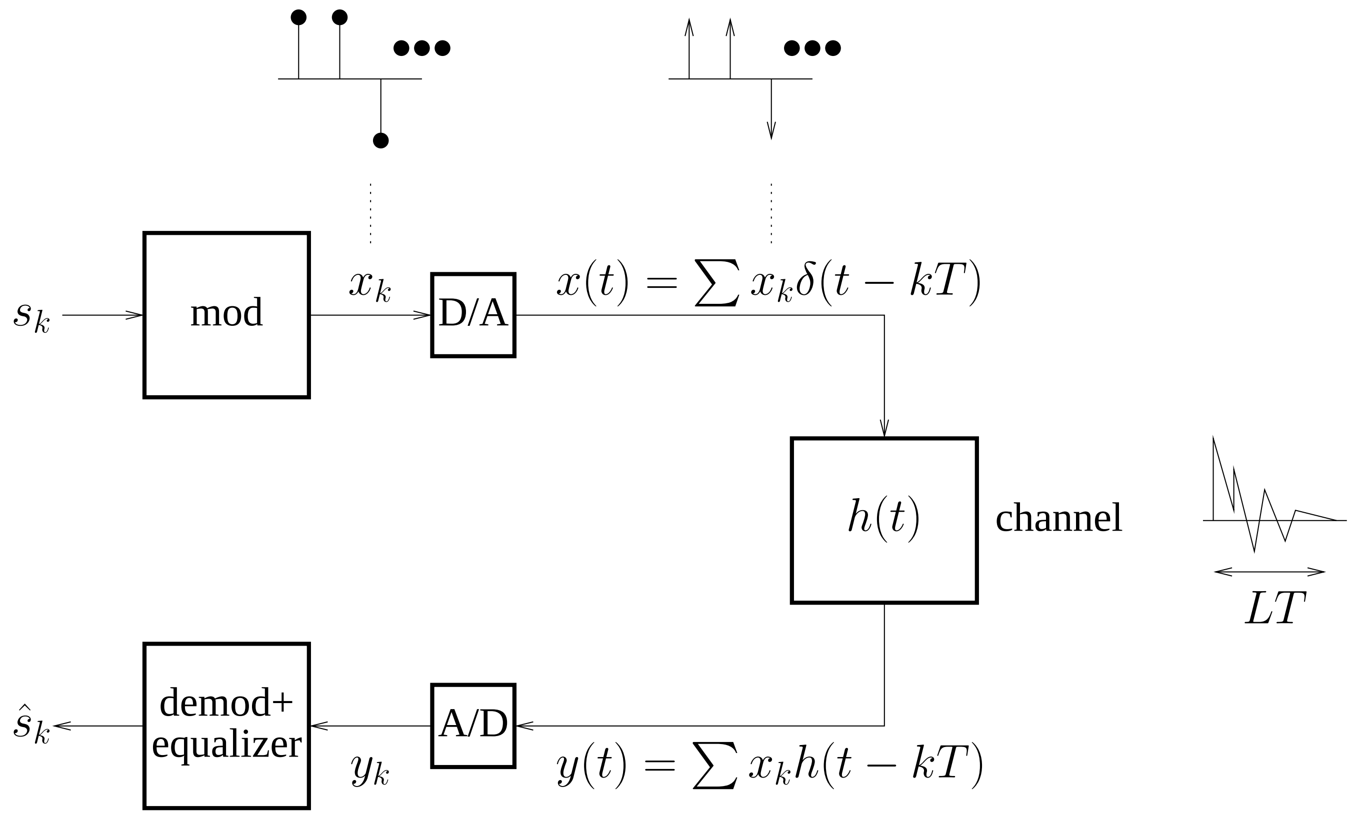
Application of FFT: OFDM

- Transmission over channels
- Equalization
- Multicarrier modulation
- OFDM
- Application to Hyperlan

Transmission over channels



Channel $h(t)$ consists of

- transmit and receive filters (sometimes calles pulse shape)
- physical channel with multipath reflections

Transmission over channels

Effect of channel, which is assumed to be FIR (finite impulse response) of length LT:

$$\begin{aligned}
y(t) &= h(t) * x(t) \\
&= \int_0^{LT} h(\tau)x(t-\tau)d\tau \\
&= \int_0^{LT} h(\tau) \sum_{i=-\infty}^{+\infty} x_i \delta(t-\tau-iT)d\tau \\
&= \sum_{i=-\infty}^{+\infty} x_i \int_0^{LT} h(\tau)\delta(t-\tau-iT)d\tau \\
&= \sum_{i=-\infty}^{+\infty} x_i h(t-iT)
\end{aligned}$$

Defining $y_k = y(kT)$ and $h_k = h(kT)$, we obtain a discrete convolution:

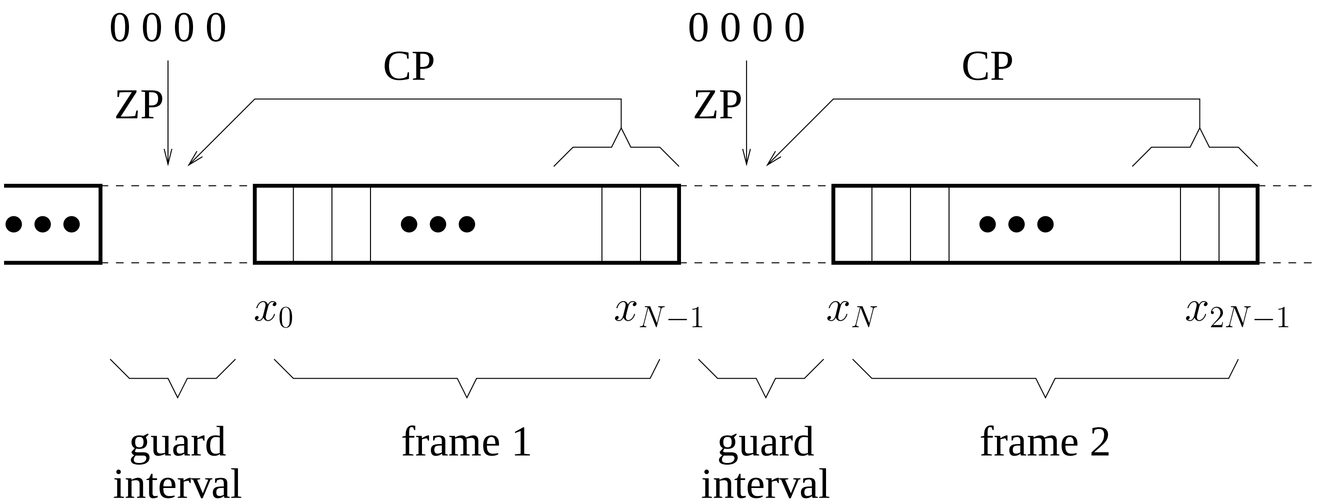
$$y_k = \sum_{i=-\infty}^{+\infty} x_i h_{k-i} = \sum_{i=0}^{L-1} h_i x_{k-i} = h_k * x_k$$

Transmission over channels

- If $L > 1$, we have *intersymbol interference* (ISI)

$$y_k = h_d x_{k-d} + \underbrace{\sum_{i=0}^{d-1} h_i x_{k-i} + \sum_{i=d+1}^{L-1} h_i x_{k-i}}_{\text{ISI}}$$

- Symbol sequences are usually transmitted in *frames*:



- In between frames, we insert a *guard interval* for the duration of the channel, i.e., $L - 1$ symbols. The goal is to make the received frames mutually independent, i.e., the received samples from one frame do not depend on the received samples from another frame.

Transmission over channels

- zero-padding: we insert $L-1$ zero guard symbols (example: $N=3$, $L=2$)

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \left[\begin{array}{c|cccc|c} h_1 & h_0 & & & \\ & h_1 & h_0 & & \\ & & h_1 & h_0 & \\ & & & h_1 & h_0 \end{array}\right] \left[\begin{array}{c} 0 \\ \hline x_0 \\ x_1 \\ x_2 \\ \hline 0 \end{array}\right] = \begin{bmatrix} h_0 & & \\ h_1 & h_0 & \\ & h_1 & h_0 \\ & & h_1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$$

- cyclic-prefixing: we repeat the $L-1$ last symbols (example: $N=3$, $L=2$)

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} = \left[\begin{array}{c|ccc} h_1 & h_0 & & \\ & h_1 & h_0 & \\ & & h_1 & h_0 \end{array}\right] \left[\begin{array}{c} x_2 \\ \hline x_0 \\ x_1 \\ x_2 \end{array}\right] = \begin{bmatrix} h_0 & & h_1 \\ h_1 & h_0 & \\ & h_1 & h_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{y} = \mathbf{H}_c \mathbf{x}$$

Equalization

If the channel is not an impulse, i.e., $h_k \neq h_d \delta_{k-d}$ with d an arbitrary delay, then there is ISI and usually equalization is needed

Objective of a *zero-forcing* equalizer:

$$g_k * h_k = \delta_{k-d}$$

Disadvantages:

- need to know/estimate h_k
- since h_k is FIR, g_k is IIR (infinite impulse response)
- stable inversion not always possible

Multicarrier modulation

- Idea that enables extremely simple equalization:

$$y_k = h_k * x_k \qquad \leftrightarrow \qquad Y(f) = H(f)X(f),\ -1/2 \leq f \leq 1/2$$

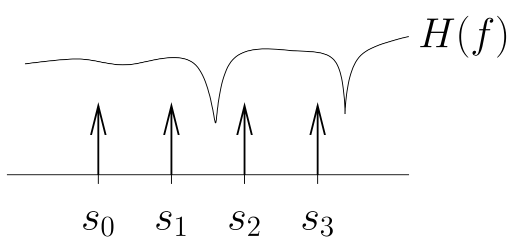
- Reconstruction in frequency domain only requires pointwise division:

$$\hat{X}(f) = G(f)H(f)X(f)\,, \qquad G(f) = H^{-1}(f)$$

- As a result, we modulate symbols on carriers with frequencies $-1/2 \leq f_i < 1/2$:

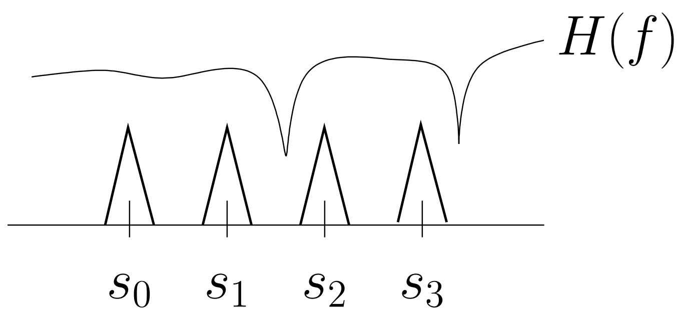
$$x_k = \sum_{i=-\infty}^{+\infty} s_i e^{j2\pi f_i k} \quad \leftrightarrow \quad X(f) = \sum_{i=-\infty}^{+\infty} s_i \delta(f - f_i),\ -1/2 \leq f \leq 1/2$$

$$y_k = \sum_{i=-\infty}^{+\infty} s_i H(f_i) e^{j2\pi f_i k} \quad \leftrightarrow \quad Y(f) = \sum_{i=-\infty}^{+\infty} s_i H(f_i)\delta(f - f_i),\ -1/2 \leq f \leq 1/2$$



- However, this relation only holds for infinite-length sequences and the FT.

Multicarrier modulation



A simple implementation is FDM (frequency-division multiplexing):

- Instead of the infinite-length complex exponentials, we use finite-length filters that split the frequency band into *non-overlapping* subbands.
- The subbands are so narrow that the channel in each subband looks *constant*: no equalization needed.
- Disadvantage: wide spacing of carriers means not efficient use of spectrum.

We next show that partially overlapping subbands are in principle allowed.

OFDM

For finite-length implementations using the DFT, we require *circular convolutions*.

- **Property 1:**

  For the *circular convolution* on finite-length sequences we have

$$y_k = h_k \circledast x_k,\ k = 0, 1, \ldots, N-1 \quad \overset{\text{DFT}}{\leftrightarrow} \quad Y[k] = H[k]X[k],\ k = 0, 1, \ldots, N-1$$

Corollary:

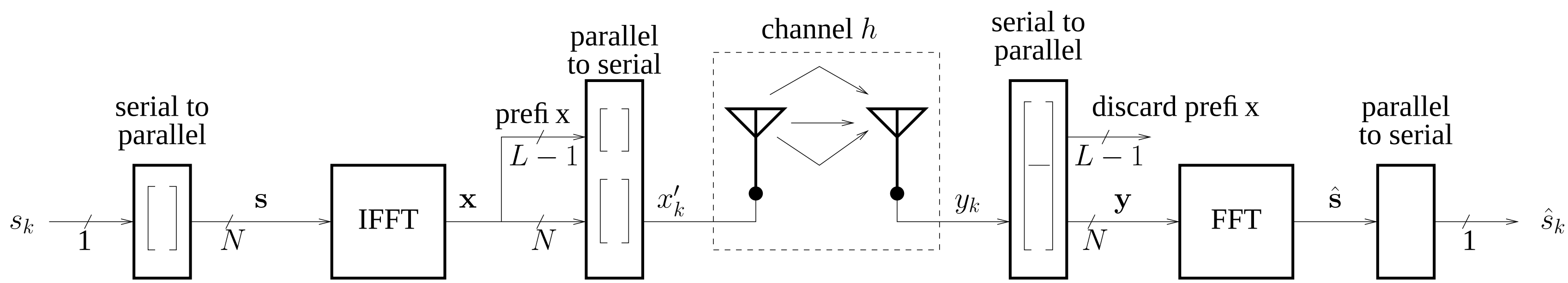
Every circulant matrix $\mathbf{H}_c$ is diagonalized by the (normalized) DFT matrix $\mathbf{F}$

$$\mathbf{H}_c = \mathbf{F}^{\mathrm{H}} \mathbf{\Lambda} \mathbf{F}, \qquad [\mathbf{F}]_{m,n} = \frac{1}{\sqrt{N}} e^{-j2\pi mn/N} \text{ and } [\mathbf{\Lambda}]_{m,n} = \begin{cases} H[m] & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$$

- **Property 2:**

  A circular convolution $y_k = h_k \circledast x_k$ can be obtained through the use of cyclic-prefixing, as explained earlier.

The idea in OFDM (orthogonal frequency division multiplexing) is to define a symbol sequence s_k in the frequency domain, transmit it in the time domain, and map the received samples back into the frequency domain.



OFDM

- **Blocking**

$$s_k \quad \longrightarrow \quad \mathbf{s} = \begin{bmatrix} s_0 \\ \vdots \\ s_{N-1} \end{bmatrix}$$

- **(normalized) IFFT**

$$\mathbf{x} = \mathbf{F}^{\mathrm{H}}\mathbf{s} = \begin{bmatrix} x_0 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

- **Cyclic-prefixing**

$$\mathbf{x}' = \begin{bmatrix} x_{N-L+1} \\ \vdots \\ x_{N-1} \\ \hline x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

OFDM

- **Unblocking**

$$\mathbf{x}' \quad \rightarrow \quad x'_k$$

- **Transmission**

$$y_k = h_k * x'_k \quad \leftrightarrow \quad y_k \; = \sum_{i=0}^{L-1} h_i \, x'_{k-i}$$

- **Blocking**

$$\mathbf{y} = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} = \mathbf{H}_c \mathbf{x}$$

- **(normalized) FFT**

$$\hat{\mathbf{s}} := \mathbf{F}\mathbf{y} = \mathbf{F}\,\mathbf{H}_c \mathbf{x} = \mathbf{F}\,\mathbf{H}_c\,\mathbf{F}^{\mathrm{H}}\mathbf{s} = \mathbf{\Lambda}\mathbf{s}$$

OFDM

OFDM is preferred for wideband data transmission over convolutive channels, since the equalization is so simple (only a scaling).

Challenges:

- synchronization in time, to find where the cyclic prefix starts
- synchronization in frequency: the carrier frequency offset must be very small
- channel zeros on the unit circle (entries of $\mathbf{\Lambda}$ not invertible): the corresponding subchannels can not be used

Often OFDM (or equivalent versions thereof) is combined with *power or bit loading*: transmit more power or more bits on the good subcarriers. This requires feedback of the channel state information.

Hyperlan

Hyperlan (high performance radio local area network) is a WLAN (wireless local area network) standard developed by ETSI (European telecom standards institute).

It is proposed to provide wireless internet connections between PCs, laptops, printers, etc within a building.

Parameters for the Hyperlan/2 standard

- RF carrier frequency: 5.2 GHz
- System consists of several adjacent OFDM channels with a bandwidth of 20 MHz, spanning at least 100 MHz in total
- 64 subcarriers with a spacing of 312.5 kHz: 48 for data, 4 for pilots (channel estimation), 6 + 6 unused (at edges of band, to reduce adjacent channel interference)
- Cyclic prefix guard interval: 800 ns = 16 samples
- Modulation alphabet: BPSK (± 1), QPSK ($\pm 1 \pm j$), 16-QAM, (64-QAM)

Hyperlan

Resulting data rate: 6–54 Mbit/s per OFDM channel