Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

exam EE2S31 SIGNAL PROCESSING
Resit: 29 July 2019 (13:30–16:30)

Closed book; two sides A4 of handwritten notes permitted

This exam consists of 5 questions (36 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (9 points)

The random variables X and Y have the joint probability density function (pdf)

$$f_{X,Y}(x,y) = \begin{cases} c\,, & \text{for} \quad 0 \leq x \leq y^2 \leq 1 \\ 0\,, & \text{otherwise.} \end{cases}$$

(2 p) (a) Make a sketch of the area where the pdf is non-zero and calculate the constant c.

(2 p) (b) Calculate the marginal pdfs $f_X(x)$ and $f_Y(y)$ as a function of parameter c.

(3 p) (c) Calculate $E[X|X \geq \frac{1}{4}]$.

(2 p) (d) Calculate $E[X|Y]$.

Question 2 (10 points)

Given is the stochastic process $X(t) = A + t$ with random variable A uniformly distributed in the (continuous) interval $[0, 2]$.

(1 p) (a) Sketch three different realizations of process $X(t)$.

(3 p) (b) Calculate the cumulative distribution function (CDF) $F_{X(t)}(x)$, as well as the probability density function (PDF) $f_{X(t)}(x)$.

(2 p) (c) Calculate the expected value $E[X(t)]$, the autocorrelation function $R(t,\tau)$, and argue whether or not this process is wide sense stationary (WSS).

Process $X(t)$ is used as an input to a system with impulse response $h(t)$, having an output $Y(t)$.

(1 p) (d) Under which conditions is the output of such a system WSS?

(1 p) (e) Under which conditions are the input and output of such a system jointly WSS?

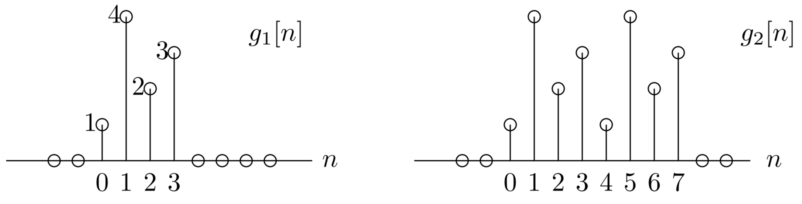
The impulse response $h(t)$ is given by

$$h(t) = \begin{cases} 1 & \text{for} \quad 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(2 p) (f) Calculate $E[Y(t)]$, i.e., the expected value of the output $Y(t)$ when the input is given by $X(t) = A + t$.

Question 3 (5 points)

Let $G_1(\omega)$ be the DTFT of the sequence $g_1[n]$ in figure (a):



(a) (b)

(a) Determine the DTFT of the other sequence, $g_2[n]$, in terms of $G_1(\omega)$.
(Note: do not evaluate $G_1(\omega)$ explicitely!)

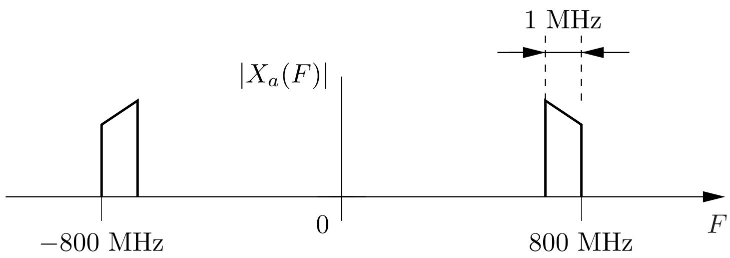
(b) Take $N = 8$. What is the DFT of $g_1[n]$ ($n = 0, \cdots, N-1$), expressed in terms of $G_1(\omega)$?

(c) How does the DFT of $g_1[n]$ change if we take $N = 16$ or $N = 4$? Explain using a drawing of the spectrum.

(d) What changes if the DFT of $g_2[n]$ is taken (using $N = 8$)? Explain using a drawing.

Question 4 (6 points)

We are given a real analog signal $x_a(t)$ with spectrum given as follows:

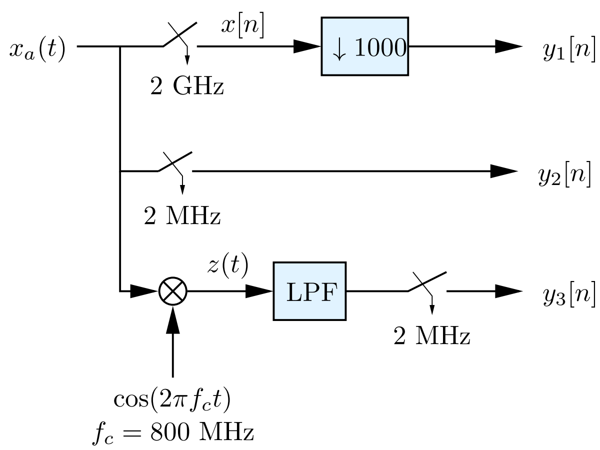


Think e.g. of a GSM signal at a carrier frequency slightly below 800 MHz.

(a) For this case, what is the Nyquist rate?

(b) What sample rate do you need at least to describe the signal?

We consider three techniques to demodulate and sample the signal.



(c) For signal $y_1[n]$, we sample the signal $x_a(t)$ at a rate of 2 GHz, and subsequently downsample by a factor 1000.

Draw the resulting spectum $Y_1(F)$. On the frequency axis, specify both ω and the corresponding 'real' frequencies F.

(d) For signal $y_2[n]$, we sample $x_a(t)$ at a rate of 2 MHz.

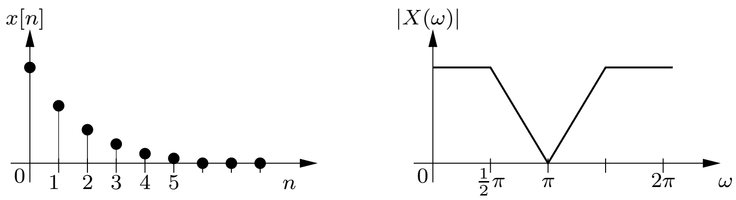
Draw the resulting spectrum $Y_2(F)$.

(e) For signal $y_3[n]$, we first apply a demodulation with $f_c = 800$ MHz, i.e., we multiply $x_a(t)$ by $\cos(2\pi f_c t)$, next we apply a suitable low pass filter (LPF), and sample at 2 MHz.

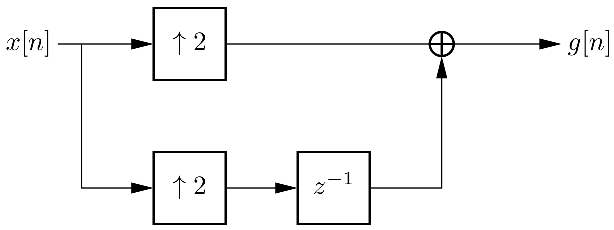
- Draw the spectrum $Z(F)$. (*Hint:* split cos into the sum of two complex exponentials.)
- What are suitable parameters for the LPF to prevent aliasing?
- Draw the resulting spectrum $Y_3(F)$.

Question 5 (6 points)

We are given a signal $x[n]$ with the following amplitude spectrum:



We consider the following system:



(a) Make a plot of the series $g[n]$.

(b) How is the spectrum $G(\omega)$ of $g[n]$ related to that of $x[n]$? (give a formula.)

Also make a drawing of the amplitude spectrum $|G(\omega)|$.

A "classical" movie projector in a cinema displays 24 images per second. Every image is projected using a short flash of light. This results in a flickering effect. To reduce this, every image is displayed twice (48 images per second).

To model the movie, we consider an arbitrary pixel and represent it by a time series $x[n]$. To model the observation by the audience, we model the eye as an ideal D/A converter, followed by a lowpass filter with a passband until 20 Hz and a stop band from 35 Hz.

(c) Give a block scheme representing the "double" projection process of $x[n]$ until the observation.

(d) Motivate why the double projection results in less flicker. (Use frequency spectra.)