Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Partial exam EE2S31 SIGNAL PROCESSING
Part 2: July 1, 2021
Block 1: Stochastic Processes (13:30-14:30)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 14:25–14:40

This block consists of three questions (25 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.
Hint: Avoid losing too much time on detailed calculations, write down the general approach first.

Question 1 (7 points)

Let $X(t) = A\cos(\Omega_0 t)$ be a random process, where $A \in \{-1, +1\}$ with equal probabilities, and Ω_0 is a given frequency.

(a) Draw two different realizations of $X(t)$.

(b) What type of random process is $X(t)$? [Think of continuous value/discrete value; continuous-time/discrete time.]

(c) Compute the probability mass function (PMF) $P_{X(t)}(x)$.

(d) Compute $\mathrm{E}[X(t)]$.

(e) Compute $R_X(t, \tau)$.

(f) Is $X(t)$ stationary? Is it WSS?

Question 2 (9 points)

Let X_n be an independent identically distributed (iid) random sequence with mean 2 and variance 3, and consider $Y_n = \frac{1}{2}(X_n + X_{n-1})$.

(a) Compute $\mathrm{E}[Y_n]$.

(b) Compute $\mathrm{var}[Y_n]$.

(c) Compute $R_X[k]$.

(d) Compute $R_{XY}[n, k]$ and $R_Y[n, k]$.

(e) Compute the average power of Y_n.

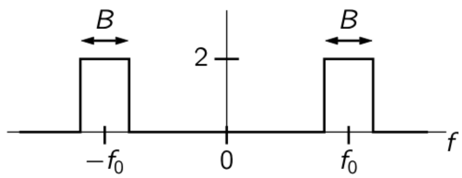
(f) Is Y_n iid? Is it WSS? Is it jointly WSS with X_n?

(g) If X_n is Gaussian, is Y_n Gaussian?

Question 3 (9 points)

The power spectral density $S_X(f)$ of a random process $X(t)$ is given by

$$S_X(f) = \begin{cases} 2 & |f \pm f_0| \leq \frac{B}{2} \\ 0 & \text{otherwise} \end{cases}$$



(a) Compute the average power of $X(t)$.

(b) Determine the autocorrelation function $R_X(\tau)$.

Hint: You may need to use Supplement table 1, 2, p. 29/30.

(c) $X(t)$ can be generated by passing white noise through a filter. Assume the noise power spectral density of the input is 1 W/Hz. Specify the filter transfer function $H(f)$.

(d) Let $Y(t) = X(t-5)$. Determine $S_Y(f)$.

Let $Z(t) = 2\,X(t) + N(t)$, where $N(t)$ is independent white noise with power spectral density N_0.

(e) Determine $S_Z(f)$.

(f) Determine $S_{XZ}(f)$ and $S_{NZ}(f)$.

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Partial exam EE2S31 SIGNAL PROCESSING
Part 2: July 1st 2021
Block 1: Digital Signal Processing (14:55-15:55)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted. Upload answers during 15:50-16:05.

This block consists of three questions (24 points), more than usual. This will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. No points will be awarded for results without a derivation. Write your name and student number on each sheet.

Question 4 (11 points)

Let us consider a first-order IIR filter with impulse response

$$h(n) = \frac{1}{4}\left(\frac{1}{2}\right)^{n} u(n) + \frac{1}{3}\left(\frac{1}{2}\right)^{(n-1)} u(n-1)$$

A Direct Form I realization of the filter is shown in Figure 1.

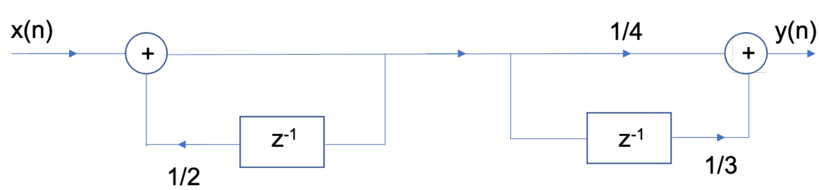

Figure 1

The outputs of the multipliers in this system are quantized using a midtread quantizer and a sign-magnitude coding scheme with 3 bits plus the sign bit. The quantizer can encode values between $(-1, 1)$.

We model the effect of quantization as an additive noise source $e(n)$, and we assume that $e(n)$ is an uncorrelated wide-sense stationary process that is uniformly distributed.

(2 p) (a) What is the variance of the quantization noise of this particular quantizer?

(3 p) (b) Now let us consider the quantization noise at the output of the filter. Compute the variance of the quantization noise at the output of the filter!

(3 p) (c) Give an alternative (Direct Form I) realization of the filter! Is this implementation better or worse in terms of quantization noise power?

(3 p) (d) What is binary code (using the specified quantizer) of the 2nd output sample of the filter in respone to the sequence $[\frac{3}{4}, 0, 0, 0, ...]$, taking into account quantization effects?

Question 5 (8 points)

Suppose that we want to record a single-channel EEG signal and store it for offline analysis. Our storage capacity is limited, therefore, we will store 100 samples per second. We are interested in analysing the signal in the frequency band $0 - 40$Hz, the signal content above 40Hz is of no interest.

We are going to pass the analog EEG signal through the system represented by the block diagram on Figure 2.

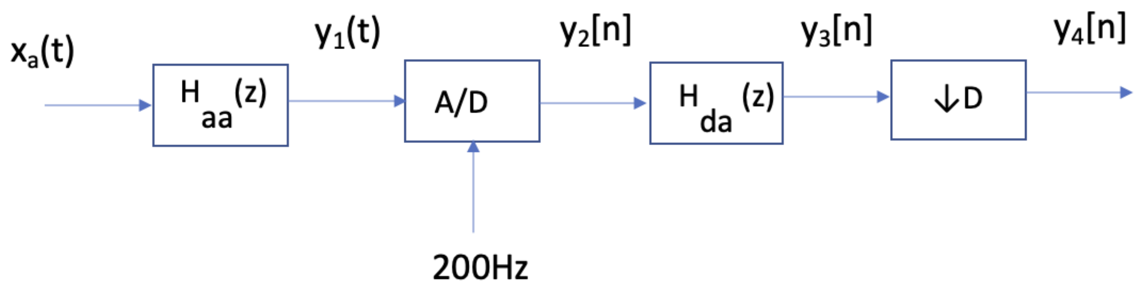

Figure 2

(1 p) (a) What is the purpose of the filters $H_{aa}(z)$ and $H_{da}(z)$?

(1 p) (b) What is the advantage of this system (compared to directly sampling at the desired sampling rate)? Explain!

(2 p) (c) What are the specifications of the filter $H_{aa}(z)$ and $H_{da}(z)$ in terms of pass, stop and transition band?

(1 p) (d) What is the value of D?

(3 p) (e) Assuming that the spectrum of the signal Y_3 is given by the sketch on Figure 3, sketch the spectrum of the signal Y_4! Indicate both physical and normalized frequencies!

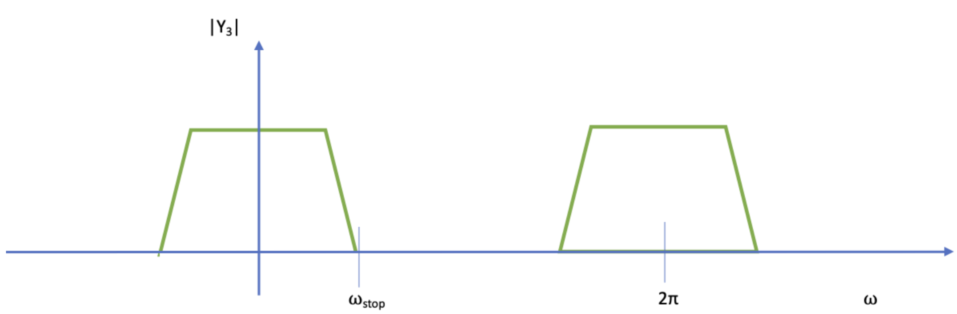

Figure 3

Question 6 (5 points)

Sigma-delta-modulators use prediction, 1-bit quantization and oversampling to achieve high SNR A/D conversion. Let us consider an input signal $x[n]$, with a bandwidth $B = 10\text{kHz}$. The first few values of its autocorrelation sequence R_{xx} are given as $R_{xx}[0] = 100$, $R_{xx}[1] = 42$, $R_{xx}[2] = 35$.

1 p (a) Assume that the signal $y[n]$ is given by:

$$y(n) = x[n] - x[n-1] \tag{1}$$

Is it advantageous to quantize the $y[n]$ instead of $x[n]$?

2 p (b) Suggest a modification to Eq. **??** to improve the quantization scheme! Be as specific as possible!

2 p (c) Recall that the quantization noise power of a first-order SDM can be expressed by

$$\sigma_n^2 \approx \frac{1}{3}\pi^2\sigma_e^2\left(\frac{2B}{F_s}\right)^3 \tag{2}$$

where σ_e^2 is the quantization noise variance F_s is the chosen sampling rate. How should we choose F_s in order to achieve an increase of 18dB in SNR?