7. CDMA: CODE DIVISION MULTIPLE ACCESS

Outline

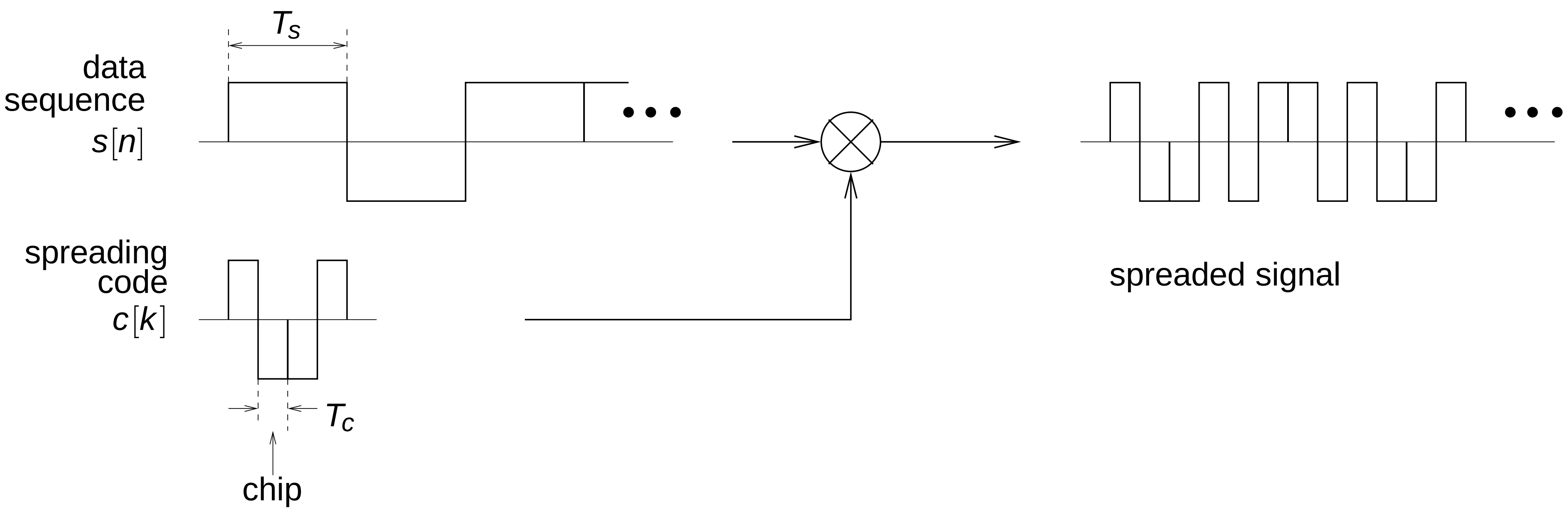
1. Data model
2. Single-user receivers:
   - Matched filter
   - Rake receiver
3. Multi-user receiver

Literature

- Hui Liu, “Signal Processing Applications in CDMA Communications”, Artech House, 2000.
- Sergio Verdu, “Multiuser Detection”, Cambridge University Press, 1998.

CDMA model

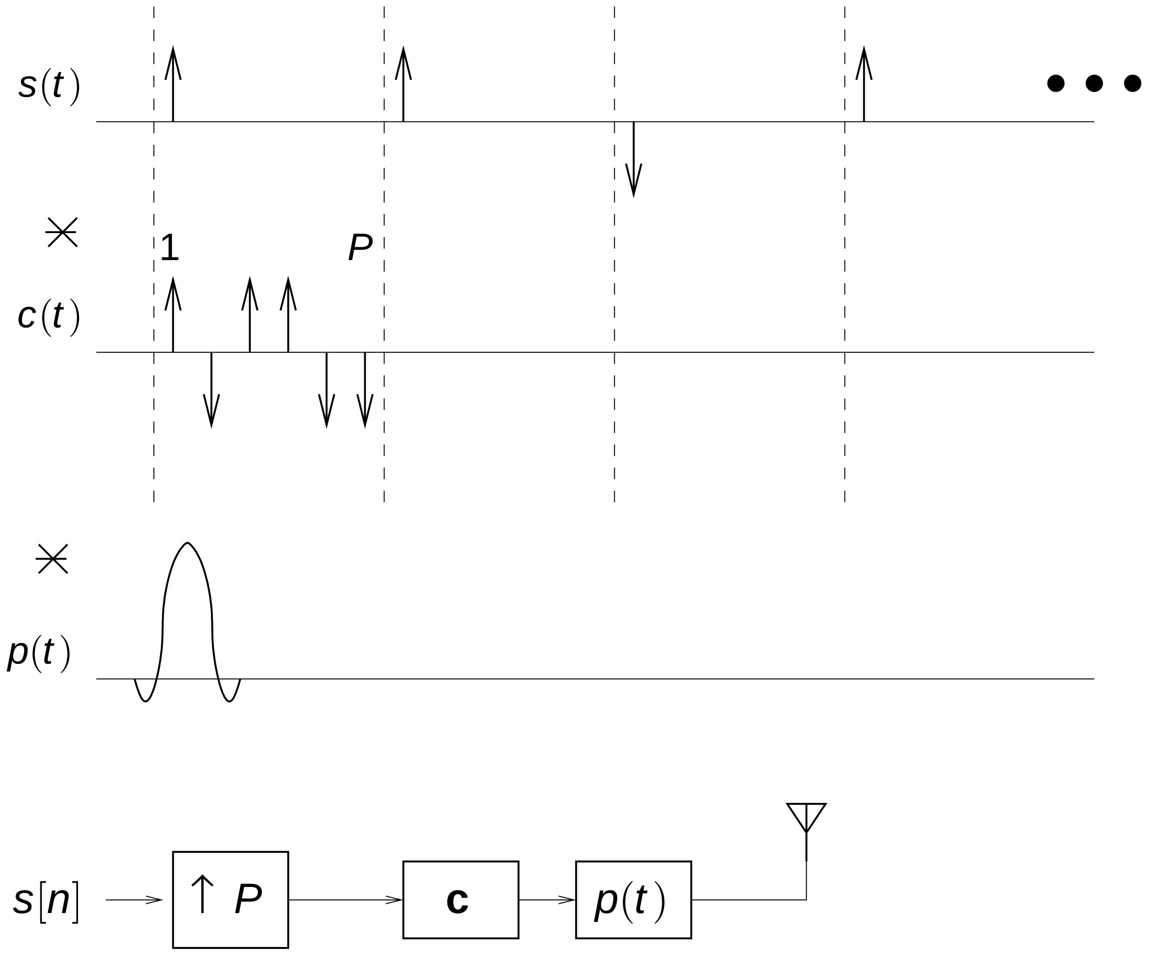
Direct sequence CDMA



- Code length P ("processing gain") such that $PT_c = T_s$, bandwidth expanded by factor P. Typical range: $P = 16, \cdots, 512$.
- Codes are 'pseudo-noise' (PN)

CDMA model

Repeated short code has a filter model:



CDMA model

$$x(t) = \sum_{n=-\infty}^{\infty} s[n]g(t-nT_s)\,, \qquad g(t) = \sum_{k=0}^{P-1} c[k]p(t-kT_c)$$

Pulse shaping filter $p(t)$ is usually a Nyquist filter, i.e., $p(t) = \begin{cases} 1\,, & t=0 \\ 0\,, & t = \pm T_c,\, \pm 2T_c \;\cdots \end{cases}$

Vector model (sampled at chip rate):

$$\mathbf{x}[n] := \begin{bmatrix} x(nT_s) \\ x(nT_s+T_c) \\ \vdots \\ x(nT_s+(P-1)T_c \end{bmatrix} = \begin{bmatrix} c[0] \\ c[1] \\ \vdots \\ c[P-1] \end{bmatrix} s[n] \;=\; \mathbf{c}s[n]$$

c will play the same role as the array response, except that it is *known*

Synchronous CDMA model

Different users transmit at same time, but use different codes.

Q synchronized users with code vectors $\mathbf{c}_1, \cdots, \mathbf{c}_Q$:

$$\mathbf{x}[n] = \sum_{i=1}^{Q} \begin{bmatrix} c_i[0] \\ c_i[1] \\ \vdots \\ c_i[P-1] \end{bmatrix} s_i[n] = \sum_{i=1}^{Q} \mathbf{c}_i s_i[n]$$

Orthogonal codes:

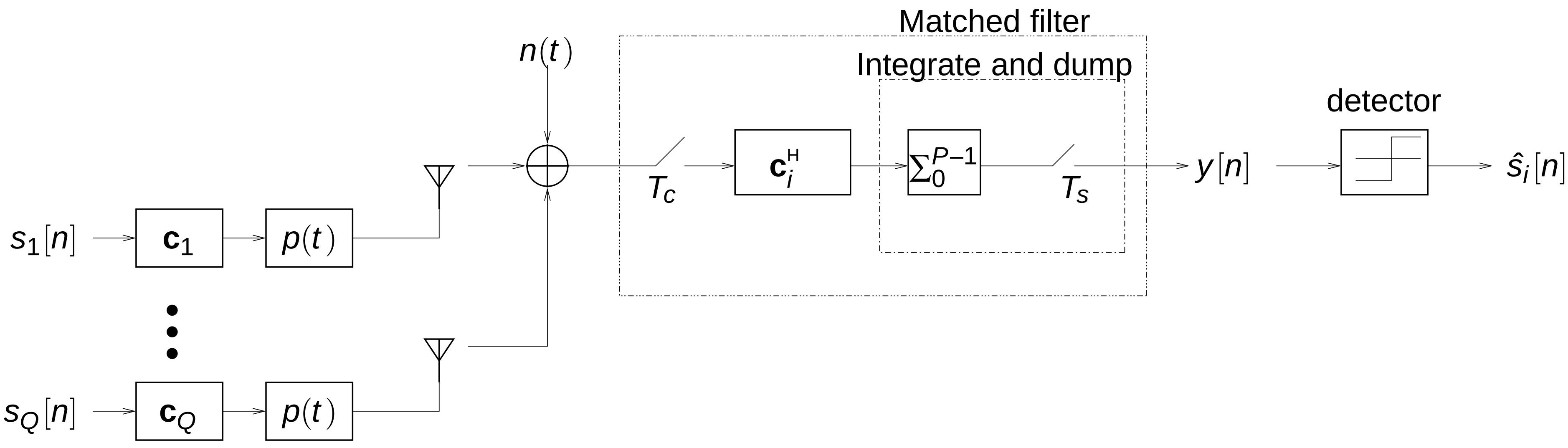
$$\sum_{k=0}^{P-1} \bar{c_i}[k] c_j[k] = 0 \quad (i \neq j) \qquad \Leftrightarrow \qquad \mathbf{c}_i^{\mathrm{H}} \mathbf{c}_j = 0 \quad (i \neq j)$$

Synchronous CDMA receiver: Matched filter

To receive user i, correlate with his code:

$$y[n] = \mathbf{c}_i^{\mathrm{H}}\mathbf{x}[n] = \sum_{j=1}^{Q} \mathbf{c}_i^{\mathrm{H}}\mathbf{c}_j s_j[n] = P^2 s_i[n]$$

This is equivalent to a matched filter



Synchronous CDMA receiver: Matched filter

Assume synchronized users with orthogonal codes.

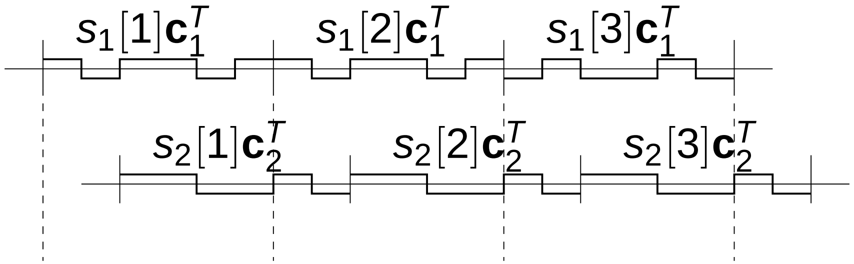
Then the matched filter removes all Multi-user Access Interference (MAI)

With added noise $n(t)$ of variance σ^2 per sample, $y[n]$ has two remaining components:

Signal of Interest	$Ps_i[n]$	variance P^2
Noise at output	$\sum_{k=0}^{P-1} \bar{c_i}[k]n(nT_s + kT_c)\,,$	variance $P\sigma^2$

The SNR at the output is P times the SNR at $x(t)$ (processing gain).

Asynchronous CDMA model



Asynchronous users with delays $\{\tau_i\}$:

$$x(t) = \sum_{i=1}^{Q} \sum_{k=0}^{P-1} s_i[n] g_i(t - nT_s - \tau_i), \qquad g_i(t) = \sum_{k=0}^{P-1} c_i[k] p(t - kT_c)$$

Assume we synchronize user 1 ($\tau_1 = 0$) then

$$\mathbf{x}[n] = \begin{bmatrix} x(nT_s) \\ x(nT_s + T_c) \\ \vdots \\ x(nT_s + (P-1)T_c \end{bmatrix} = \begin{bmatrix} c_1[0] \\ c_1[1] \\ \vdots \\ c_1[P-1] \end{bmatrix} s_1[n] + \begin{bmatrix} 0 \\ 0 \\ \hline c_2[0] \\ \vdots \end{bmatrix} s_2[n] + \begin{bmatrix} \vdots \\ c_2[P-1] \\ \hline 0 \\ 0 \end{bmatrix} s_2[n-1] + \cdots$$

Matched filter: $y[n] = \mathbf{c}_1^{\mathrm{H}} \mathbf{x}[n] = P s_1[n] + \mathrm{MAI}$

Asynchronous CDMA receiver

The MAI is small only if $\mathbf{c}_1$ has small cross-correlations with arbitrary shifts of the other users' codes.

- Code design (Gold codes, $\cdots$)
- This is impossible unless $Q \ll P$

Also requires power control to receive all users equally strong (± 3 dB or even less)

- This usually requires feedback
- fast fading: power has to be adjusted every ms.

Better: multi-user detector

Multi-user detector

$$\mathbf{x}[n] = [\mathbf{c}_1 \;\; \mathbf{c}_2^{\downarrow} \;\; \mathbf{c}_2^{\uparrow} \;\; \cdots] \begin{bmatrix} s_1[n] \\ s_2[n] \\ s_2[n-1] \\ \vdots \end{bmatrix} + \mathbf{n}[n] = \mathbf{C}\mathbf{s}[n] + \mathbf{n}[n]$$

- ZF: $\hat{\mathbf{s}}[n] = \mathbf{C}^{\dagger}\mathbf{x}[n] = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{x}[n]$
- MMSE: $\hat{\mathbf{s}}[n] = \mathbf{W}^H\mathbf{x}[n]$ where $\mathbf{W} = \mathbf{R}_x^{-1}\mathbf{C} = (\mathbf{C}\mathbf{C}^H + \sigma^2\mathbf{I})^{-1}\mathbf{C}$

  $\Rightarrow \quad s_1[n] = \mathbf{c}_1^H(\mathbf{C}\mathbf{C}^H + \sigma^2\mathbf{I})^{-1}\mathbf{x}[n]$

Note:

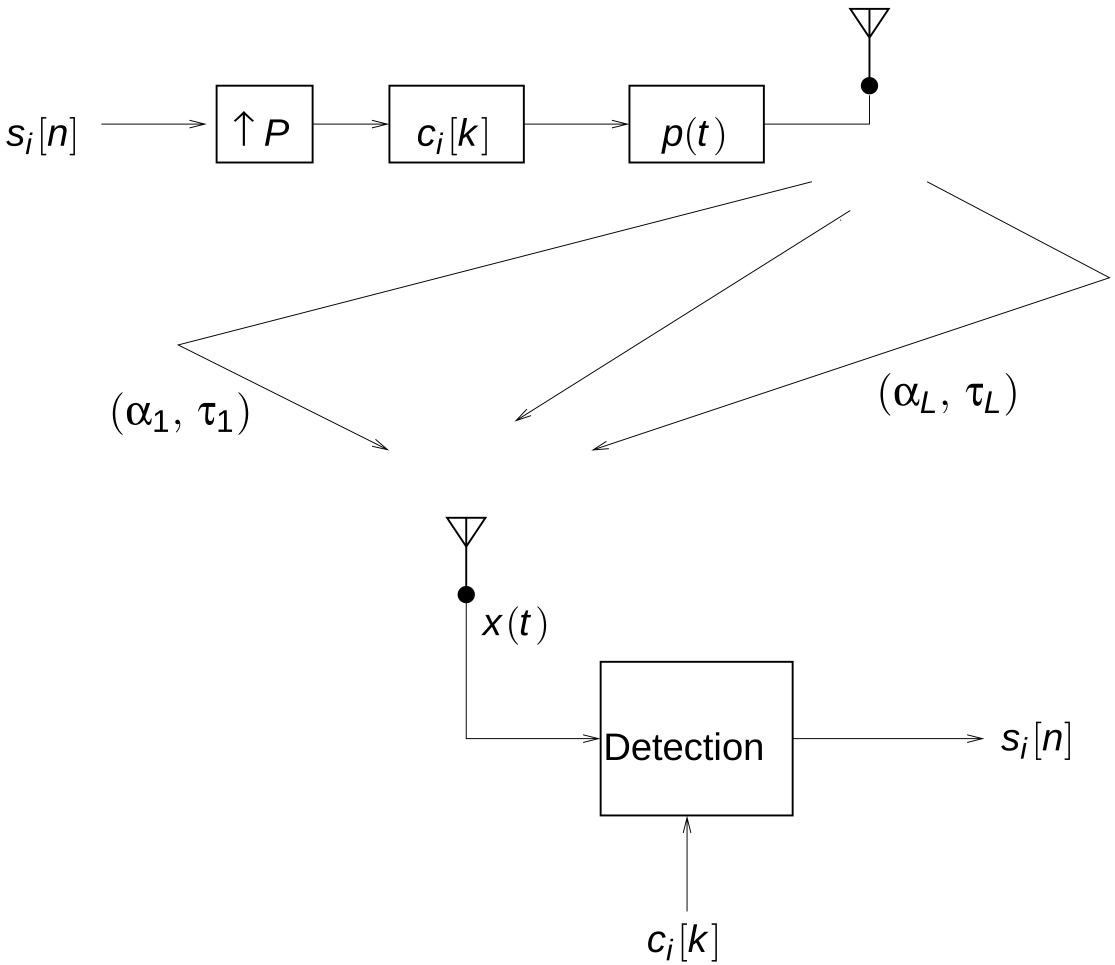
– In this data model, the estimates of $s_i[n]$ and $s_i[n-1]$ are not connected (better is MLSE: max likelihood sequence estimator — Viterbi)

– Inversion requires $2Q-1 \leq P$

– All codes, delays (and amplitudes) need to be known.

Multipath CDMA model

Multipath channel model



Multipath CDMA model

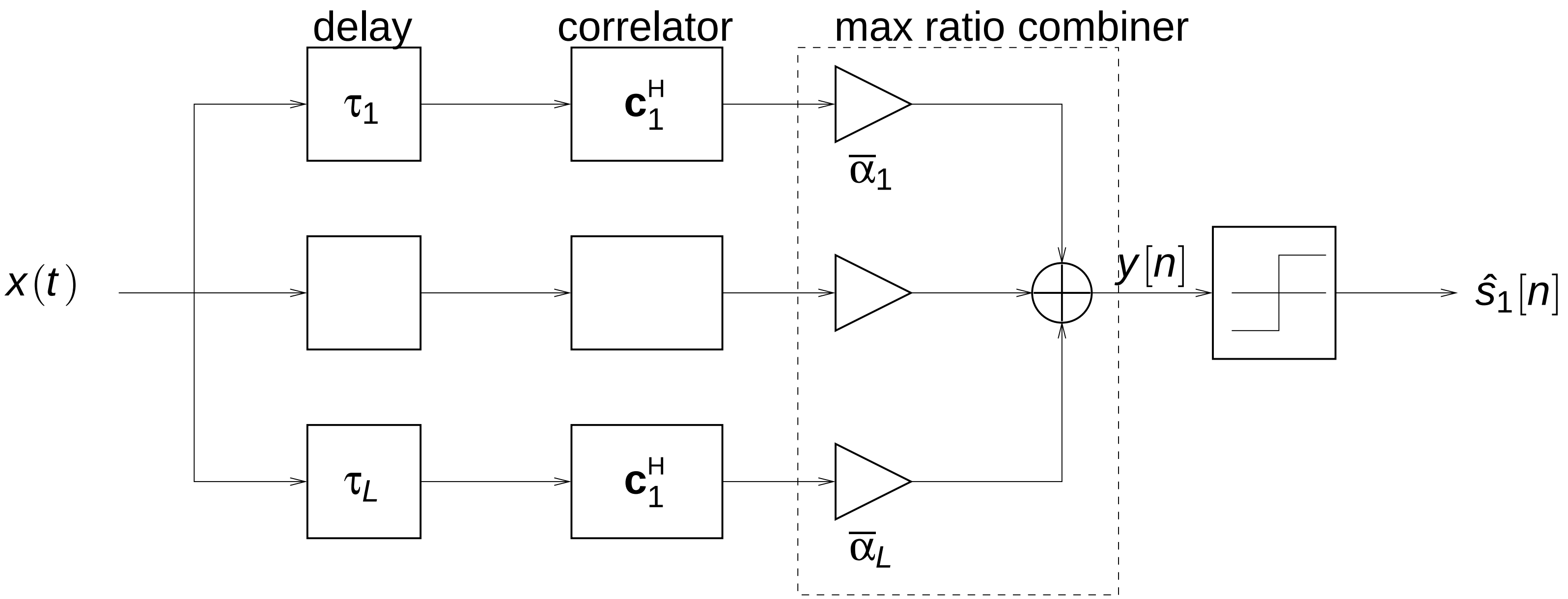
Discrete multipath model:

$$x(t) = \sum_{i=1}^{L}\sum_{k=0}^{P-1} s_1[n]\alpha_i g_1(t - nT_s - \tau_i) + \cdots$$

$$\mathbf{x}[n] := \begin{bmatrix} x(nT_s) \\ x(nT_s + T_c) \\ \vdots \\ \hline x((n+1)T_s) \\ x((n+1)T_s + T_c) \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \\ \\ \mathbf{C}_1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_L \end{bmatrix}}_{\mathbf{h}_1} s_1[n] \quad + [\cdots] s_1[n-1] + \cdots$$

RAKE receiver

RAKE receiver with maximum ratio combining



$$y[n] = \mathbf{h}_1^H \mathbf{x}[n] = [\bar{\alpha}_1 \cdots \bar{\alpha}_L]\mathbf{C}_1^H \mathbf{x}[n]$$

This is simply a matched filter for the first component. Suboptimal since other users and ISI assumed to be cancelled out. Performance is interference limited.

The delays τ_i and complex gains α_i have to be estimated (via training or decision directed; not easy). Simpler alternative: equal gain combining ($\alpha_i = 1$)

Multiuser receivers

- **Optimal**: joint estimation of all user bits and channels (MLSE)

  Complexity is exponential

- **Linear multi-user receivers**:

  – estimate channels of all users, e.g. using training symbols

  – invert joint channel: decorrelating (=zero-forcing) and MMSE receiver

  Amount of required training symbols is linear in the number of users ...

- interference cancellation

  – hard decision vs. soft decision

  – serial vs. parallel

- **Blind multi-user receivers**:

  Use *code structure*, no training symbols.

  Estimate channels, or try to estimate equalizer directly

Blind CDMA channel estimation

Assume $\tau_i = iT_c$ and use channel structure:

$$\mathbf{h}_1 = \begin{bmatrix} h(0) \\ h(T_c) \\ \vdots \\ h(LT_c) \end{bmatrix} = \begin{bmatrix} c_1[0] & & \\ \vdots & \ddots & \\ c_1[P-1] & & c_1[0] \\ & \ddots & \vdots \\ & & c_1[P-1] \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \vdots \\ \alpha_L \end{bmatrix} = \mathbf{C}_1\mathbf{a}_1$$

The code matrix $\mathbf{C}_1$ is known, and tall.

Algorithm

– Find the column span $\hat{\mathbf{U}}$ of $[\mathbf{x}[0] \;\; \mathbf{x}[1] \;\; \cdots]$. It contains the vector $\mathbf{h}_1$.

– Find which vector in $\hat{\mathbf{U}}$ can be written as $\mathbf{C}_1\mathbf{a}_1$. This identifies $\mathbf{h}_1$.

– Find a suitable equalizer (MMSE).

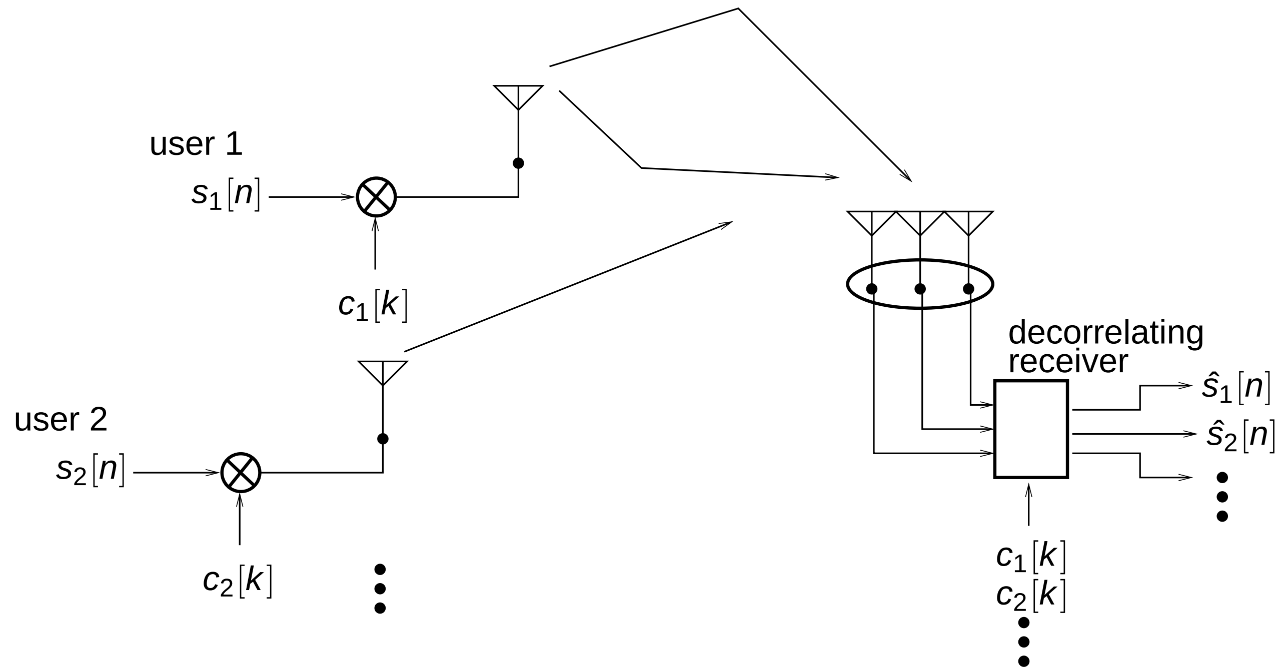
Without noise, this algorithm finds all user channels *exactly*.

Long-code CDMA

Long-code CDMA is CDMA where the user codes are not periodic. It is used in UMTS (called Wideband CDMA). Suppression by matched filtering is supposed to be better than for periodic codes.

- Long-code wideband CDMA receivers are for computational reasons usually based on simple matched-filter techniques, and hence suffer from multiaccess interference.
- To mitigate this problem, we propose decorrelating RAKE and MMSE receivers for the uplink of long-code CDMA systems.

Scenario



- Users are asynchronous
- Can have multipath
- Receiver knows all codes (base station)

Data model

The system is frame-oriented.

User i transmits $\mathbf{s}_i = [s_{i1}, \cdots, s_{i,N}]^T$.

The signal is spread by aperiodic codes $\mathbf{c}_{i1}$, $\mathbf{c}_{i2}$, $\cdots$.

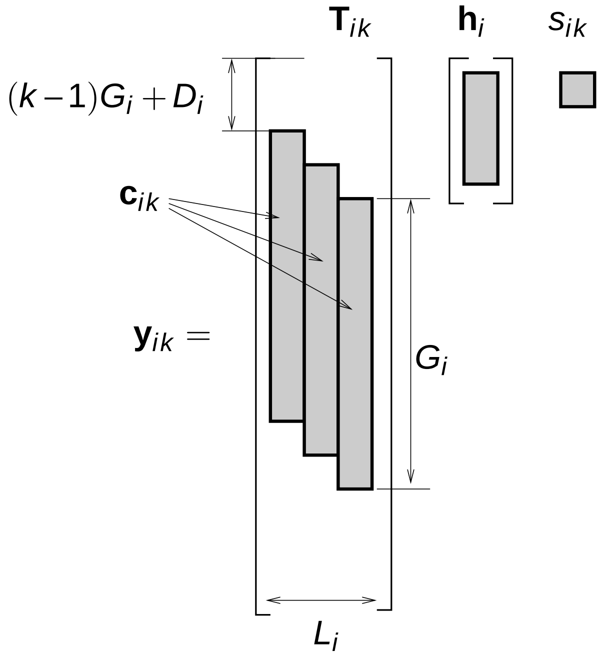
$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{i,N} \end{bmatrix} \quad \Rightarrow \quad \mathbf{u}_i = \begin{bmatrix} s_{i1} \begin{bmatrix} c_{i1} \\ c_{i2} \\ \vdots \\ c_{iG} \end{bmatrix} \\ s_{i2} \begin{bmatrix} c_{i,G+1} \\ c_{i,G+2} \\ \vdots \\ c_{i,2G} \end{bmatrix} \\ \vdots \end{bmatrix}$$

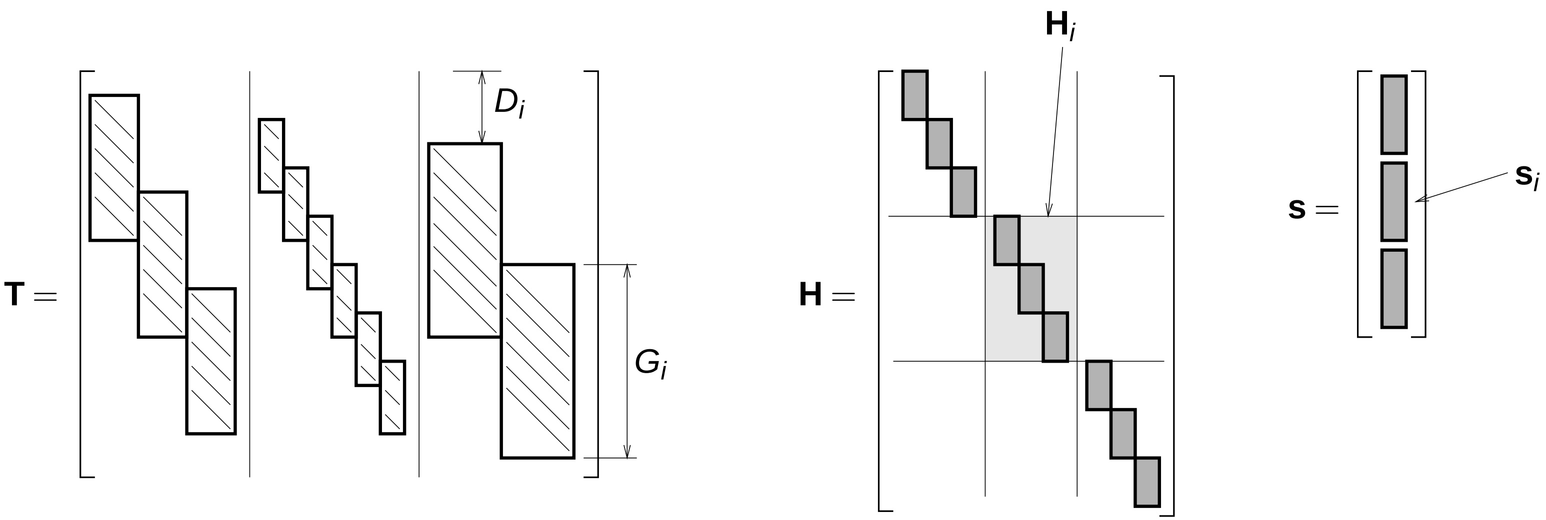
Data model

During transmission, the spreaded signal is convolved by a channel with coefficients $\mathbf{h}_i = [h_{i1}, \cdots, h_{i,L}]^T$.

$$\mathbf{y}_i = \begin{bmatrix} s_{i1}\begin{bmatrix} c_{i1} \\ c_{i2} \\ \vdots \\ c_{iG} \end{bmatrix} \\ s_{i2}\begin{bmatrix} c_{i,G+1} \\ c_{i,G+2} \\ \vdots \\ c_{i,2G} \end{bmatrix} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} h_{i1} \quad + \begin{bmatrix} 0 \\ s_{i1}\begin{bmatrix} c_{i1} \\ c_{i2} \\ \vdots \\ c_{iG} \end{bmatrix} \\ s_{i2}\begin{bmatrix} c_{i,G+1} \\ c_{i,G+2} \\ \vdots \\ c_{i,2G} \end{bmatrix} \\ \vdots \\ \vdots \end{bmatrix} h_{i2} \quad + \begin{bmatrix} 0 \\ 0 \\ s_{i1}\begin{bmatrix} c_{i1} \\ c_{i2} \\ \vdots \\ c_{iG} \end{bmatrix} \\ s_{i2}\begin{bmatrix} c_{i,G+1} \\ c_{i,G+2} \\ \vdots \\ c_{i,2G} \end{bmatrix} \\ \vdots \end{bmatrix} h_{i3} + \cdots$$

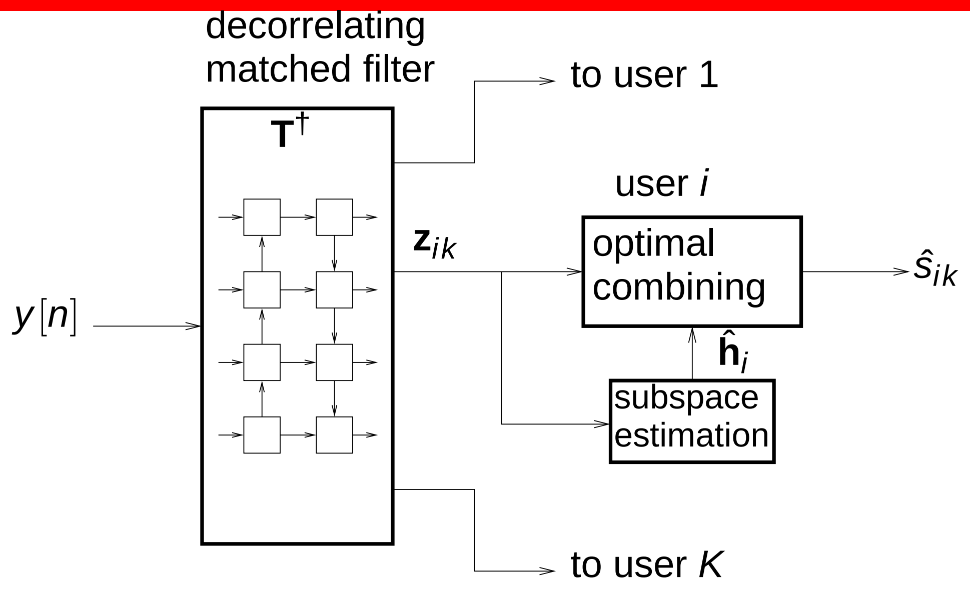
Data model





$$\mathbf{y} = \mathbf{T}\mathbf{H}\mathbf{s} + \mathbf{w}$$

Receiver structure



$$\mathbf{y} = \mathbf{T}\mathbf{H}\mathbf{s} \qquad \Rightarrow \qquad \mathbf{z} := \mathbf{T}^\dagger \mathbf{y} = \mathbf{H}\mathbf{s} = \begin{bmatrix} \mathbf{H}_1\mathbf{s}_1 \\ \mathbf{H}_2\mathbf{s}_2 \\ \vdots \end{bmatrix}$$

$$\mathbf{z}_i = \mathbf{H}_i\mathbf{s}_i = \begin{bmatrix} \mathbf{h}_i & & \\ & \mathbf{h}_i & \\ & & \ddots \end{bmatrix} \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \end{bmatrix} \qquad \Rightarrow \qquad \mathbf{Z}_i = \mathbf{h}_i [s_{i1} \quad s_{i2} \quad \cdots]$$

Receiver structure

$$\mathbf{Z}_i = \mathbf{h}_i [s_{i1} \quad s_{i2} \quad \cdots]$$

Hence, $\mathbf{Z}_i$ is rank 1.

A singular value decomposition (SVD) provides $\mathbf{h}_i$ and the symbols in the frame, up to an unknown scaling.

- *Blind*, no training needed
- This is a *decorrelating receiver*: without noise, all users are precisely separated (decorrelated)
- Need $\mathbf{T}$ to be tall, i.e. code length $>$ number of users $\times$ number of paths.
- Instead of $\mathbf{F} = \mathbf{T}^\dagger$, it appears better to use a *regularized decorrelating receiver*,

$$\mathbf{F} = (\mathbf{T}^H \mathbf{T} + \sigma^2 \mathbf{I})^{-1} \mathbf{T}^H$$