ET4 147: SIGNAL PROCESSING FOR COMMUNICATIONS

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Course outline

1. Introduction, applications
2. Data models, linear algebra
3. Beamforming and filtering concepts
4. Wiener filter, adaptive filtering
5. Direction finding and delay estimation using ESPRIT
6. Constant-modulus algorithm
7. Applications to CDMA

1. THE WIRELESS CHANNEL

Outline

1. Introduction, motivation
2. Antenna arrays
3. Multipath channel models
4. Signal modulation
5. Macroscopic channel model
6. Applications

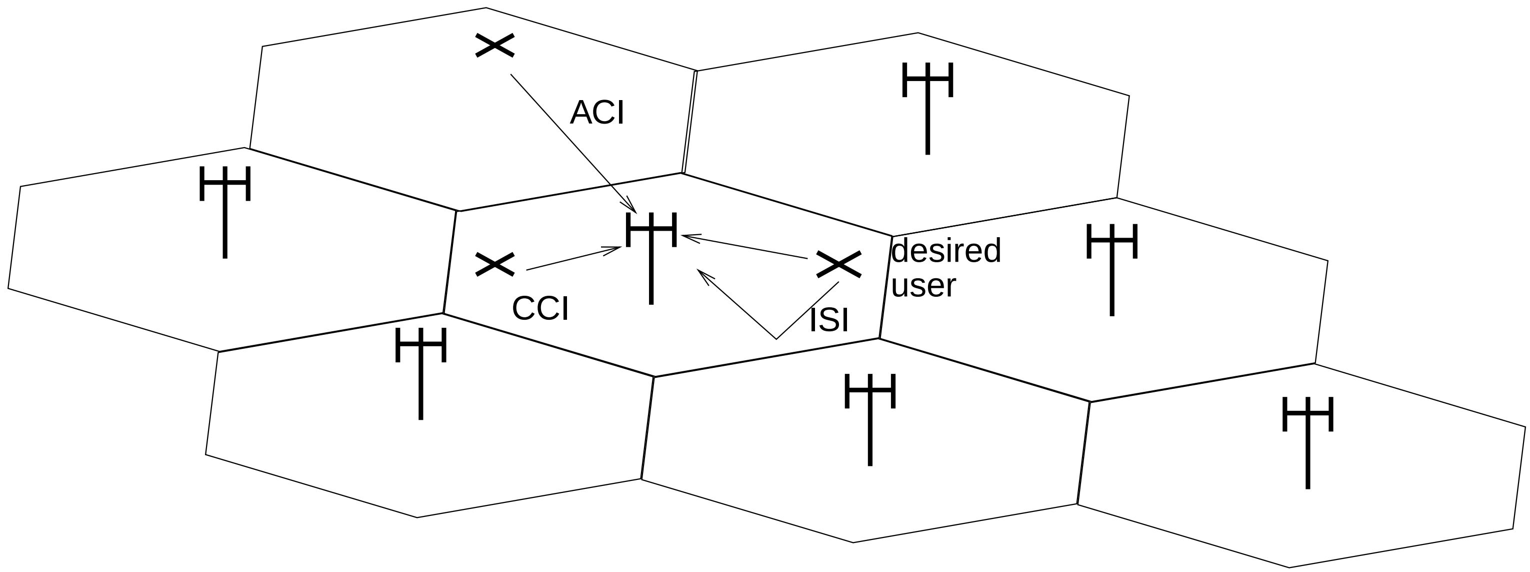
Introduction

Applications of wireless technology

Personal Communications Syst. (PCS) Mobile communications Cordless phones Messaging systems Hand pagers	*Satellite* Mobile Satellite Services GPS VSAT Cellular communications
Wireless Data Wireless local-area networks Wide-area networks	*Dual-Use Applications* Direction-finding radar Commercial spread-spectrum
RF Identifications (RFID) Inventory control and tracking Personnel and plant security Automatic vehicle identification Debit/credit systems	*Automotive* Collision-avoidance/warning In-vehicle navigation systems Intelligent-vehicle highway syst.

Introduction

The cellular network concept



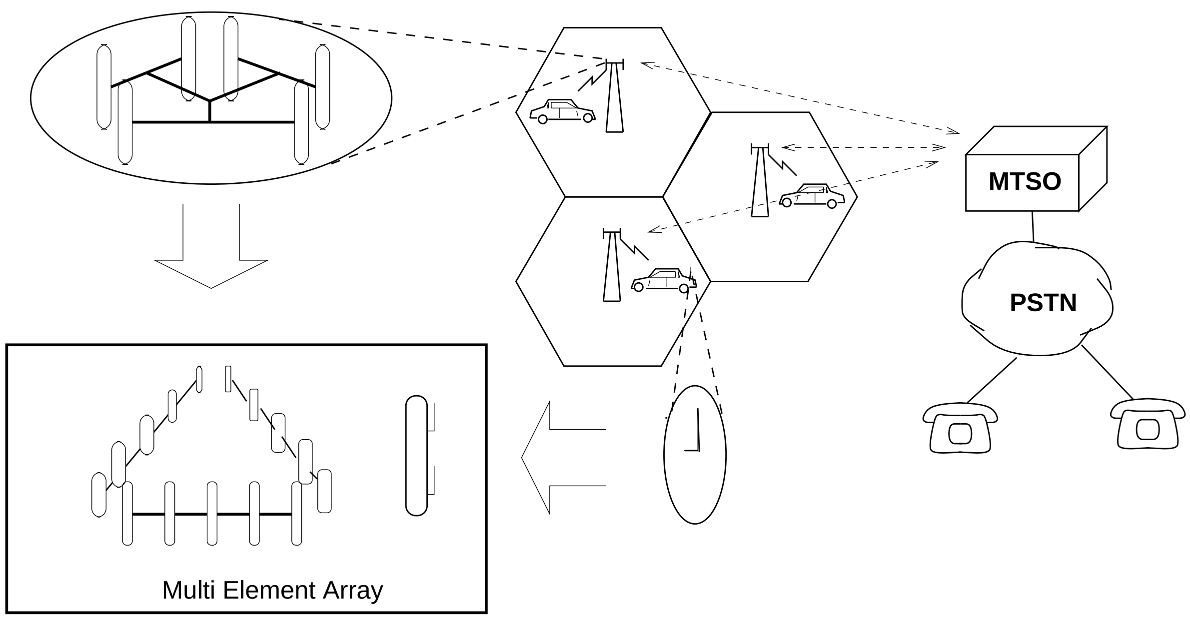
Sources of interference:

- ACI: adjacent channel interference
- CCI: cochannel interference
- ISI: intersymbol interference (time dispersion)

Introduction — applications

Space-time processing

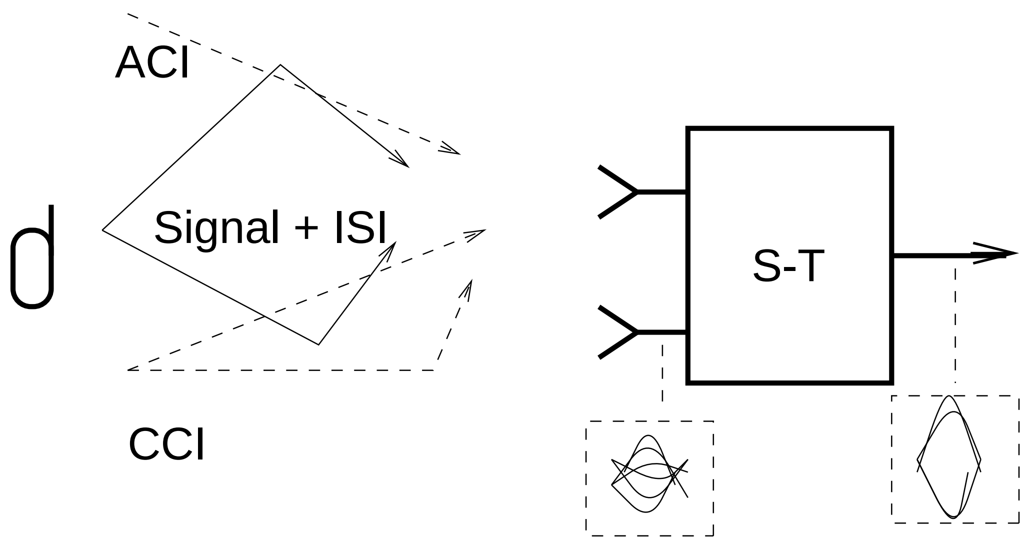
- Use multiple antennas and space-time processing to enhance performance.
- Can provide diversity and interference reduction



Introduction — applications

Why Space-Time processing?

- **Enhance signal** (increase average power and reduce effect of fades)
- **Reduce** adjecent channel, co-channel, and intersymbol **interference**



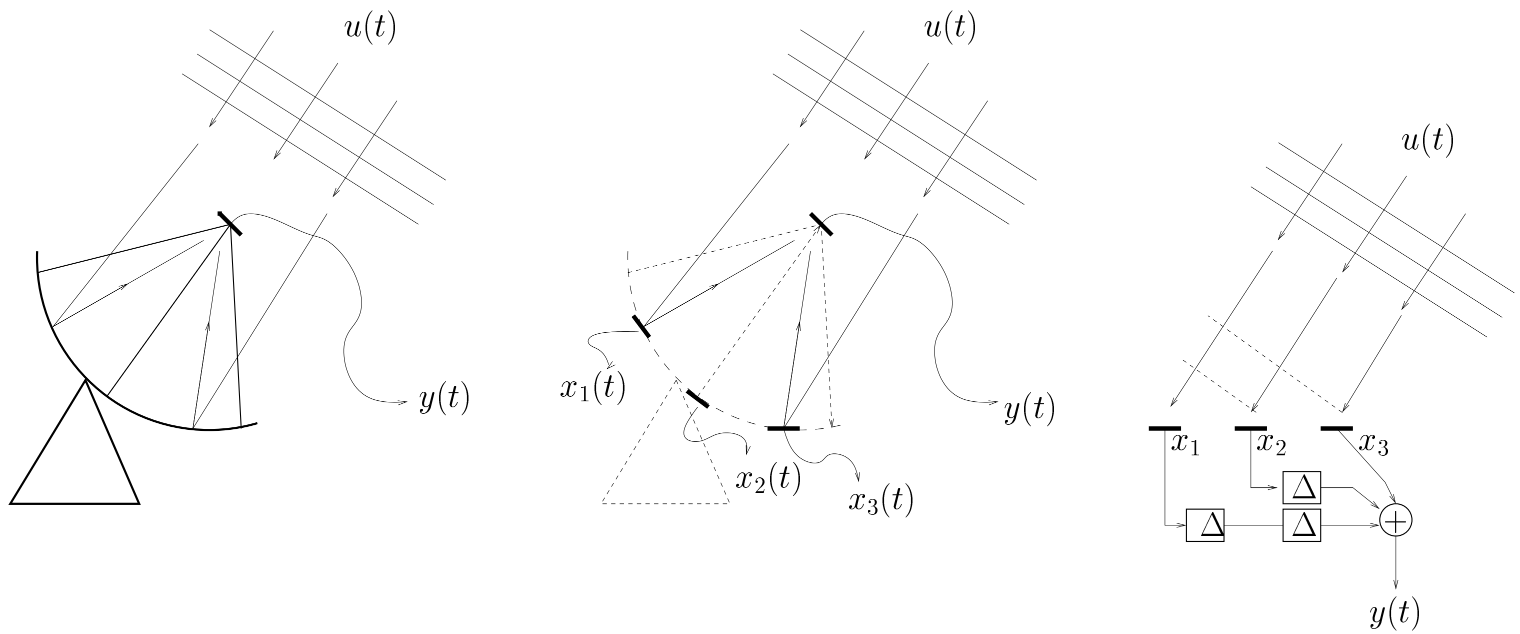
Conclusion: S-T processing promises significant improvements in coverage, capacity, data rate and quality of wireless networks

Introduction — objectives

- **Signal processing** tries to extract information from measured signals
- **signal processing for communications**:
  - Detection
  - Signal enhancement/noise suppression
    - coherent addition
    - spatial-temporal filtering
  - Source/channel characterizations:
    - number of sources
    - location
    - waveforms (information from the sources)
  - Localization and tracking of moving users

Introduction — array processing

Coherent adding



With an array of sensors ($m = 1, \cdots, M$):

$$x_m(t) \;=\; u(t) + n_m(t)\;; \qquad \text{noise variance: } \sigma^2$$

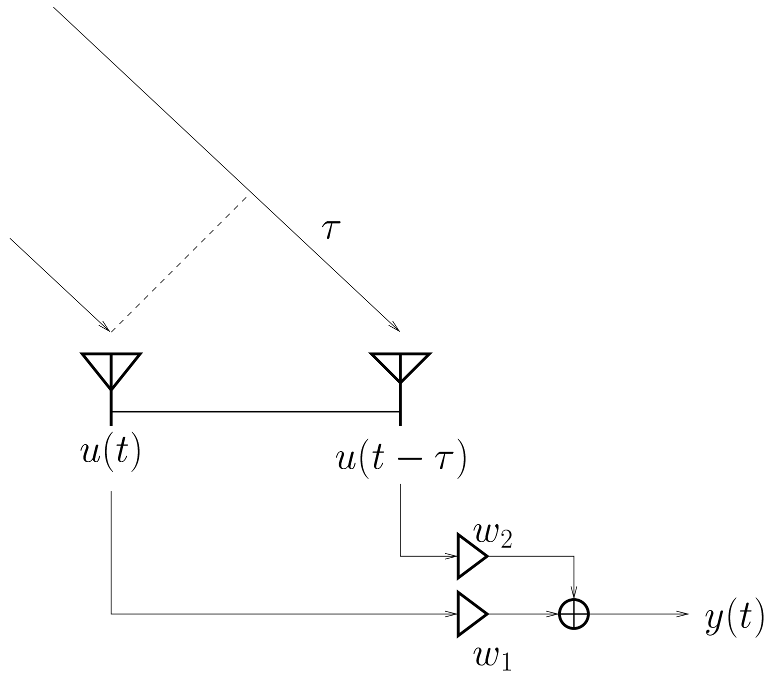
If the noise on the antennas is uncorrelated, then

$$y(t) \;=\; \frac{1}{M}\sum_1^M x_m(t) \;=\; u(t) + \frac{1}{M}\sum_1^M n_m(t) \;=\; u(t) + n(t)\;; \qquad \text{noise variance: } \frac{1}{M}\sigma^2$$

TU Delft

Introduction — array processing

Null-steering

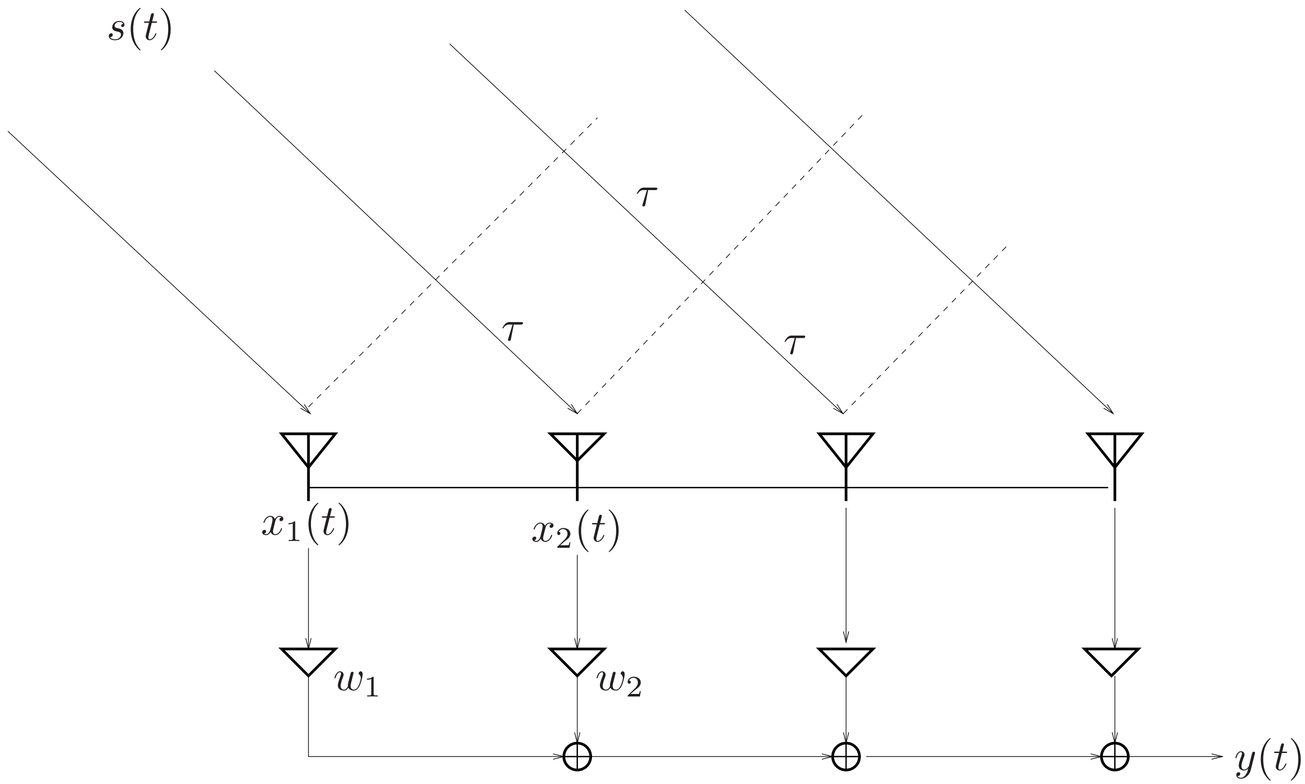


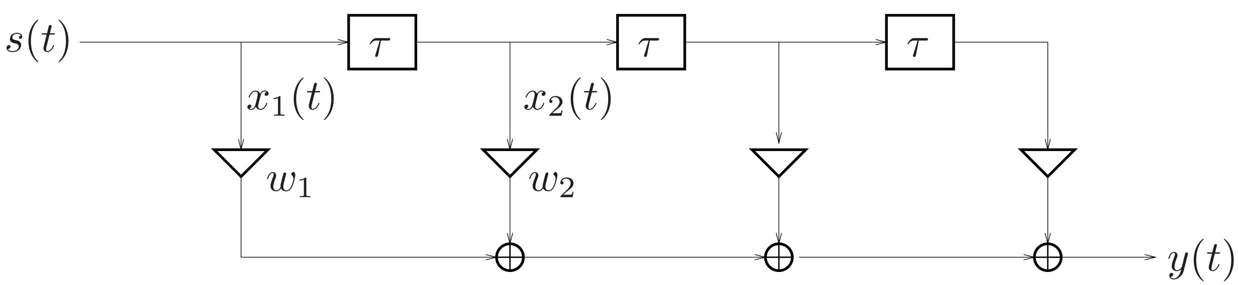
$$
\begin{aligned}
y(t) &= w_1 u(t) + w_2 u(t-\tau) \\
Y(\omega) &= U(\omega)\left(w_1 + w_2 e^{-j\omega\tau}\right)
\end{aligned}
$$

The signal is nulled out, $U(\omega)=0$, at a certain frequency ω_0 if

$$w_2 = -w_1 e^{j\omega_0\tau}$$

Introduction — array processing





Data model

Baseband signal

An antenna receives a real-valued bandpass signal with center frequency f_c

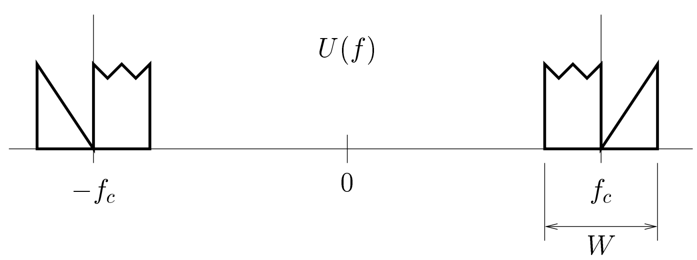
$$u(t) \;=\; \text{real}\{s(t)e^{j2\pi f_c t}\} \;=\; x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t$$

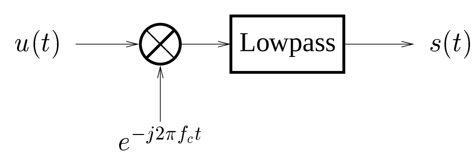
The *baseband signal* or *complex envelope* is

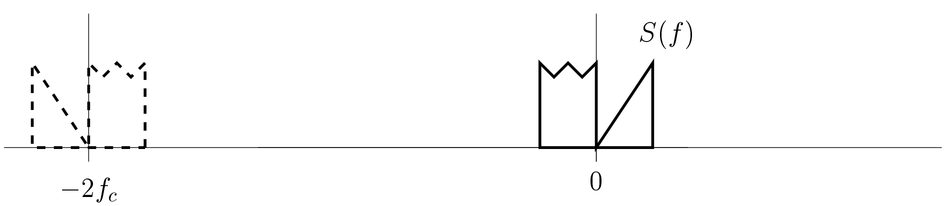
$$s(t) \;=\; x(t) + jy(t)$$

$s(t)$ is recovered from $u(t)$ by *demodulation*:

multiply $u(t)$ with $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ and low-pass filter the resulting signals







Data model

Small delays of narrow band signals

We are interested in the effect of small delays of $u(t)$ on the baseband signal $s(t)$

$$u_\tau(t) \;:=\; u(t-\tau) \;=\; \text{real}\{s(t-\tau)e^{-j2\pi f_c\tau}e^{j2\pi f_c t}\}$$

The complex envelope of the delayed signal is

$$s_\tau(t) = s(t-\tau)e^{-j2\pi f_c\tau}$$

Let W be the bandwidth of $s(t)$. If $\exp(j2\pi f\tau) \approx 1$ for all frequencies $|f| \le \frac{W}{2}$, then

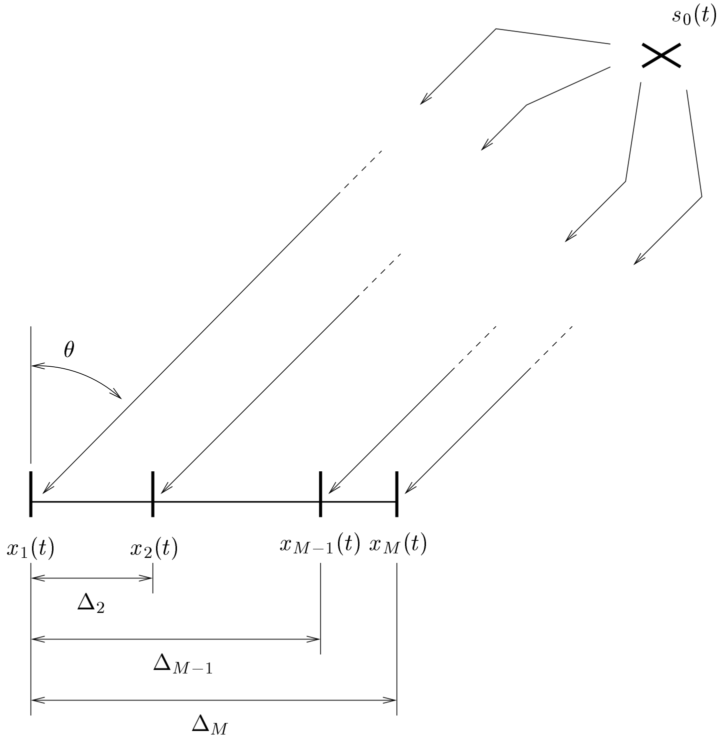
$$s(t-\tau) \;=\; \int_{-W/2}^{W/2} S(f)e^{j2\pi f(t-\tau)}\mathrm{d}f \;\approx\; \int_{-W/2}^{W/2} S(f)e^{j2\pi ft}\mathrm{d}f \quad = s(t)$$

$$\Rightarrow \quad s_\tau(t) \;\approx\; s(t)e^{-j2\pi f_c\tau} \qquad \text{for } W\tau \ll 1$$

Conclusion: for narrowband signals, time delays shorter than the inverse bandwidth amount to phase shifts of the complex envelope.

Data model

Antenna array response



- Far field assumption: planar waves
- θ is the direction of arrival
- A is the attenuation
- T_i is propagation time to i-th element

$$x_i(t) = a(\theta) A s_0(t - T_i) e^{-j2\pi f_c T_i}$$

Define $s(t) = s_0(t - T_1)$, $\tau_i = T_i - T_1$, and $\beta = Ae^{-j2\pi f_c T_1}$, then

$$x_i(t) = a(\theta)\beta s(t - \tau_i) e^{-j2\pi f_c \tau_i}$$

Data model

Antenna array response

τ_i can be expressed as a function of θ and Δ_i (distance in wavelengths)

$$2\pi f_c \tau_i = -2\pi f_c \frac{\delta_i \sin(\theta)}{c} = -2\pi \frac{\delta_i}{\lambda} \sin(\theta) = -2\pi \Delta_i \sin(\theta)$$

If τ_i is small compared to the inverse bandwidth of $s(t)$, then

$$s_{\tau_i}(t) \;=\; s(t)e^{-j2\pi f_c \tau_i} \;=\; s(t)e^{j2\pi \Delta_i \sin(\theta)}$$

Collect the received signals into a vector $\mathbf{x}(t)$:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j2\pi\Delta_2 \sin(\theta)} \\ \vdots \\ e^{j2\pi\Delta_M \sin(\theta)} \end{bmatrix} a(\theta)\beta s(t) =: \mathbf{a}(\theta)\beta s(t)$$

Data model

Antenna array response

$$\mathbf{x}(t) = \begin{bmatrix} 1 \\ e^{j2\pi\Delta_2 \sin(\theta)} \\ \vdots \\ e^{j2\pi\Delta_M \sin(\theta)} \end{bmatrix} a(\theta)\beta s(t) =: \mathbf{a}(\theta)\beta s(t)$$

$\mathbf{a}(\theta)$ is the **array response vector**

For a uniform linear array, $\Delta_i = (i-1)\Delta$, we obtain

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{j2\pi\Delta \sin(\theta)} \\ \vdots \\ e^{j2\pi(M-1)\Delta \sin(\theta)} \end{bmatrix} a(\theta) = \begin{bmatrix} 1 \\ \phi \\ \vdots \\ \phi^{M-1} \end{bmatrix} a(\theta), \quad \phi = e^{j2\pi\Delta \sin(\theta)}$$

The factor $a(\theta)$ is often ommitted (*omnidirectional* and *normalized* antennas)

Data model

Array manifold

The *array manifold* is the curve that the vector $\mathbf{a}(\theta)$ describes when θ is varied:

$$\mathcal{A} = \{\mathbf{a}(\theta) : 0 \leq \theta < 2\pi\}$$

- one source

$$\mathbf{x}(t) \;=\; \mathbf{a}(\theta)\beta s(t)$$

For varying $s(t)$, the vector $\mathbf{x}(t)$ is confined to a **line**

➠ θ can be estimated from $\mathbf{x}(t)$ (direction finding)

- two sources

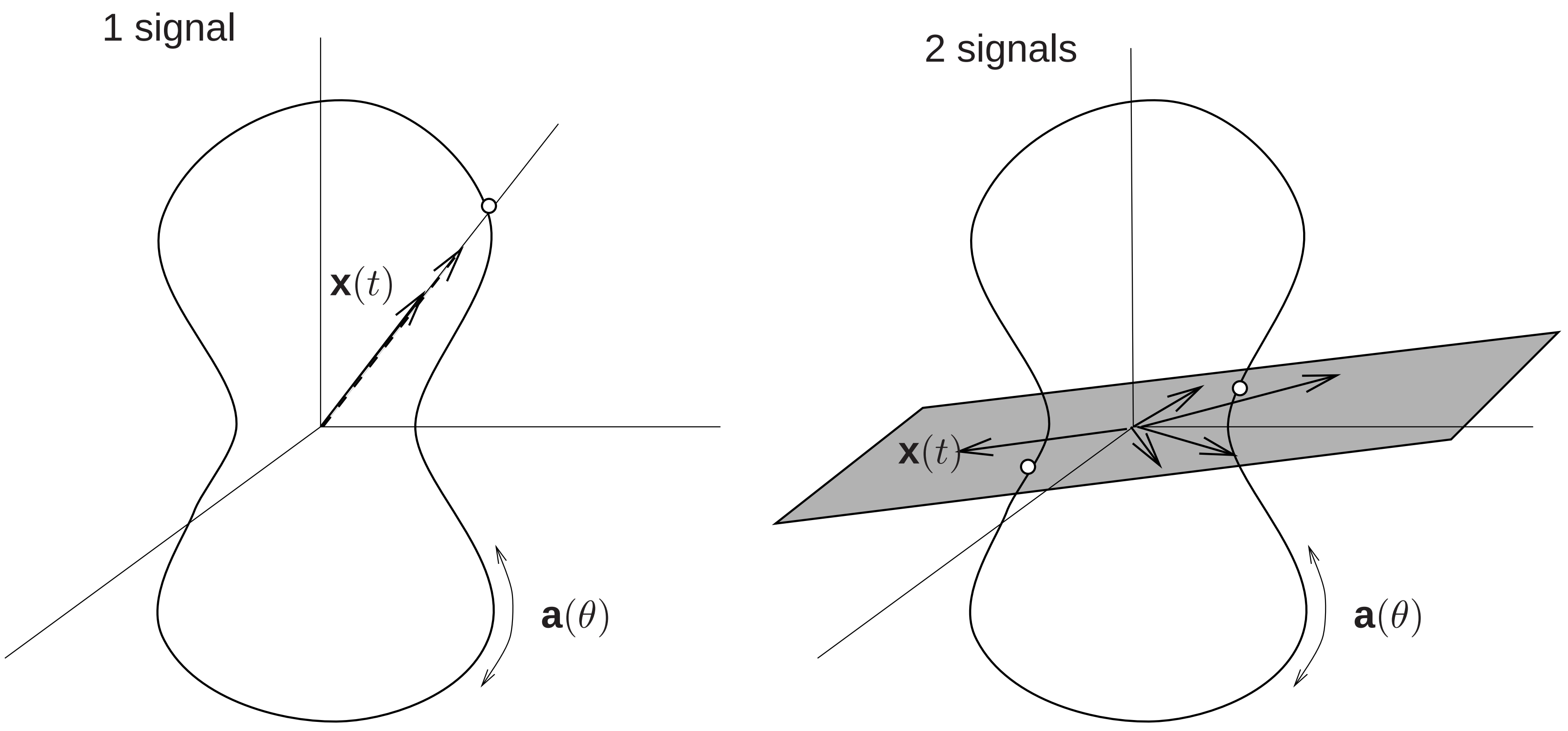
$$\mathbf{x}(t) = \mathbf{a}(\theta_1)\beta_1 s_1(t) \;+\; \mathbf{a}(\theta_2)\beta_2 s_2(t) = [\mathbf{a}(\theta_1) \quad \mathbf{a}(\theta_2)] \begin{bmatrix} \beta_1 & \\ & \beta_2 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix}$$

For varying $s(t)$, the vector $\mathbf{x}(t)$ is now confined to a **plane**

➠ θ_1 and θ_2 can be estimated from $\mathbf{x}(t)$ (direction finding)

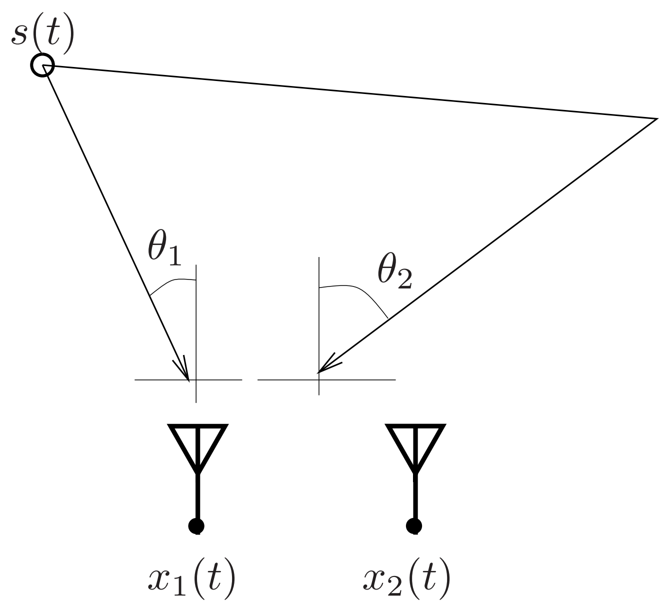
Data model

Principle of direction finding



Data model

Multipath



$$
\begin{aligned}
\mathbf{x}(t) &= \mathbf{a}(\theta_1)\beta_1 s(t) + \mathbf{a}(\theta_2)\beta_2 s(t) \\
&= \{\beta_1 \mathbf{a}(\theta_1) + \beta_2 \mathbf{a}(\theta_2)\}\, s(t) \quad = \mathbf{a}\, s(t)
\end{aligned}
$$

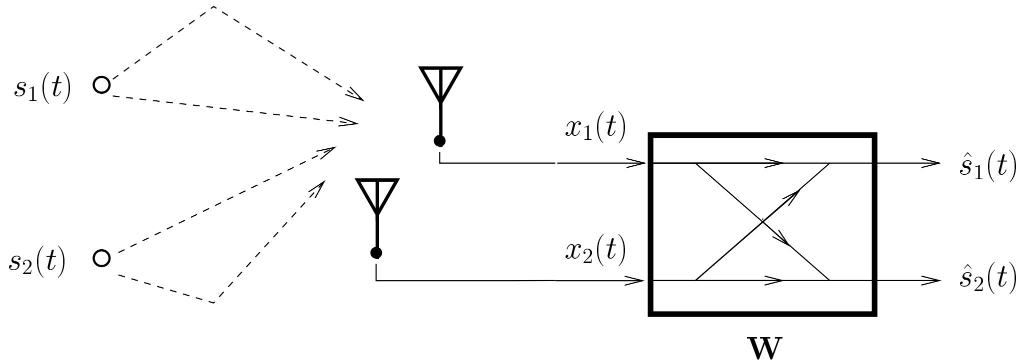
In this case, the combined vector **a** is **not** on the array manifold

➠ direction finding gets much more complicated

At any rate, $\mathbf{x}(t)$ contains an *instantaneous multiple* of $s(t)$

Data model

Instantaneous mixtures



- For narrowband signals, a delay translates into a phase shift
  - ➠ the received data is an **instantaneous linear mixture**:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$

- Collect N samples: $\mathbf{X} = [\mathbf{x}(0), \cdots, \mathbf{x}(N-1)]$ and $\mathbf{S} = [\mathbf{s}(0), \cdots, \mathbf{s}(N-1)]$

$$\mathbf{X} = \mathbf{A}\mathbf{S}$$

- We look for a beamformer such that

$$\mathbf{W}^{\mathrm{H}}\mathbf{x}(t) = \mathbf{s}(t) \qquad \Leftrightarrow \qquad \mathbf{W}^{\mathrm{H}}\mathbf{A} = \mathbf{I}$$

Array response vector

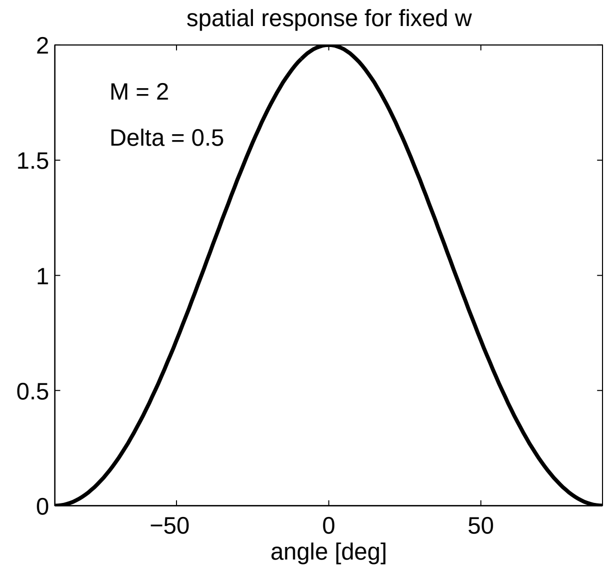
Array response graph

Suppose we choose a beamforming vector $\mathbf{w}$, e.g., $\mathbf{w} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$:

$$y(t) \;=\; \mathbf{w}^{\mathrm{H}}\mathbf{x}(t) \;=\; \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta)\beta s(t)$$

The response of the array to a unit-amplitude signal, $|\beta s(t)| = 1$, from direction θ is

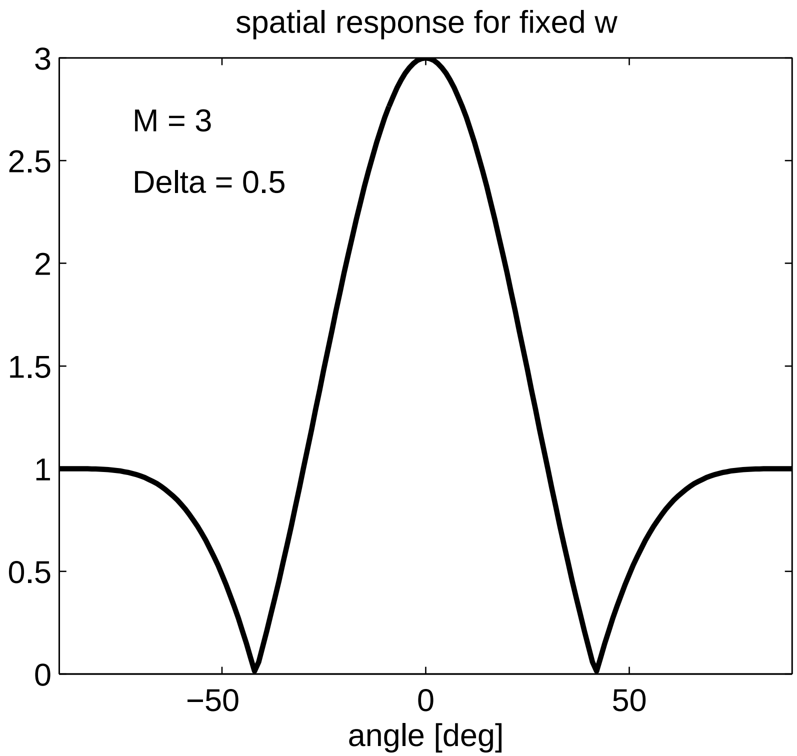
$$|y(t)| = |\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta)|$$

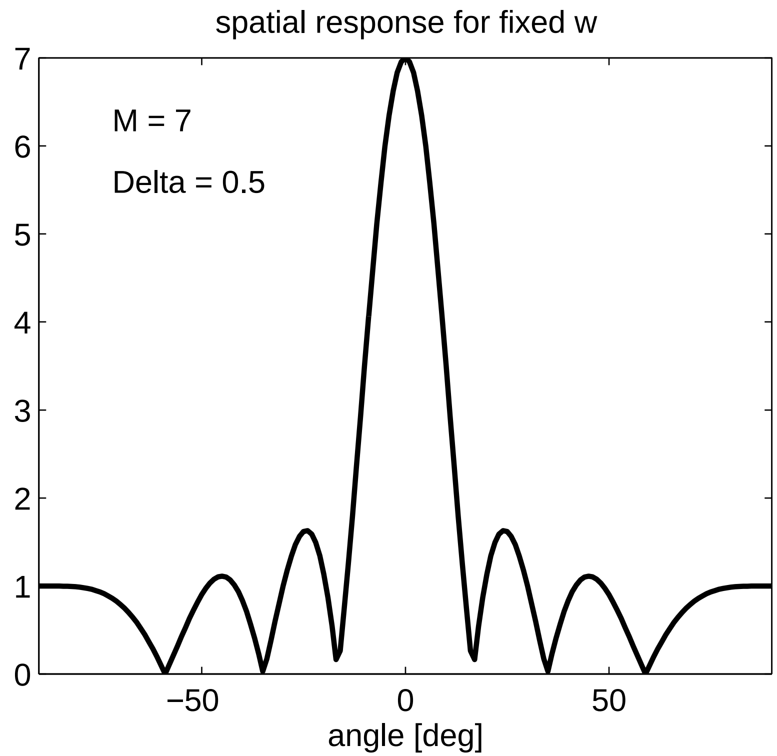


Array response vector

Sidelobes

With a larger number of antennas, resolution improves but sidelobes occur:





Array response vector

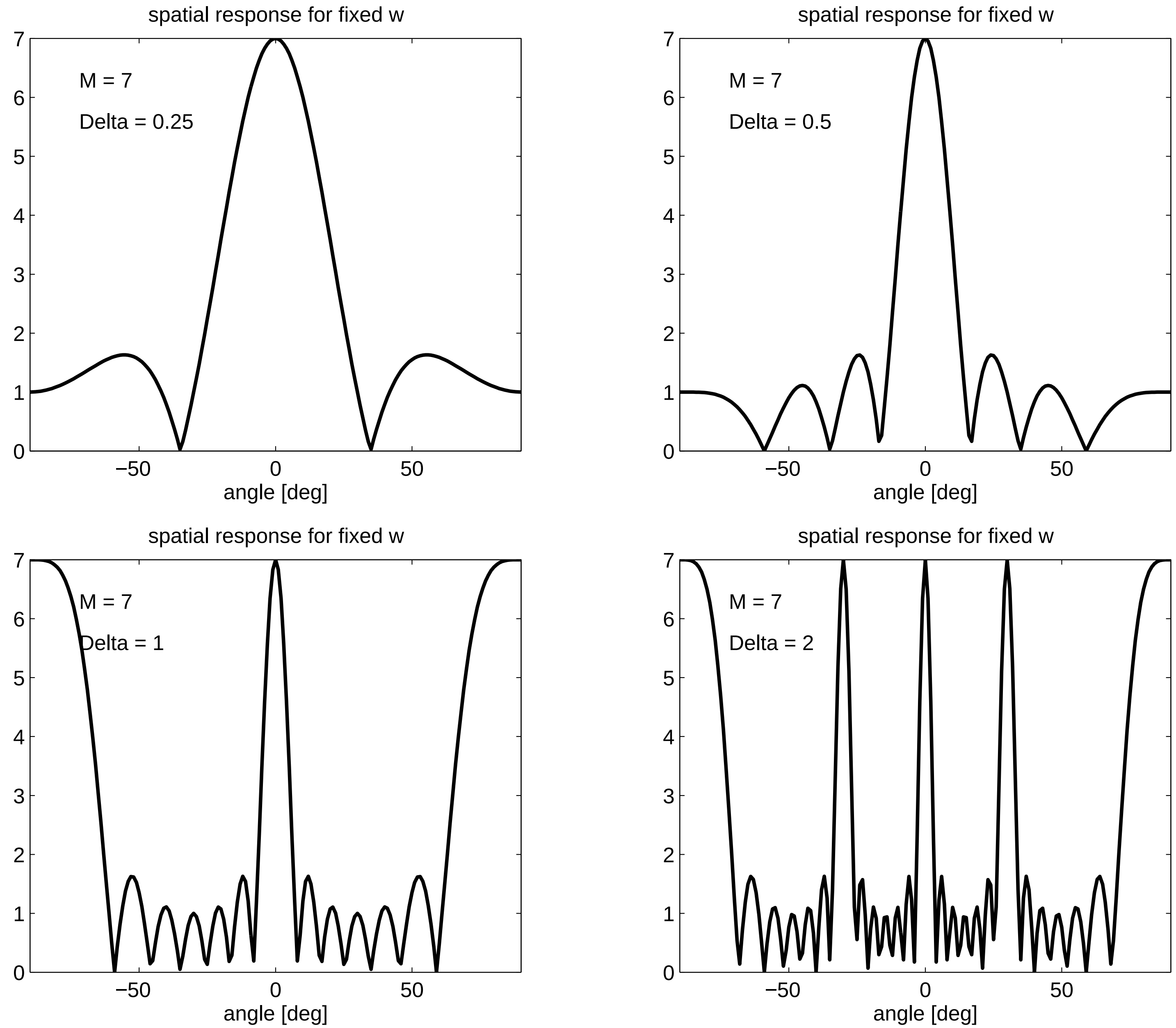
Ambiguity

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ \phi \\ \vdots \\ \phi^{M-1} \end{bmatrix}, \qquad \phi = e^{j2\pi\Delta\sin(\theta)}.$$

$$\sin(\theta) \in [-1, 1] \quad \Rightarrow \quad 2\pi\Delta\sin(\theta) \in [-2\pi\Delta, 2\pi\Delta]$$

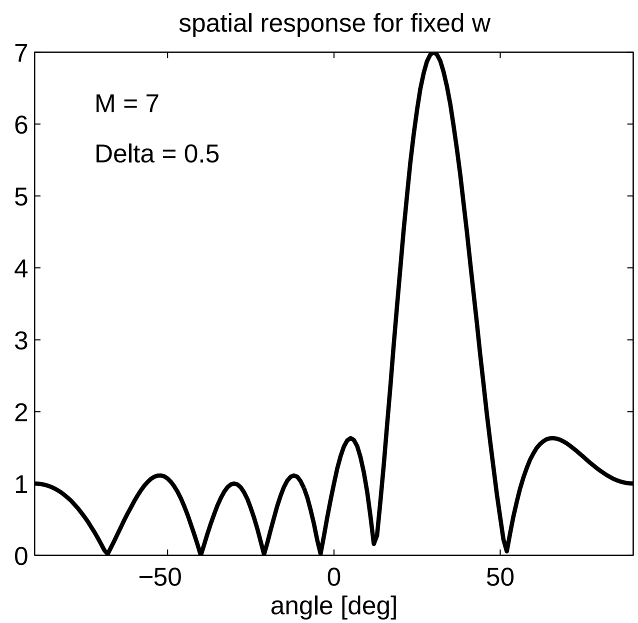
- θ determines ϕ uniquely iff $\Delta \leq \frac{1}{2}$ wavelengths
- For $\Delta > \frac{1}{2}$ there is *spatial aliasing*, and *grating lobes* occur
- We can still estimate **A** and do e.g., nullsteering

Array response vector



Array response vector

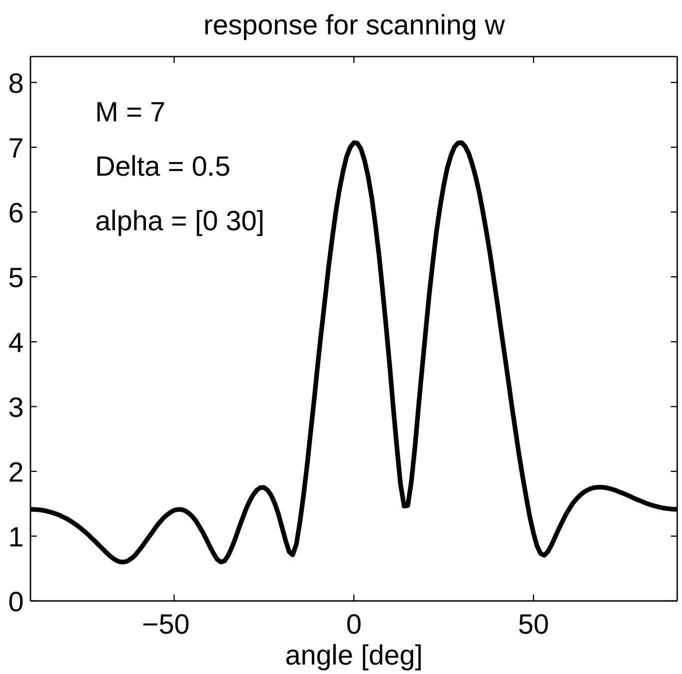
- $\mathbf{w}$ can be used to steer the beam in other directions
- Choose e.g., $\mathbf{w} = \mathbf{a}(30°)$ and look at $|y(t)| = |\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta)|$

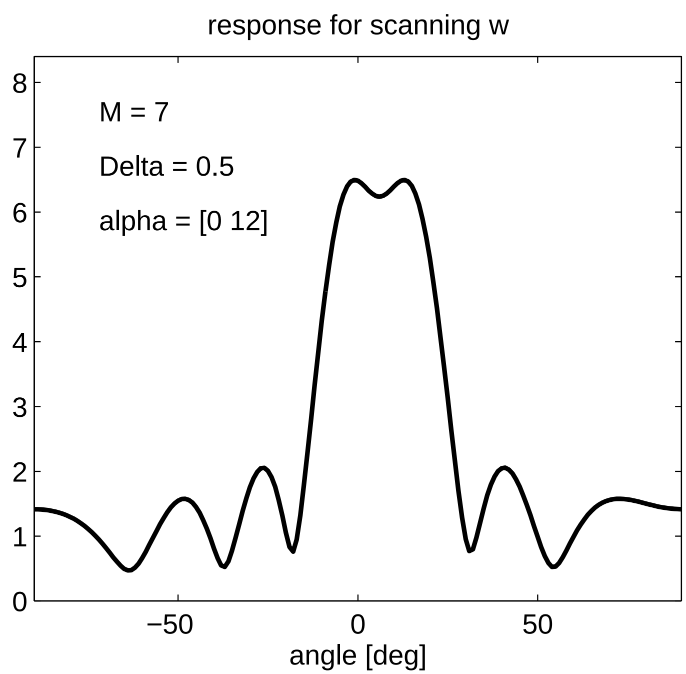


TUDelft

Array response vector

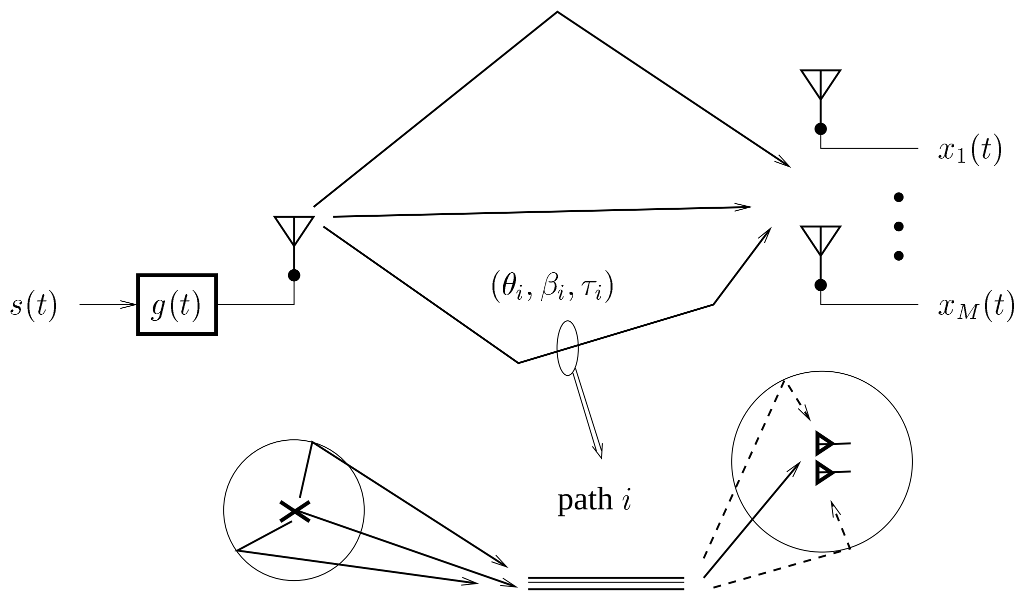
- To estimate directions of sources, we can scan **w**
- Simple scan would be $\{\mathbf{w} = \mathbf{a}(\theta);\ -\pi/2 \leq \theta \leq \pi/2\}$ and look at $|y(t)| = |\mathbf{w}^{\mathrm{H}}\mathbf{x}(t)|$.
- For a single source, this produces precisely the same plots as before
- If two sources are well separated, they can be resolved





Multipath channel

Jakes' model



- scatterers local to mobile
- remote scatterers
- scatteres local to base

Multipath channel

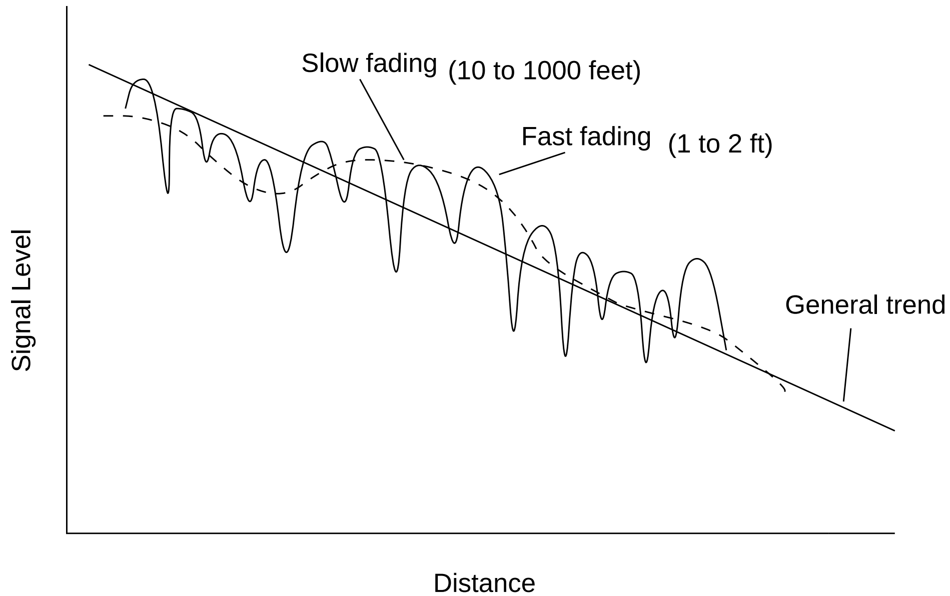
Jakes' model

$$\mathbf{x}(t) = \left[\sum_{i=1}^{r} \mathbf{a}(\theta_i)\beta_i g(t-\tau_i)\right] * s(t)$$

- We bring out the temporal effects $g(t)$ (previously included in $s(t)$)
- We only consider scattering local to mobile
  – Each ray has a (small) angular and delay spread
  – These spreads are usually but not always ignored
  – Creates major effects on the gains β_i ➠ *fading*

Multipath channel

Fading



- **General trend:** ≈ 35 - 50 dB / decade (path loss)
- **Slow fading:** caused by shadowing; typically log-normal distributed
- **Fast fading:** caused by scatterers near mobile; typically Rayleigh distributed

Multipath channel

Derivation of Rayleigh fading

- Gain for single mini-ray is $Ae^{-j2\pi f_c T}$; A: attenuation; T: propagation delay

- Gain β is the result of many such mini-rays:

$$\beta = \sum_{n=1}^{N} A_n e^{-j2\pi f_c T_n}$$

  A_n and T_n are attenuation and delay related to the n-th mini-ray

- If relative delays are independent and uniformly distributed over their range, then $\phi_n = 2\pi f_c T_n \bmod 2\pi$ are independent and uniformly distributed over $[0, 2\pi)$

- The A_n's are usually assumed to be i.i.d. as well

- For large N, the central limit theorem gives

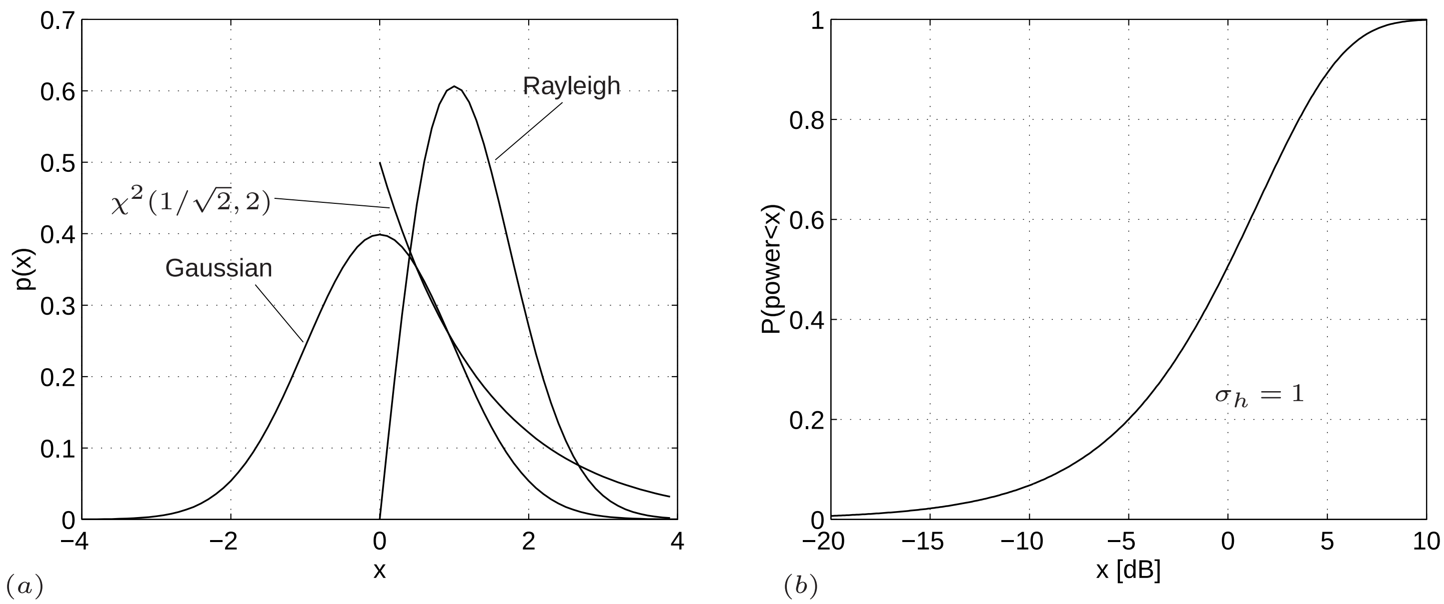
$$\beta \sim \mathcal{CN}(0, \sigma_\beta^2) \qquad \Leftrightarrow \qquad p(\beta) = \frac{1}{\sqrt{2\pi}\sigma_\beta} e^{-\frac{|\beta|^2}{\sigma_\beta^2}}$$

$$\sigma_\beta^2 = \mathrm{E}[|\beta|^2] = N\mathrm{E}[|A_n|^2]$$

Multipath channel

Rayleigh fading (cont'd)

- β has a complex normal distribution with zero mean
- $|\beta|$ is *Rayleigh* distributed
- $|\beta|^2$ is *Chi-square* distributed



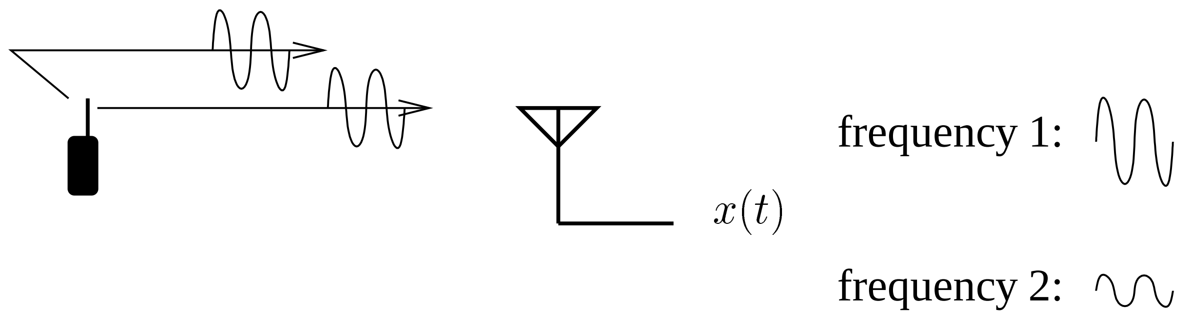
TUDelft

Multipath channel

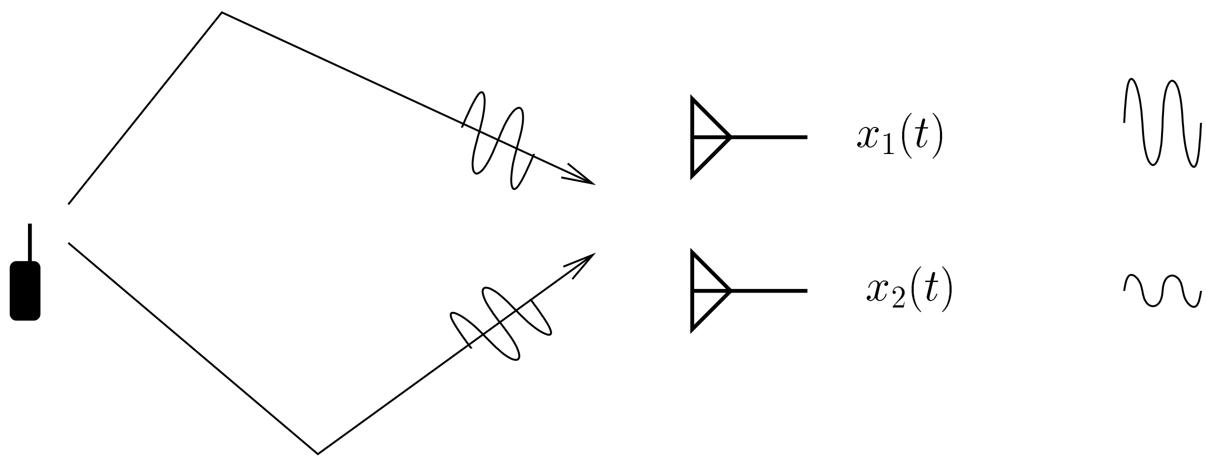
Fading

Time-selective fading (Doppler)

Frequency-selective fading (long delay)



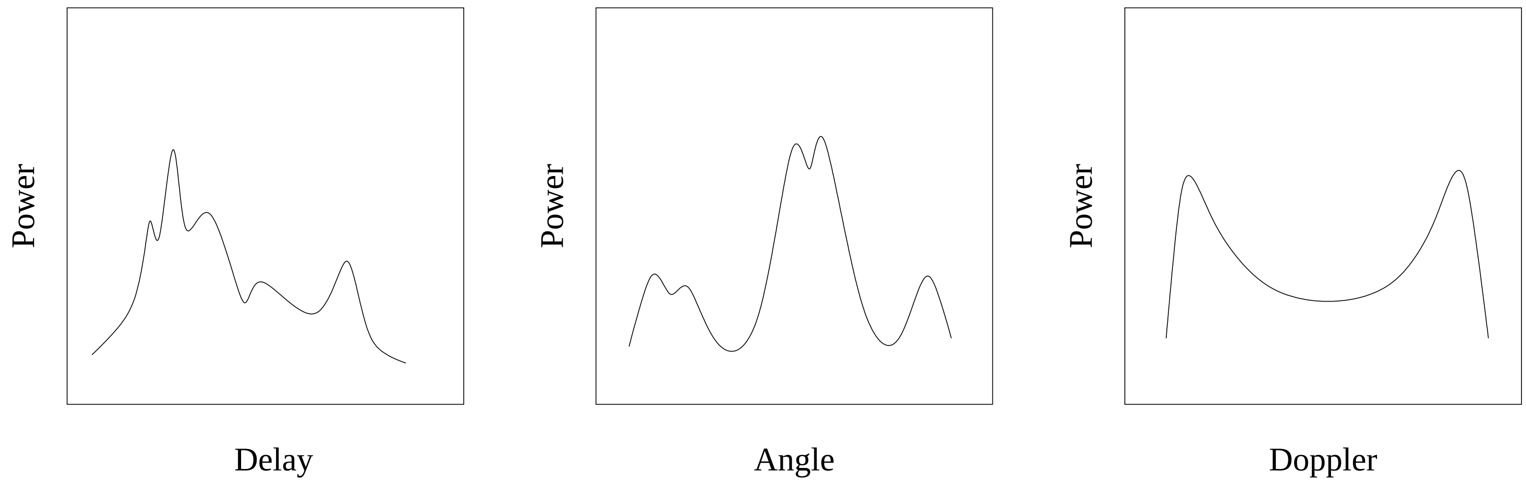
Space-selective fading (large angle)



Multipath channel

Delay, Angle and Doppler Spreads

Local and remote scattering, and mobile motion *spreads* the signal



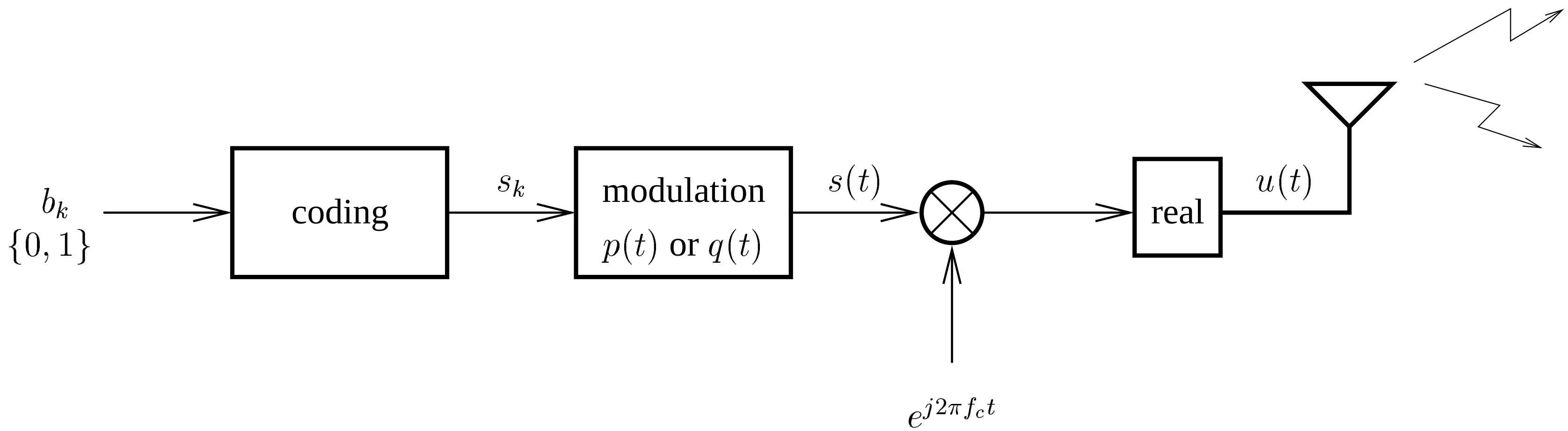
- Delay spread ranges from 0.1 to 20 microsecs
- Angle spread ranges from 1 to 360 degrees
- Doppler spread ranges from 5 to 190 Hz.

Multipath channel

Typical parameter values

Environment	delay spread	angle spread	Doppler spread
Flat rural (macro)	0.5 μs	1°	190 Hz
Urban (macro)	5 μs	20°	120 Hz
Hilly (macro)	20 μs	30°	190 Hz
Mall (micro)	0.3 μs	120°	10 Hz
Indoors (pico)	0.1 μs	360°	5 Hz

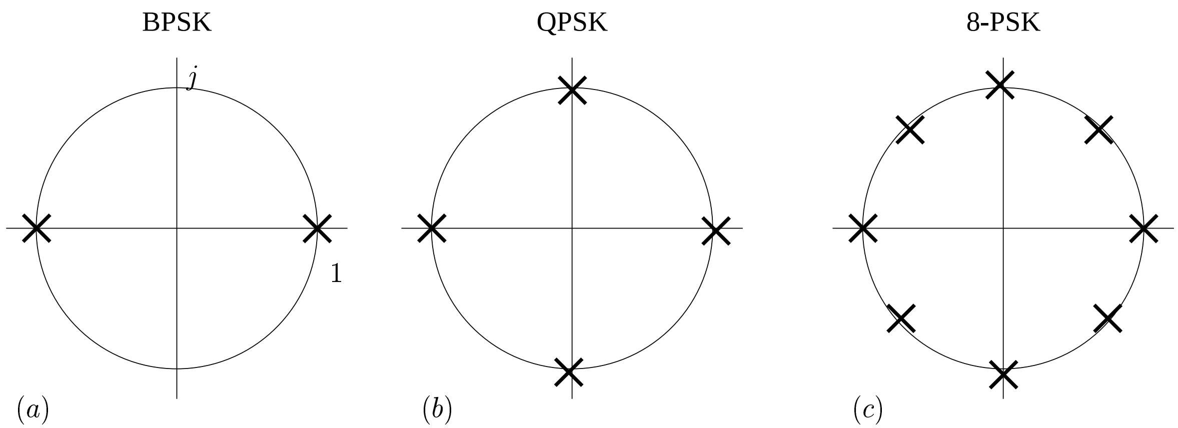
Signal modulation



- Digital alphabets
- Modulation
  - – Linear modulation
  - – Phase modulation

Signal modulation

Digital constellations



$b_k \in \{0,1\} \quad \rightarrow \quad s_k$ chosen from (up to a possible scaling):	
BPSK	$\{1, -1\}$
QPSK	$\{1, j, -1, -j\}$
m-PSK	$\{1, e^{j2\pi/m}, e^{j2\pi 2/m}, \ldots, e^{j2\pi(m-1)/m}\}$
m-PAM	$\{\pm 1, \pm 3, \ldots, \pm(m-1)\}$
m-QAM	$\{\pm 1 \pm j, \pm 1 \pm 3j, \ldots, \pm 1 \pm (\sqrt{m}-1)j, \pm 3 \pm j, \ldots, \pm(\sqrt{m}-1) \pm (\sqrt{m}-1)j\}$

Signal modulation

Linear modulation

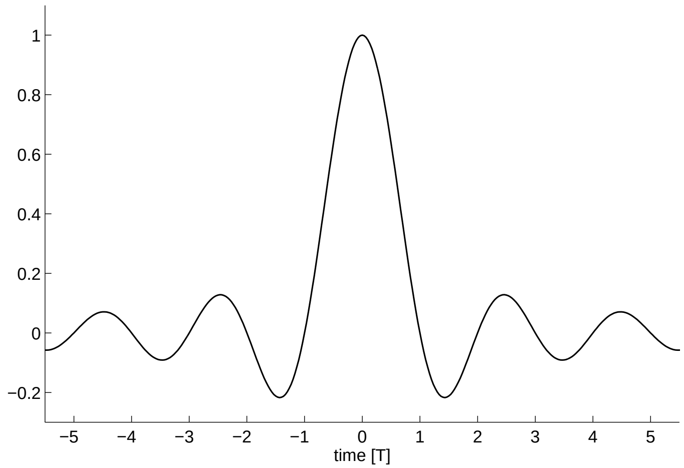
- Amplitude is modulated:

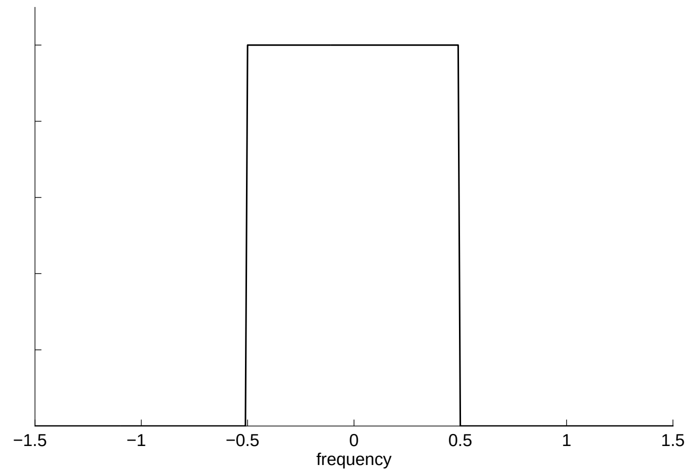
$$s_\delta(t) = \sum_{k=-\infty}^{\infty} s_k \delta(t-k) \qquad s(t) = p(t) * s_\delta(t) = \sum_{k=-\infty}^{\infty} s_k p(t-k)$$

- Optimum waveform is both localized in time and frequency (does not exist)

- One possible choice: sinc pulse shape

$$p(t) = \frac{\sin \pi t}{\pi t}, \qquad P(f) = \begin{cases} 1, & |f| < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

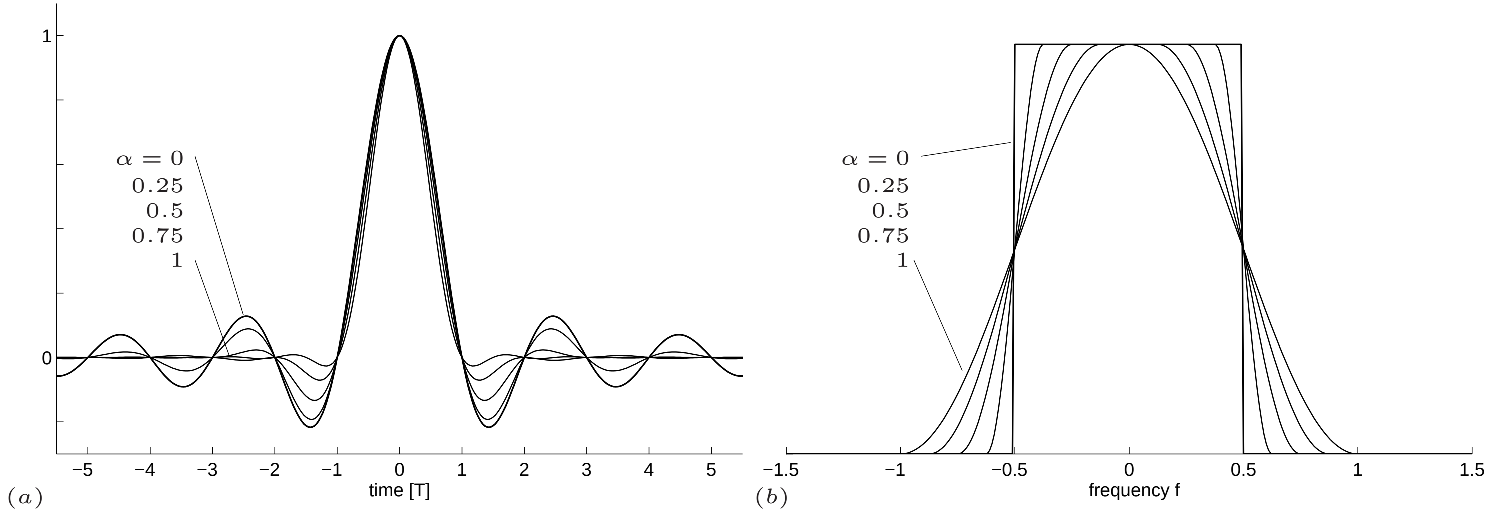




Linear modulation

- Modification: *raised-cosine* pulse shape

$$p(t) = \frac{\sin \pi t}{\pi t} \cdot \frac{\cos \alpha \pi t}{1 - 4\alpha^2 t^2} \qquad P(f) = \begin{cases} 1\,, & |f| < \frac{1}{2}(1-\alpha) \\ \frac{1}{2} - \frac{1}{2}\sin(\frac{\pi}{\alpha}(|f| - \frac{1}{2}))\,, & \frac{1}{2}(1-\alpha) < |f| < \frac{1}{2}(1+\alpha) \\ 0\,, & \text{otherwise} \end{cases}$$



α: excess bandwidth (or rolloff) parameter; common choice is $\alpha = 0.35$

Signal modulation

Linear modulation

- Sampling raised-cosine pulses at integer time instants k: $s(k) \neq 0$ only for $k = 0$
- As a result, $s_k = s(k)$, which means we can recover s_k from $s(t)$ (if synchronized)
- Sometimes $p(t)$ is considered part of the filter $g(t)$:

$$\begin{aligned}\mathbf{x}(t) &= \left[\sum_{i=1}^{r} \mathbf{a}(\theta_i)\beta_i g(t-\tau_i)\right] * s_\delta(t) \\ &= \sum_{k=-\infty}^{\infty} \left[\sum_{i=1}^{r} \mathbf{a}(\theta_i)\beta_i g(t-k-\tau_i)\right] s_k\end{aligned}$$

Signal modulation

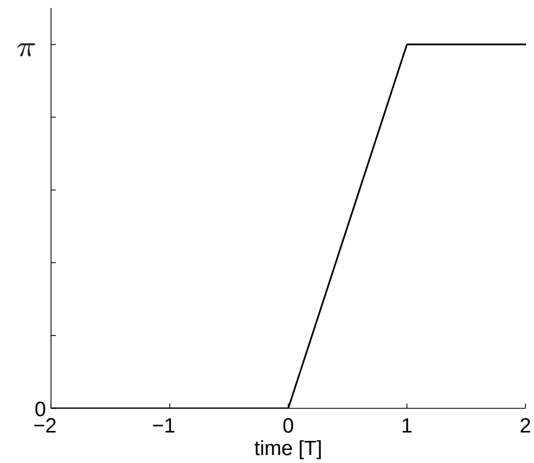
Phase modulation

- Phase is modulated:

$$s(t) = e^{j\phi(t)} \qquad \phi(t) = q(t) * s_\delta(t) = \sum_{k=-\infty}^{\infty} s_k q(t-k)$$

- A simple choice for $q(t)$, used for BPSK, is

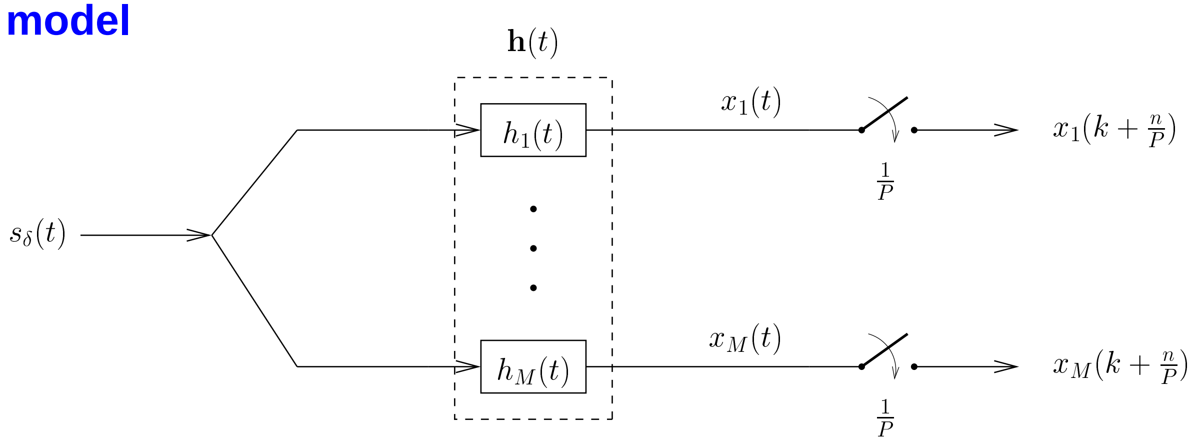
$$q(t) = \begin{cases} 0 & t < 0\,, \\ \pi t & 0 \le t < 1\,, \\ \pi & t \ge 1 \end{cases}$$



- Abrupt changes in phase can also be avoided

Macroscopic channel model

FIR channel model



- We collect all temporal effects in $\mathbf{h}(t)$
  - Filtering effects at transmitter and receiver
  - propagation channel
  - pulse shape for linear modulation
- We obtain the data model

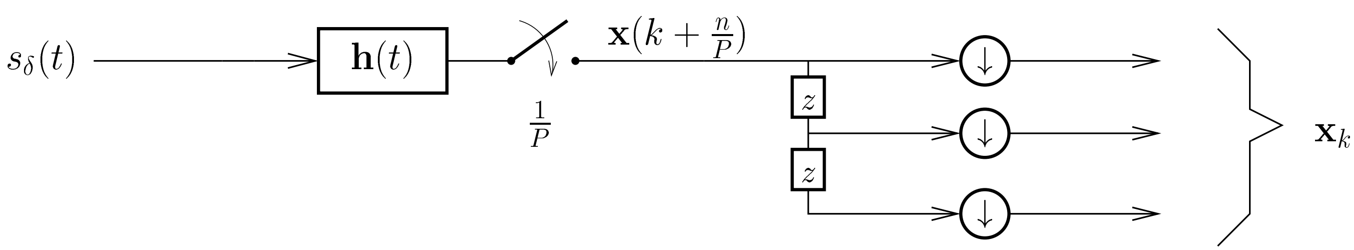
$$\mathbf{x}(t) = \mathbf{h}(t) * s_\delta(t) \qquad \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix}, \quad \mathbf{h}(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_M(t) \end{bmatrix}$$

Macroscopic channel model

- We sample $\mathbf{x}(t)$ at a rate of P samples per symbol (P is the oversampling factor):

$$\mathbf{x}(k+\tfrac{n}{P}) = \left[\mathbf{h}(\tfrac{n}{P}) \;\; \mathbf{h}(1+\tfrac{n}{P}) \;\; \ldots \;\; \mathbf{h}(L-1+\tfrac{n}{P})\right] \begin{bmatrix} s_k \\ s_{k-1} \\ \vdots \\ s_{k-L+1} \end{bmatrix}, \qquad n = 0, 1, \ldots, P-1$$

$$\mathbf{x}_k := \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+\frac{1}{P}) \\ \vdots \\ \mathbf{x}(k+\frac{P-1}{P}) \end{bmatrix} = \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(1) & \cdots & \mathbf{h}(L-1) \\ \mathbf{h}(\frac{1}{P}) & \mathbf{h}(1+\frac{1}{P}) & \cdots & \mathbf{h}(L-1+\frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ \mathbf{h}(\frac{P-1}{P}) & \mathbf{h}(1+\frac{P-1}{P}) & \cdots & \mathbf{h}(L-1+\frac{P-1}{P}) \end{bmatrix} \begin{bmatrix} s_k \\ s_{k-1} \\ \vdots \\ s_{k-L+1} \end{bmatrix}$$



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Macroscopic channel model

- Construct a data matrix $\mathbf{X}$ as

$$\mathbf{X} = [\mathbf{x}_0 \quad \cdots \quad \mathbf{x}_{N-1}] := \begin{bmatrix} \mathbf{x}(0) & \mathbf{x}(1) & \cdots & \mathbf{x}(N-1) \\ \mathbf{x}(\frac{1}{P}) & \mathbf{x}(1+\frac{1}{P}) & \cdots & \mathbf{x}(N-1+\frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ \mathbf{x}(\frac{P-1}{P}) & \mathbf{x}(1+\frac{P-1}{P}) & \cdots & \mathbf{x}(N-1+\frac{P-1}{P}) \end{bmatrix} : \quad MP \times N$$

- $\mathbf{X}$ has a factorization

$$\mathbf{X} = \mathbf{H}\mathcal{S}_L$$

$$:= \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(1) & \cdots & \mathbf{h}(L-1) \\ \mathbf{h}(\frac{1}{P}) & \mathbf{h}(1+\frac{1}{P}) & \cdots & \mathbf{h}(L-1+\frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ \mathbf{h}(\frac{P-1}{P}) & \mathbf{h}(1+\frac{P-1}{P}) & \cdots & \mathbf{h}(L-1+\frac{P-1}{P}) \end{bmatrix} \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \\ s_{-1} & s_0 & \cdots & s_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{-L+1} & s_{-L+2} & \cdots & s_{N-L} \end{bmatrix}$$

$$\mathbf{H} : MP \times L \qquad \mathcal{S}_L : L \times N$$

Macroscopic channel model

- For space-time equalization over m symbol intervals, construct

$$\mathcal{X}_m = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \cdots & \mathbf{x}_{N-1} \\ \mathbf{x}_{-1} & \mathbf{x}_0 & \cdots & \mathbf{x}_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_{-m+1} & \mathbf{x}_{-m+2} & \cdots & \mathbf{x}_{N-m} \end{bmatrix} : \quad mMP \times N$$

- $\mathcal{X}_m$ has a factorization

$$\mathcal{X}_m = \mathcal{H}_m \mathcal{S}_{L+m-1}$$

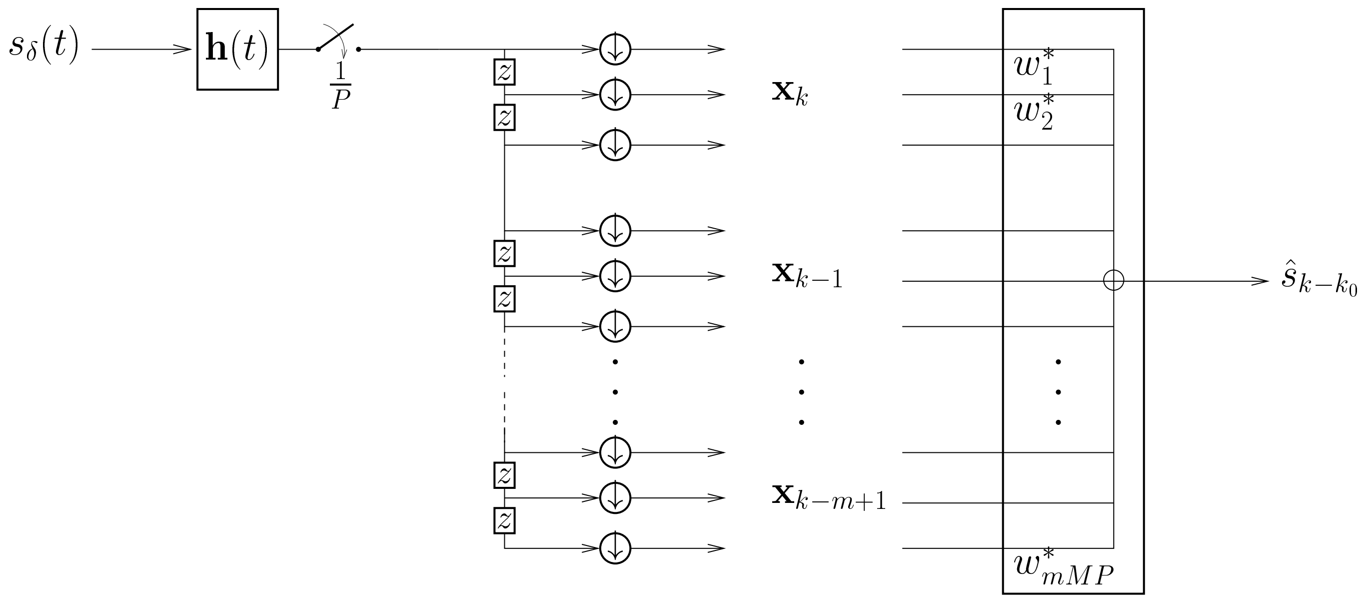
$$:= \begin{bmatrix} \boxed{\mathbf{H}} & & \mathbf{0} \\ & \boxed{\mathbf{H}} & \\ & \ddots & \\ \mathbf{0} & & \boxed{\mathbf{H}} \end{bmatrix} \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \\ s_{-1} & s_0 & \cdots & s_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{-L-m+2} & s_{-L-m+3} & \cdots & s_{N-L-m+1} \end{bmatrix}$$

$$\mathcal{H}_m : mMP \times (L+m-1) \qquad \mathcal{S}_{L+m-1} : (L+m-1) \times N$$

Macroscopic channel model

- A space-time equalizer is a vector **w** which combines the rows of $\mathcal{X}_m$:

$$\mathbf{w}^H \mathcal{X}_m = [\hat{s}_{-k_0}\ \hat{s}_{1-k_0}\ \cdots\ \hat{s}_{N-1-k_0}]$$



- If we increase m by 1: 1 new row in $\mathcal{S}_{L+m-1}$ BUT MP new rows in $\mathcal{X}_m$

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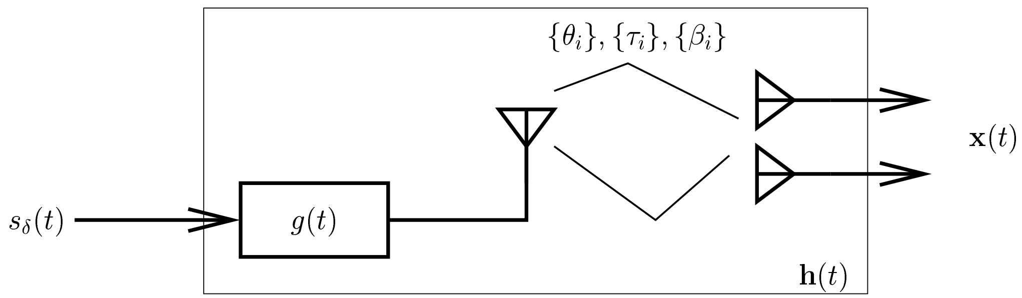
Macroscopic channel model

Connection to the multiray model

- The multiray propagation model is (for specular multipath)

$$\mathbf{h}(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_M(t) \end{bmatrix} = \sum_{i=1}^{r} \mathbf{a}(\theta_i)\,\beta_i\, g(t-\tau_i)$$

$g(t)$:	temporal effects (filtering + raised-cosine pulse shape)
θ_i:	direction-of-arrival
τ_i:	propagation delay
$\beta_i \in \mathbb{C}$:	complex path attenuation (fading)



Macroscopic channel model

Connection to multiray model

- Collect samples of $h_i(t)$ into a row vector

$$\mathbf{h}_i = [h_i(0) \quad h_i(\tfrac{1}{P}) \quad \cdots \quad h_i(L-\tfrac{1}{P})]$$

- Similarly, collect the samples of $g(t-\tau)$ into a row vector

$$\mathbf{g}(\tau) = [g(-\tau) \quad g(\tfrac{1}{P}-\tau) \quad \cdots \quad g(L-\tfrac{1}{P}-\tau)]$$

- The channel model can be written as

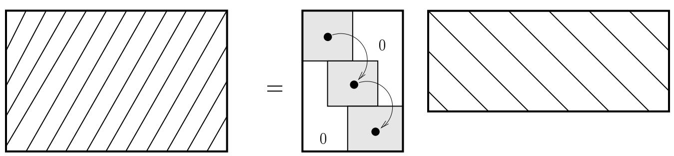
$$\mathbf{H}' = \begin{bmatrix} \text{—}\mathbf{h}_1\text{—} \\ \vdots \\ \text{—}\mathbf{h}_M\text{—} \end{bmatrix} = \begin{bmatrix} | & & | \\ \mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_r) \\ | & & | \end{bmatrix} \begin{bmatrix} \beta_1 & & \\ & \ddots & \\ & & \beta_r \end{bmatrix} \begin{bmatrix} \text{—}\mathbf{g}(\tau_1)\text{—} \\ \vdots \\ \text{—}\mathbf{g}(\tau_r)\text{—} \end{bmatrix} =: \mathbf{ABG}$$

- **H** and **H**′ are "the same", but reorganized: **H** is $MP \times L$ and **H**′ is $M \times LP$.

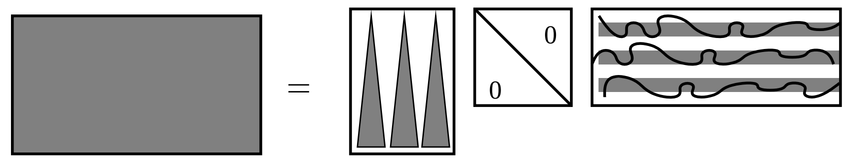
Macroscopic channel model

Summary of properties

$\mathcal{X} = \mathcal{H}\mathcal{S}$



$\mathbf{H}' = \mathbf{ABG}$



Properties		$\mathcal{H}$	$\mathcal{S}$
macro	matrix	block Toeplitz	Toeplitz
		col$(\mathcal{H})=$ col$(\mathcal{X})$	row$(\mathcal{S})=$ row$(\mathcal{X})$
	modulation		FA, CM, non-Gaussian, $\cdots$
	temporal	cyclostationarity	independence
parametric	temporal	known $\mathbf{g}(\tau)$	training: known $\{s_k\}$
	spatial	known $\mathbf{a}(\theta)$	

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