wijnholds, A.M. Sardarabadi, **Signal Processing Tools for Radio Astronomy**, in *S. Bhattacharryya e.a. (Ed.), Handbook of Signal Processing Systems*, 3rd ed., Springer, pp. (40 pp.), July 2018.