Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Total points (both blocks): 55 pt. The grade has been computed relative to 38 pt = grade 10.

Partial exam EE2S31 SIGNAL PROCESSING
Part 1: May 27, 2020
Block 1: Stochastic Processes (13:30-14:30)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted. Upload answers during 14:25–14:35

This block consists of three questions (30 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.
Hint: Avoid losing too much time on detailed calculations, write down the general approach first.

Question 1 (12 points)

Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} c(x+y) & \text{for} \quad x \geq 0,\ y \geq 0,\ x+y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c.

(b) Find $f_X(x)$, $f_Y(y)$, and $f_{X|Y}(x|y)$.

(c) Find $\mathrm{E}[X]$ and $\mathrm{E}[Y]$.

(d) Find $\hat{X}_M(Y)$, the minimum mean square error (MMSE) estimator for X given a single sample of Y.

(e) Find the PDF of $W = X + Y$.

Question 2 (8 points)

In a test of a circuit, the probability that it functions correctly is $p = 0.8$. We test n circuits; the outcomes of the tests are independent. Let X be a RV that describes the outcome of a single test (0 or 1), and let K be the number of circuits that successfully pass the test.

(a) Derive that the moment generating function (MGF) of X is $\phi_X(s) = 1 - p + p\,e^s$.

(b) What is the MGF of K?

(c) Use the MGF to derive $\mathrm{E}[K]$, $\mathrm{E}[K^2]$, and compute from this $\mathrm{Var}[K]$.

(d) Use the central limit theorem to estimate the probability of finding at least 500 acceptable circuits in a batch of 600 circuits.

(e) Using the Chebyshev inequality, what is an upper bound on the probability of finding at least 500 acceptable circuits in a batch of 600 circuits?

Note: You may express your answer for (d) in terms of $\Phi(z)$ or $Q(z)$.

Question 3 (10 points)

A continuous random variable X has PDF

$$f_X(x) = \begin{cases} \dfrac{1}{12} & \text{for} \quad -6 \le x \le 6, \\ 0 & \text{otherwise.} \end{cases}$$

We observe a noisy version of X, namely $Y = X + N$, where N is an independent zero mean Gaussian random variable with variance 3.

Let $\hat{X}_{\mathrm{ML}}(Y)$ be the maximum likelihood (ML) estimate of X based on a single observation of Y, and let $\hat{X}_L(Y) = aY + b$ be the linear minimum mean square error (LMMSE) estimate of X.

(a) What is $\mathrm{E}[Y]$, $\mathrm{Var}[Y]$, and $\mathrm{Cov}[X, Y]$?

(b) What is $\hat{X}_{\mathrm{ML}}(Y)$?

(c) Is $\hat{X}_{\mathrm{ML}}(Y)$ an unbiased estimator?

(d) Compute e_{ML}, the mean square error of the ML estimator.

(e) Find a and b, the optimum coefficients for the LMMSE estimate.

(f) Is $\hat{X}_L(Y)$ an unbiased estimator?

(g) What is e_L, the mean square error of the optimum linear estimate.

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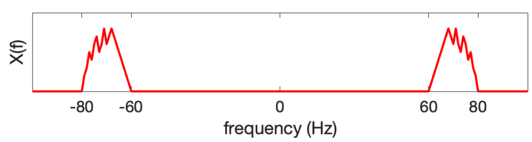
Partial exam EE2S31 SIGNAL PROCESSING
Part 1: May 27th 2020
Block 2: Digital Signal Processing (14:35-15:35)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted. Upload answers during 15:30-15:40.

This block consists of three questions (25 points), more than usual. This will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. No points will be awarded for results without a derivation. Write your name and student number on each sheet.

Question 4 (9 points)

Given an analog signal $x_a(t)$ with a spectrum shown below. I want to digitize the signal.



(1 p) (a) What is the bandwidth of the signal?

(1 p) (b) What is the minimum sampling rate F_s to avoid aliasing?

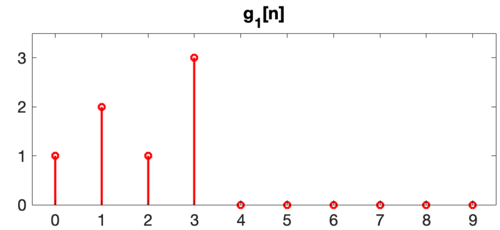
(2 p) (c) I have 2 A/D converters available, one operating at 50Hz, the other operating at 60Hz. Can I use any of the two to sample the signal without aliasing? Sketch and explain!

(3 p) (d) I would like to down-convert the signal $x_a(t)$, i.e. reconstruct the equivalent baseband signal. Is that possible given any of the sampling rates in point (b) and (c), assuming ideal interpolation? Give the equation to reconstruct the baseband signal using ideal interpolation!

(2 p) (e) What changes if I use sample-and-hold interpolation for the same purpose as in (d)? Sketch and explain in case 60Hz sampling rate is chosen!

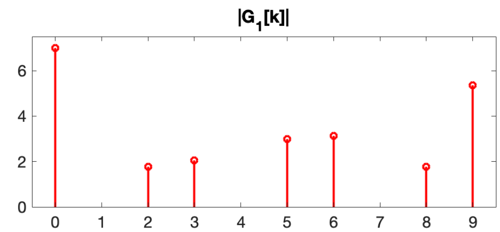
Question 5 (7 points)

Given a discrete sequence $g_1[n]$ as shown below.



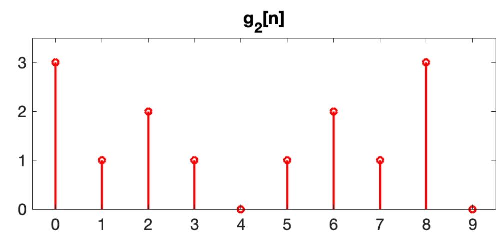
(1 p) (a) How is $G_1[k]$, the 10-point DFT of $g_1[n]$ related to the $G_1(\omega)$, the DTFT of $g_1[n]$? Explain!

(2 p) (b) The disk where we stored $G_1[k]$ was corrupted. We did not store $g_1[n]$ We could retrieve the magnitude of some of the DFT coefficients, these are shown in the figure below. Is it possible to reconstruct the missing values $|G_1[1]|$, $|G_1[4]|$ and $|G_1[7]|$? Explain and sketch!



(1 p) (c) What is the 5-point DFT of $g_1[n]$? Explain and sketch/express in terms of $G_1[k]$!

(3 p) (d) Given $g_2[n]$ as shown below. What is its 10-point DFT, denoted by $G_2[k]$, expressed in terms of $G_1[k]$?



Question 6 (9 points)

Given a short discrete sequence $x[n]$ and a filter with impulse response $h[n]$, as shown below:
$x[n] = [1, 2, 1, 0, -1, -2]$ $h = [1, 2, 3, 3, 2, 1]$

(2 p) (a) Describe the 4 consecutive steps of performing the filtering using DFT. Be as specific as possible, but no need to make any computations at this point!

(1 p) (b) What is the value of the 6th sample of the filtered signal?

(2 p) (c) Now, consider the circular convolution of $x[n]$ and $h[n]$! What is the value of the 6th sample? Is it different or identical to the solution in (b)? Explain!

(4 p) (d) Given the 3-point DFTs of the sequences $x_1[n] = [1, 1, -1]$ and $x_2[n] = [2, 0, -2]$ as $X_1[n] = [1, 1 - 1.73j, 1 + 1.73j]$ and $X_2[n] = [0, 3 - 1.73j, 3 + 1.73j]$. Explain how to compute $X[k]$, the 6-point DFT of $x[n]$ using these 3-point DFTs! Be as specific as possible. Compute the coefficients $X[0]$ and $X[3]$!