3. SPATIAL PROCESSING TECHNIQUES

Outline

1. Blind channel estimation

2. Blind symbol estimation

Data model

- Let us consider the general single-user model (see introduction)

$$\mathcal{X} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \cdots & \mathbf{x}_{N-1} \\ \mathbf{x}_{-1} & \mathbf{x}_0 & \cdots & \mathbf{x}_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_{-m+1} & \mathbf{x}_{-m+2} & \cdots & \mathbf{x}_{N-m} \end{bmatrix}$$

$$:= \begin{bmatrix} \mathbf{h}_0 & \cdots & \mathbf{h}_{L-1} & & & & \mathbf{0} \\ & \mathbf{h}_0 & \cdots & \mathbf{h}_{L-1} & & & \\ & & \ddots & & \ddots & & \\ \mathbf{0} & & & \mathbf{h}_0 & \cdots & \mathbf{h}_{L-1} \end{bmatrix} \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \\ s_{-1} & s_0 & \cdots & s_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{-L-m+2} & s_{-L-m+3} & \cdots & s_{N-L-m+1} \end{bmatrix} = \mathcal{H}\mathcal{S}$$

- The model for multiple users can then be written as

$$\mathcal{X} = \mathcal{H}^{(1)}\mathcal{S}^{(1)} + \cdots + \mathcal{H}^{(d)}\mathcal{S}^{(d)} = \left[\mathcal{H}^{(1)}, \ldots, \mathcal{H}^{(d)}\right] \begin{bmatrix} \mathcal{S}^{(1)} \\ \vdots \\ c\mathcal{S}^{(d)} \end{bmatrix}$$

Blind estimation

- The starting point of our blind estimation methods is the SVD of $\mathcal{X}$:

$$\mathcal{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H = \mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}_s^H + \mathbf{U}_n\mathbf{\Sigma}_n\mathbf{V}_n^H$$

  where $\mathbf{\Sigma}_n$ is only non-zero if there is some noise.

- $\mathbf{U}_n$ describes the subspace orthogonal to the columns of $\mathcal{X}$ and $\mathbf{V}_n$ describes the subspace orthogonal to the rows of $\mathcal{X}$:

$$\mathbf{U}_n^H\mathcal{X} = \mathbf{0} \qquad \mathcal{X}\mathbf{V}_n = \mathbf{0}$$

- Because the columns of $\mathcal{X}$ are linear combinations of the columns of $\mathcal{H}$, we have

$$\mathbf{U}_n^H\mathcal{X} = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{U}_n^H\mathcal{H} = \mathbf{0}$$

- Because the rows of $\mathcal{X}$ are linear combinations of the rows of $\mathcal{S}$, we have

$$\mathcal{X}\mathbf{V}_n = \mathbf{0} \quad \Leftrightarrow \quad \mathcal{S}\mathbf{V}_n = \mathbf{0}$$

- The blind methods are now obtained by exploiting the structure in $\mathcal{H}$ and $\mathcal{S}$.

Blind channel estimation

- For a single user, we can transform $\mathbf{U}_n^H \mathcal{H} = \mathbf{0}$ into

$$\begin{bmatrix} \mathbf{U}_{n,1}^H & \cdots & \mathbf{U}_{n,m}^H \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 & \cdots & \mathbf{h}_{L-1} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}_0 & \cdots & \mathbf{h}_{L-1} \end{bmatrix} = \mathbf{0}$$

$$\Leftrightarrow \quad \underbrace{\begin{bmatrix} \mathbf{U}_{n,1}^H & & \mathbf{0} \\ \vdots & \ddots & \\ \mathbf{U}_{n,m}^H & & \mathbf{U}_{n,1}^H \\ & \ddots & \vdots \\ \mathbf{0} & & \mathbf{U}_{n,m}^H \end{bmatrix}}_{\mathbf{U}_{n,T}} \begin{bmatrix} \mathbf{h}_0 \\ \vdots \\ \mathbf{h}_{L-1} \end{bmatrix} = \mathbf{0}$$

- This can be solved by finding the right null-space of $\mathbf{U}_{n,T}$ (through the SVD).
- If the null-space has dimension one, there is a solution up to a scalar ambiguity.
- If the null-space has a larger dimension, there are too many solutions.

Blind symbol estimation

- For a single user, we can transform $\mathcal{S}\mathbf{V}_n = \mathbf{0}$ into

$$\begin{bmatrix} s_0 & \cdots & s_{N-1} \\ \vdots & & \vdots \\ s_{-L-m+2} & \ddots & s_{N-L-m+1} \end{bmatrix} \mathbf{V}_n = \mathbf{0}$$

$$\Leftrightarrow \quad \begin{bmatrix} s_{-L-m+2} & \cdots & s_{N-1} \end{bmatrix} \underbrace{\begin{bmatrix} \boxed{\mathbf{V}_n} & & & \\ & \boxed{\mathbf{V}_n} & & \\ & & \ddots & \\ & & & \boxed{\mathbf{V}_n} \end{bmatrix}}_{\mathbf{V}_{n,T}} = \mathbf{0}$$

- This can be solved by finding the left null-space of $\mathbf{V}_{n,T}$ (through the SVD).
- If the null-space has dimension one, there is a solution up to a scalar ambiguity.
- If the null-space has a larger dimension, there are too many solutions.

Blind estimation for multiple users

- If we have multiple users, then the blind methods will become

$$\mathbf{U}_{n,T}\begin{bmatrix}\mathbf{H}_0\\ \vdots\\ \mathbf{H}_{L-1}\end{bmatrix}=\mathbf{0} \qquad \begin{bmatrix}\mathbf{s}_{-L-m+2} & \cdots & \mathbf{s}_{N-1}\end{bmatrix}\mathbf{V}_{n,T}=\mathbf{0}$$

where $\mathbf{H}_l$ ($MP \times d$) and $\mathbf{s}_n$ ($d \times 1$) stack the channels and symbols of the d users.

- If $\mathbf{U}_{n,T}$ has a d-dimensional right null-space, then we find a solution for $\mathbf{H}_l$ up to a $d \times d$ transformation matrix $\mathbf{T}$: $\hat{\mathbf{H}}_l = \mathbf{H}_l\mathbf{T}$.

- If $\mathbf{V}_{n,T}$ has a d-dimensional left null-space, then we find a solution for $\mathbf{s}_n$ up to a $d \times d$ transformation matrix $\mathbf{T}$: $\hat{\mathbf{s}}_n = \mathbf{T}\mathbf{s}_n$.

- This transformation matrix $\mathbf{T}$ has to be resolved using for instance the finite alphabet property of the data symbols.