Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Partial exam EE2S31 SIGNAL PROCESSING
Part 2: 24 June 2022 (13:30-15:30)

Closed book; two sides of one A4 with handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of four questions (34 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.
Hint: Avoid losing too much time on detailed calculations, write down the general approach first.

Question 1 (8 points)

An analog signal $x_a(t)$ is a linear combination of 4 sinusoids with frequency components at 300 Hz, 400 Hz, 1.3 kHz and 4.2 kHz, as indicated in Figure 1. As depicted in Figure 2A, the signal is sampled at 2 kHz. The sampled signal is converted back to analog again using an ideal D/A converter (DAC), followed by a low-pass filter with cut-off frequency at 900 Hz. The result is $y_a(t)$.

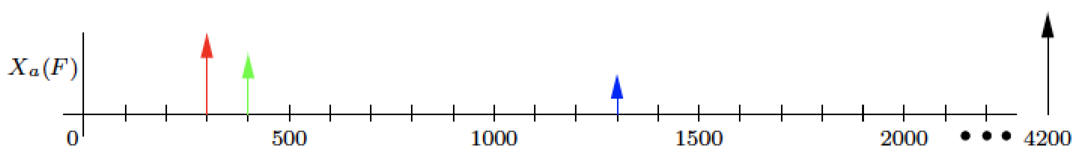


Figure 1

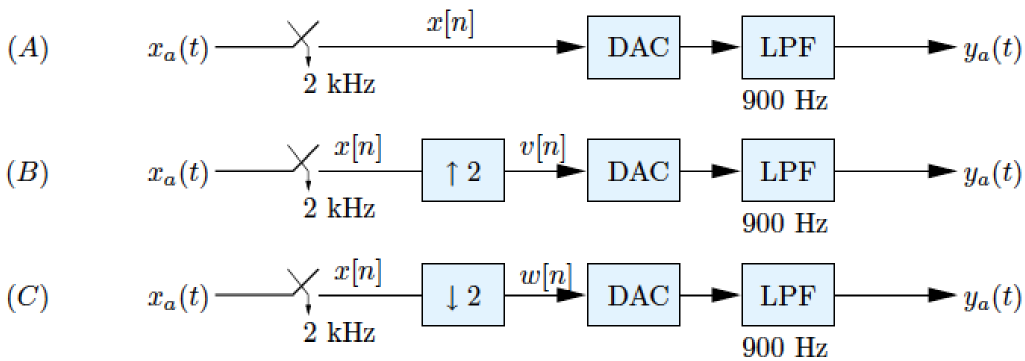


Figure 2

2 p (a) Draw the spectrum of $x[n]$ and $y_a(t)$. Where applicable, indicate both physical and normalized frequencies! What are the frequency components in the output signal?

2 p (b) Now we introduce upsampling, as shown in Figure 2B. Draw the spectum of $v[n]$ and $y_a[n]$. What are the frequency components of the output now?

2 p (c) Now we introduce downsampling instead of upsampling, as shown in Figure 2C. Draw the spectum of $w[n]$ and $y_a[n]$. What are the frequency components of the output now?

2 p (d) Modify the system (C) in order to avoid aliasing. You are allowed to use an A/D converter with an arbitrary sampling rate and a digital filter. No other components in system (C) can be changed.

Question 2 (9 points)

Given a signal $x_a(t)$ with bandwidth $B = 120$ Hz and unit variance with a range between $[-1, 1]$. The signal is digitized using an A/D converter using a binary representation with 4 bits plus a sign bit.

1 p (a) What is the maximum possible value of the quantization error?

1 p (b) Assuming that the error is zero mean and is uniformly distributed, what is the power of the quantization noise?

2 p (c) The signal to quantization noise ratio is given by SQNR $= 10\log_{10}\frac{\sigma_x^2}{\sigma_e^2}$. Show that increasing the number of bits in the A/D converter, the SQNR reduces with 6dB.

2 p (d) Name two other strategies to increase the SQNR!

2 p (e) The figure below shows the frequency response of the noise ($H_e(\omega)$) and signal transfer functions ($H_s(\omega)$) of a 1st and 2nd order sigma-delta modulator (SDM). Based on this figure, explain the concept of noise shaping!

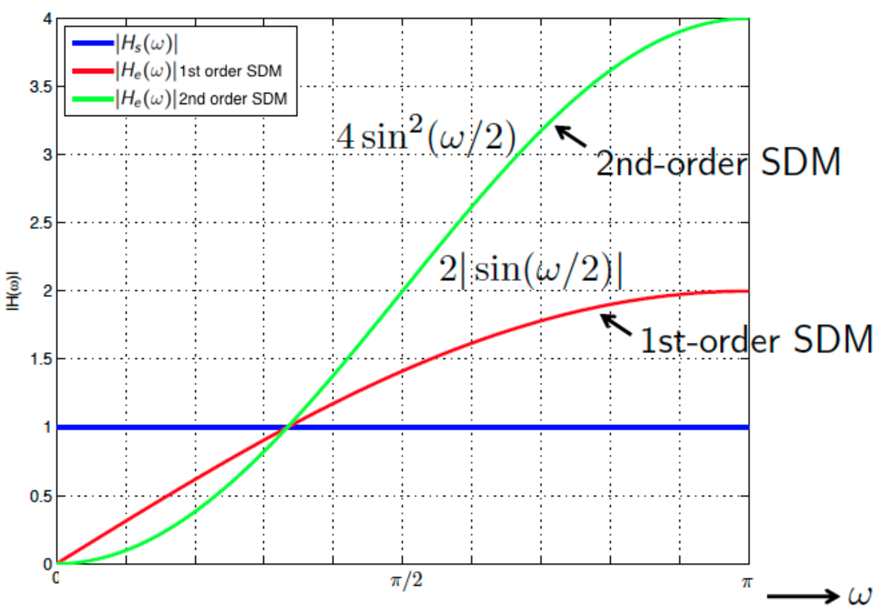


Figure 3

1 p (e) At which sampling rate (roughly) should we sample the signal $x_a(t)$ in order to efficiently suppress noise in the signal band?

Question 3 (8 points)

Consider the random process $X(t) = A\cos(2\pi f_0 t)$, where A is a random variable with zero mean and variance σ^2, and f_0 is a non-random frequency in Hz.

(a) Draw two realizations of $X(t)$.

(b) Determine $\mathrm{E}[X(t)]$.

(c) Compute the autocorrelation function $R_X(t, \tau)$.

(d) Is $X(t)$ a WSS random process? (Motivate)

Now, let A and B be two independent random variables with zero mean and variance σ^2, and consider $Z(t) = A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t)$.

(e) Compute the autocorrelation function $R_Z(t, \tau)$.

(f) Is $Z(t)$ a WSS random process? (Motivate)

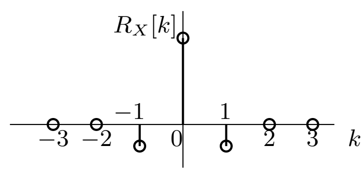
Hint: Recall $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$, $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$.

Question 4 (9 points)

For this question you might want to make use of Table 3, included at the end of this exam.

Consider a WSS random process $X[n]$ with autocorrelation sequence

$$R_X[k] = -\frac{1}{2}\delta[k+1] + 2\delta[k] - \frac{1}{2}\delta[k-1]$$

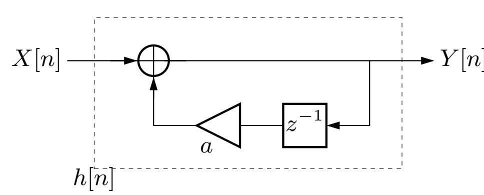


(a) Compute the power spectral density, $S_X(\phi)$, and make a plot of $S_X(\phi)$ (specify the values on the axes).

(b) Is this a valid autocorrelation sequence? (Motivate)

$X[n]$ is filtered by a first-order IIR filter with impulse response

$$h[n] = a^n u[n], \qquad |a| < 1$$



resulting in the output $Y[n]$. For the moment, take $a = \frac{1}{2}$.

(c) Is $Y[n]$ an AR process? (Motivate)

(d) Find $R_{XY}[k]$, the cross-correlation sequence.

(e) Find $S_Y(\phi)$, the power spectral density of the output.

(f) Find $R_Y[k]$, the auto-correlation sequence of the output.

(g) Find the average power of the output.

(h) Is it possible to select a such that $Y[n]$ is white? If so, compute a.

Discrete Time function	Discrete Time Fourier Transform
$\delta[n] = \delta_n$	1
1	$\delta(\phi)$
$\delta[n-n_0] = \delta_{n-n_0}$	$e^{-j2\pi\phi n_0}$
$u[n]$	$\dfrac{1}{1-e^{-j2\pi\phi}} + \dfrac{1}{2}\sum_{k=-\infty}^{\infty}\delta(\phi+k)$
$e^{j2\pi\phi_0 n}$	$\sum_{k=-\infty}^{\infty}\delta(\phi-\phi_0-k)$
$\cos 2\pi\phi_0 n$	$\frac{1}{2}\delta(\phi-\phi_0) + \frac{1}{2}\delta(\phi+\phi_0)$
$\sin 2\pi\phi_0 n$	$\frac{1}{2j}\delta(\phi-\phi_0) - \frac{1}{2j}\delta(\phi+\phi_0)$
$a^n u[n]$	$\dfrac{1}{1-ae^{-j2\pi\phi}}$
$a^{\lvert n\rvert}$	$\dfrac{1-a^2}{1+a^2-2a\cos 2\pi\phi}$
g_{n-n_0}	$G(\phi)e^{-j2\pi\phi n_0}$
$g_n e^{j2\pi\phi_0 n}$	$G(\phi-\phi_0)$
g_{-n}	$G^*(\phi)$
$\sum_{k=-\infty}^{\infty} h_k g_{n-k}$	$G(\phi)H(\phi)$
$g_n h_n$	$\int_{-1/2}^{1/2} H(\phi')G(\phi-\phi')\,d\phi'$

Note that $\delta[n]$ is the discrete impulse, $u[n]$ is the discrete unit step, and a is a constant with magnitude $|a| < 1$.

Table 3 Discrete-Time Fourier transform pairs and properties.