Subspace intersection tracking using GSVD and the Signed URV algorithm

Mu Zhou and Alle-Jan van der Veen

TU Delft, The Netherlands

TUDelft

Outline

- **Part I: Application**

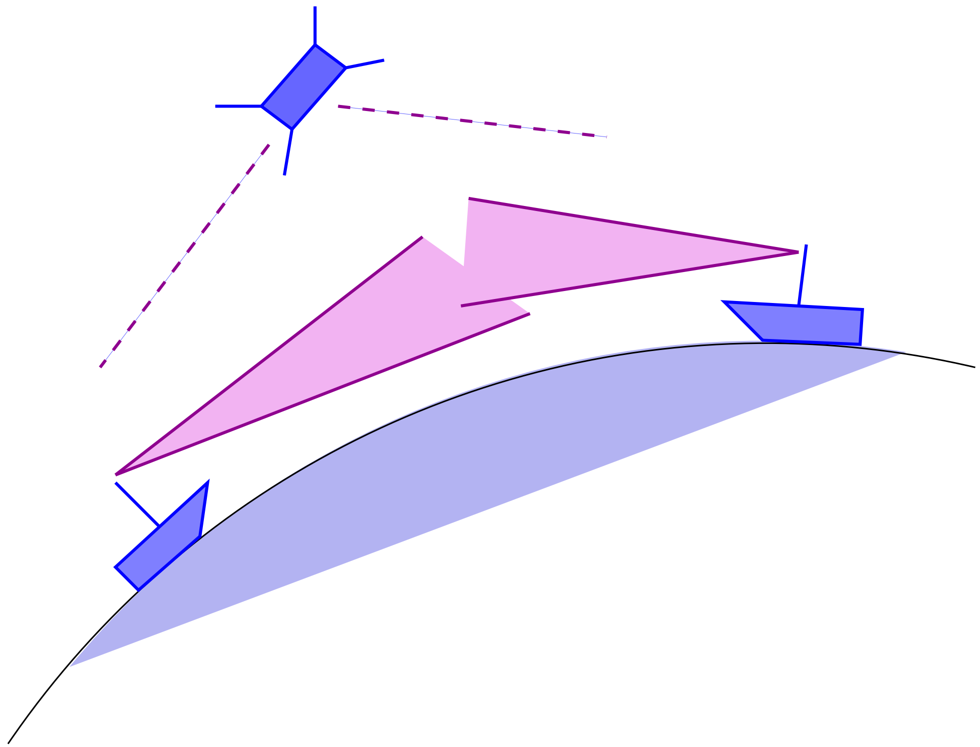
  1. AIS ship transponder signal separation
  2. Algorithm based on Generalized SVD (GSVD)

- **Part II: Subspace tracking**

  1. Signed (hyperbolic) URV to approximate the GSVD
  2. Updating the SURV

AIS signal separation

Automatic Idenfication of Ships (AIS)



- Short data packets in a TDMA system, only partial synchronization
- On surface: $\approx$ 50 km range; from satellite: $\approx$ 500 km range
- Many partially overlapping signals, need blind source separation

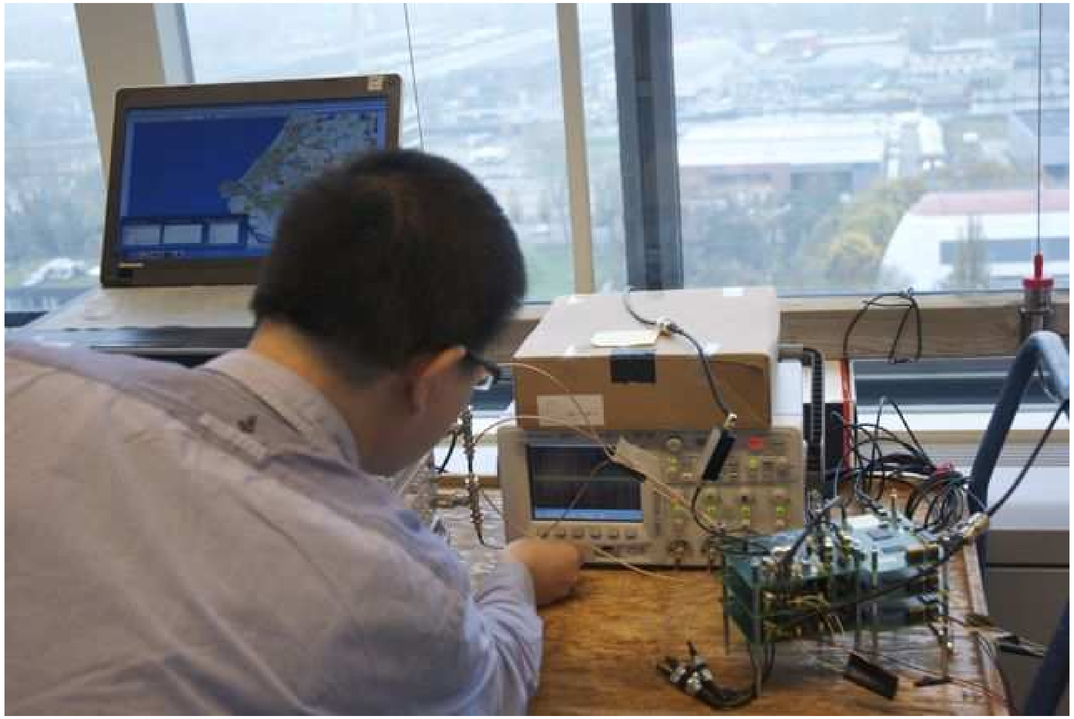
AIS signal separation

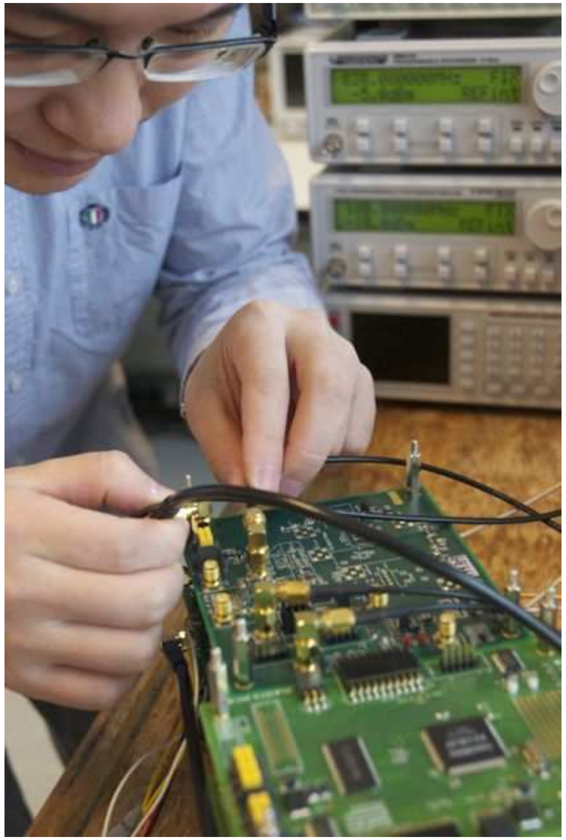
ISIS AIS satellite prototype (Triton-1 mission)

Launched 2013

AIS signal separation

TU Delft AIS 4-channel receiver





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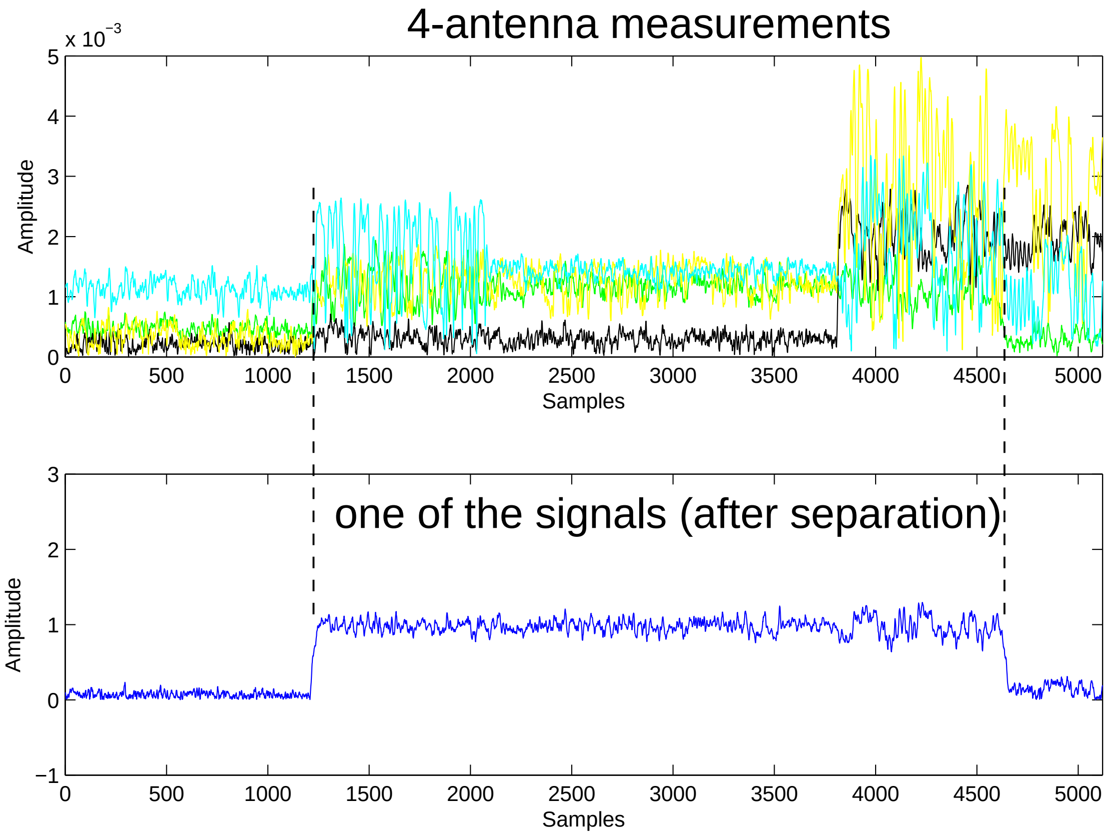
AIS signal separation

AIS overlapping signals

Example of a measurement



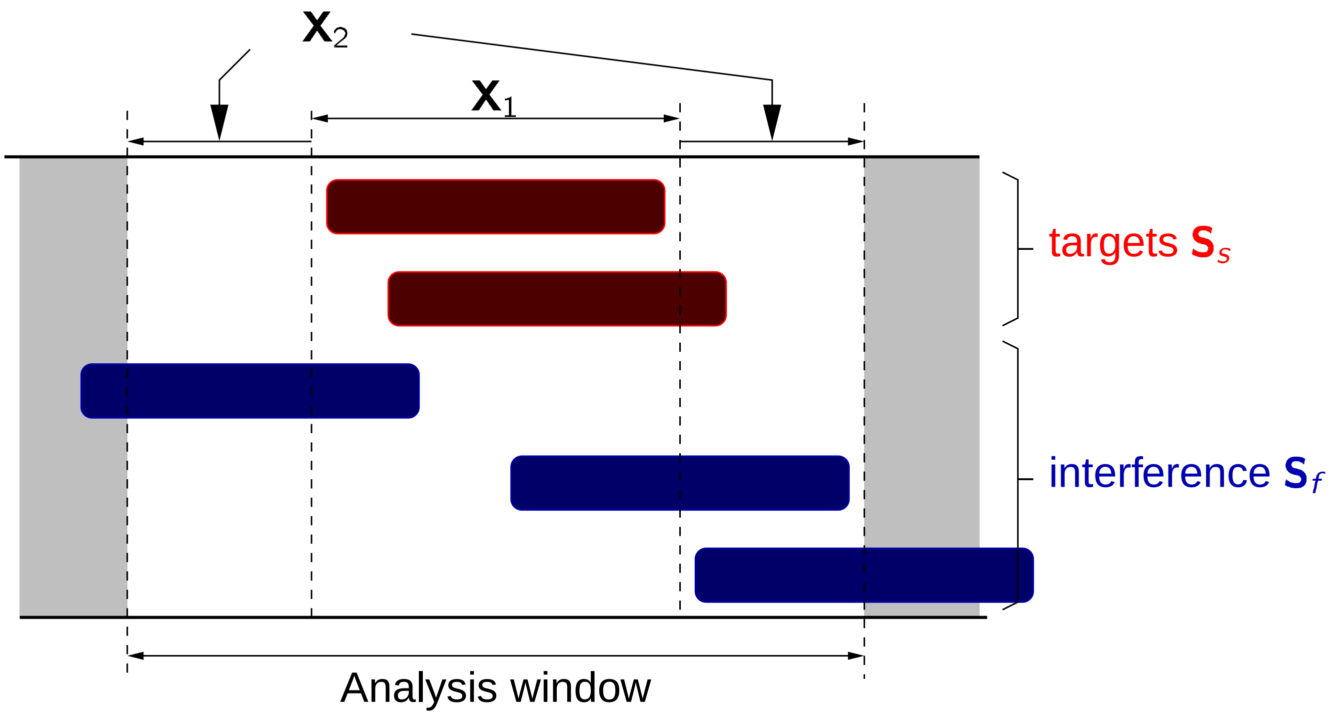
Data model

Received signal

Assume M antennas, stack received signals $x_i[k]$ into column $\mathbf{x}[k]$:

$$\mathbf{x}[k] = \mathbf{h}_1 s_1[k] + \cdots + \mathbf{h}_d s_d[k] + \mathbf{n}[k] = \mathbf{H}\mathbf{s}[k] + \mathbf{n}[k]$$

$\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_d]$: tall, full column rank; columns normalized to $\|\mathbf{h}_i\| = 1$



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Data model

Covariance model

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{H}_s \mathbf{R}_{s1} \mathbf{H}_s^H + \mathbf{H}_f \mathbf{R}_{f1} \mathbf{H}_f^H + \sigma^2 \mathbf{I} \\ \mathbf{R}_2 &= \mathbf{H}_s \mathbf{R}_{s2} \mathbf{H}_s^H + \mathbf{H}_f \mathbf{R}_{f2} \mathbf{H}_f^H + \sigma^2 \mathbf{I} \end{aligned}$$

The distinction between target signals and interfering signals is defined by

$$\mathbf{R}_{s1} > \mathbf{R}_{s2}\,, \qquad \mathbf{R}_{f1} < \mathbf{R}_{f2}$$

I.e., target signals are stronger (more samples present) in the first data block than in the second data block.

Objective

Compute a separating beamforming matrix $\mathbf{W}$ of size $m \times d_s$, such that

$$\mathbf{W}^H \mathbf{H}_s = \mathbf{M}_s\,, \qquad \mathbf{W}^H \mathbf{H}_f = \mathbf{0}$$

where $\mathbf{M}_s$ is any $d_s \times d_s$ full rank matrix.

Tools from linear algebra

Generalized SVD

For two matrices $\mathbf{Y}_1$, $\mathbf{Y}_2$ (both $m \times n$, 'wide'), the GSVD is

$$\mathrm{GSVD}(\mathbf{Y}_1, \mathbf{Y}_2) \quad \Leftrightarrow \quad \begin{cases} \mathbf{Y}_1 & = \quad \mathbf{FCU}^H \\ \mathbf{Y}_2 & = \quad \mathbf{FSV}^H \end{cases}$$

- $\mathbf{F}$: $m \times m$ is an invertible matrix, $\mathbf{C}$ and $\mathbf{S}$ are square positive diagonal matrices,
- $\mathbf{U}$, $\mathbf{V}$ are semi-unitary matrices of size $n \times m$.
- Columns of $\mathbf{F}$ are scaled to norm 1.

(This definition is ‘transposed’ compared to the Matlab definition. Also scaling is different.)

Generalized SVD (cont'd)

$\mathbf{Y}_1 = \mathbf{FCU}^H$, $\mathbf{Y}_2 = \mathbf{FSV}^H$. Given some tolerance $\epsilon \geq 0$, partition $\mathbf{C}$ and $\mathbf{S}$ as

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & & & \\ & \mathbf{C}_2 & & \\ & & \mathbf{C}_3 & \\ & & & \mathbf{C}_4 \end{bmatrix} \begin{matrix} \mathbf{C}_1 > \epsilon \mathbf{I} \\ \mathbf{C}_2 > \epsilon \mathbf{I} \\ \mathbf{C}_3 < \epsilon \mathbf{I} \\ \mathbf{C}_4 < \epsilon \mathbf{I} \end{matrix}, \qquad \mathbf{S} = \begin{bmatrix} \mathbf{S}_1 & & & \\ & \mathbf{S}_2 & & \\ & & \mathbf{S}_3 & \\ & & & \mathbf{S}_4 \end{bmatrix} \begin{matrix} \mathbf{S}_1 > \epsilon \mathbf{I} \\ \mathbf{S}_2 < \epsilon \mathbf{I} \\ \mathbf{S}_3 > \epsilon \mathbf{I} \\ \mathbf{S}_4 < \epsilon \mathbf{I} \end{matrix}$$

and $\mathbf{F}$ correspondingly as $\mathbf{F} = [\mathbf{F}_1 \quad \mathbf{F}_2 \quad \mathbf{F}_3 \quad \mathbf{F}_4]$

- $\mathrm{ran}(\mathbf{F}_1)$ contains the common column span, i.e., $\mathrm{ran}(\mathbf{Y}_1) \cap \mathrm{ran}(\mathbf{Y}_2)$
- $\mathrm{ran}(\mathbf{F}_2)$ is the subspace of columns that are in $\mathrm{ran}(\mathbf{Y}_1)$ but not in $\mathrm{ran}(\mathbf{Y}_2)$,
- $\mathrm{ran}(\mathbf{F}_3)$ is the subspace of columns that are in $\mathrm{ran}(\mathbf{Y}_2)$ but not in $\mathrm{ran}(\mathbf{Y}_1)$,
- $\mathrm{ran}(\mathbf{F}_4)$ is a common left null space.

Tools from linear algebra

Generalized Eigenvalue Decomposition (GEV)

Squaring the GSVD, we obtain (for positive definite matrices $\mathbf{R}_1, \mathbf{R}_2$)

$$\mathrm{GEV}(\mathbf{R}_1, \mathbf{R}_2) \quad \Leftrightarrow \quad \begin{cases} \mathbf{R}_1 &= \mathbf{F}\mathbf{D}\mathbf{F}^H \\ \mathbf{R}_2 &= \mathbf{F}\mathbf{K}\mathbf{F}^H \end{cases}$$

where $\mathbf{F}$ is invertible and $\mathbf{D}$, $\mathbf{K}$ are diagonal and positive.

- Unclear if the decomposition exists if $\mathbf{R}_1$ and $\mathbf{R}_2$ indefinite ($\mathbf{D}$ and $\mathbf{K}$ may become complex).
- Can partition $\mathbf{D}, \mathbf{K}, \mathbf{F}$ in the same way as for the GSVD.

Tools from linear algebra

Oblique projections

A square matrix $\mathbf{E}$ is an oblique projection if $\mathbf{E}^2 = \mathbf{E}$. Let $\mathbf{H} = [\mathbf{H}_s \;\; \mathbf{H}_f]$ be of full column rank, then

$$\mathbf{E}_{\mathrm{H}_s,\mathrm{H}_f} := \mathbf{H} \begin{bmatrix} \mathbf{I} & \\ & \mathbf{0} \end{bmatrix} \mathbf{H}^\dagger$$

is an oblique projection. It is such that $\mathbf{E}\mathbf{H}_s = \mathbf{H}_s$ and $\mathbf{E}\mathbf{H}_f = \mathbf{0}$

Beamforming

A “zero-forcing” beamformer $\mathbf{W}$ is a full-rank factor of an oblique projection:

$$\mathbf{W}^H\mathbf{H}_s = \mathbf{M} \quad \text{invertible}\,, \qquad \mathbf{W}^H\mathbf{H}_f = \mathbf{0}$$

Example: $\mathbf{W}^H = [\mathbf{I} \;\; \mathbf{0}]\mathbf{H}^\dagger$.

Source separation

Noise-free case

- Model:

$$\mathbf{R}_1 = [\mathbf{H}_s \quad \mathbf{H}_f] \begin{bmatrix} \mathbf{R}_{s1} & \\ & \mathbf{R}_{f1} \end{bmatrix} \begin{bmatrix} \mathbf{H}_s^H \\ \mathbf{H}_f^H \end{bmatrix}, \qquad \mathbf{R}_2 = [\mathbf{H}_s \quad \mathbf{H}_f] \begin{bmatrix} \mathbf{R}_{s2} & \\ & \mathbf{R}_{f2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_s^H \\ \mathbf{H}_f^H \end{bmatrix}$$

$$\mathbf{R}_{s1} > \mathbf{R}_{s2}\,, \qquad \mathbf{R}_{f1} < \mathbf{R}_{f2}$$

- The GEV of $(\mathbf{R}_1, \mathbf{R}_2)$ is

$$\begin{cases} \mathbf{R}_1 = \mathbf{F}\mathbf{D}\mathbf{F}^H \\ \mathbf{R}_2 = \mathbf{F}\mathbf{K}\mathbf{F}^H \end{cases}$$

For a given threshold $\epsilon \geq 0$, partition **F**, **D**, **K** as

$$\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3]\,, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & & \\ & \mathbf{D}_2 & \\ & & \mathbf{D}_3 \end{bmatrix} \begin{matrix} \mathbf{D}_1 > \epsilon\mathbf{I} \\ \\ \mathbf{D}_3 < \epsilon\mathbf{I} \end{matrix}\,, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & & \\ & \mathbf{K}_2 & \\ & & \mathbf{K}_3 \end{bmatrix} \begin{matrix} \\ \mathbf{K}_2 > \epsilon\mathbf{I} \\ \mathbf{K}_3 < \epsilon\mathbf{I} \end{matrix}$$

and moreover $\mathbf{D}_1 > \mathbf{K}_1\,, \quad \mathbf{D}_2 < \mathbf{K}_2$

Source separation

Then

$$\text{ran}(\mathbf{F}_1) = \text{ran}(\mathbf{H}_s)\,, \qquad \text{ran}(\mathbf{F}_2) = \text{ran}(\mathbf{H}_f)\,.$$

Using **F**, we can construct the oblique projector to cancel the interference:

$$\mathbf{E} = \mathbf{F}\begin{bmatrix} \mathbf{I} & & \\ & \mathbf{0} & \\ & & \mathbf{0} \end{bmatrix}\mathbf{F}^{-1}$$

whereas a separating beamformer is $\mathbf{W}^H = [\mathbf{I}\;\;\mathbf{0}\;\;\mathbf{0}]\mathbf{F}^{-1}$.

White noise with known covariance $\sigma^2\mathbf{I}$

F from $\text{GEV}(\mathbf{R}_1, \mathbf{R}_2)$ changes (unlike EVD of a single matrix in white noise which will shift eigenvalues but not change the eigenvectors).

Could compute $\text{GEV}(\mathbf{R}_1 - \sigma^2\mathbf{I}, \mathbf{R}_2 - \sigma^2\mathbf{I})$; but risk that matrices become indefinite.

First need to remove the noise subspace.

Source separation

Algorithm using SVD and GEV

1. *Preprocessing:* compute the SVD:

$$[\mathbf{Y}_1 \;\; \mathbf{Y}_2] = [\mathbf{U}_1 \quad \mathbf{U}_2] \begin{bmatrix} \mathbf{\Sigma}_1 & \\ & \mathbf{\Sigma}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^H \\ \mathbf{V}_2^H \end{bmatrix}, \qquad \begin{matrix} \mathbf{\Sigma}_1 > \sigma \mathbf{I} \\ \mathbf{\Sigma}_2 < \sigma \mathbf{I} \end{matrix}$$

   Then apply a rank and dimension reduction: $\hat{\mathbf{Y}}_1 = \mathbf{U}_1^H \mathbf{Y}_1$, $\hat{\mathbf{Y}}_2 = \mathbf{U}_1^H \mathbf{Y}_2$

2. Compute the rank-reduced covariance matrices $\hat{\mathbf{R}}_1 = \hat{\mathbf{Y}}_1 \hat{\mathbf{Y}}_1^H$, $\hat{\mathbf{R}}_2 = \hat{\mathbf{Y}}_2 \hat{\mathbf{Y}}_2^H$

3. Compute the GEV of the *noise-shifted* rank-reduced covariance matrices,

$$\mathrm{GEV}(\hat{\mathbf{R}}_1 - \sigma^2 \mathbf{I}, \hat{\mathbf{R}}_2 - \sigma^2 \mathbf{I}) \quad \Leftrightarrow \quad \begin{cases} \hat{\mathbf{R}}_1 - \sigma^2 \mathbf{I} = \mathbf{F}\mathbf{D}\mathbf{F}^H \\ \hat{\mathbf{R}}_2 - \sigma^2 \mathbf{I} = \mathbf{F}\mathbf{K}\mathbf{F}^H \end{cases}$$

4. Sort the entries of $\mathbf{D}, \mathbf{K}$ and correspondingly partition $\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2]$.

   The term $\mathbf{F}_3$ should be absent as the noise subspace has been removed.

5. The separating beamformer is $\mathbf{W}^H = [\mathbf{I} \quad \mathbf{0}][\mathbf{F}_1 \quad \mathbf{F}_2]^{-1} \mathbf{U}_1^H$