Partial exam EE2S31 SIGNAL PROCESSING
Part 1: 28 May 2018, 13:30–15:30

Closed book; two sides A4 of handwritten notes permitted

This exam consists of four questions (28 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (8 points)

Consider the following two probability density functions (PDFs) of the independent random variables X and Y:

$$f_X(x) = \begin{cases} \lambda_1 e^{-\lambda_1 x}, & \text{for} \quad x \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$f_Y(y) = \begin{cases} \lambda_2 e^{-\lambda_2 y}, & \text{for} \quad y \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

In this question we are interested in the random variable $Z = X + Y$.

a) Determine the distribution $f_Z(z)$ and show that it equals

$$f_Z(z) = \begin{cases} \dfrac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 z} - e^{-\lambda_2 z} \right), & \text{for} \quad z \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

   Hint: In the derivation you might want to use a partial fraction expansion.

b) Calculate the cumulative distribution function (CDF) $F_Z(z)$.

c) Determine the PDF $f_{Z|Z \geq 4}(z)$.

d) Calculate $E[Z|Z \geq 4]$.

Question 2 (6 points)

Consider the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} \lambda e^{-\lambda x} & \text{for} \quad 0 \leq y \leq x \\ 0 & \text{otherwise.} \end{cases}$$

a) Calculate the PDFs $f_X(x)$ and $f_Y(y)$ and use these marginal PDFs to argue whether or not X and Y are dependent.

b) Calculate the MMSE estimator $\hat{Y} = E[Y|X]$.

c) Calculate the MMSE estimator $\hat{X} = E[X|Y]$.

Question 3 (8 points)

Consider the signal $g_1[n]$ in figure (a) and the signal $g_2[n]$ in figure (b). A plot of the DTFT $G_1(\omega)$ of $g_1[n]$ is shown in figure (c).

(a) (b) (c)
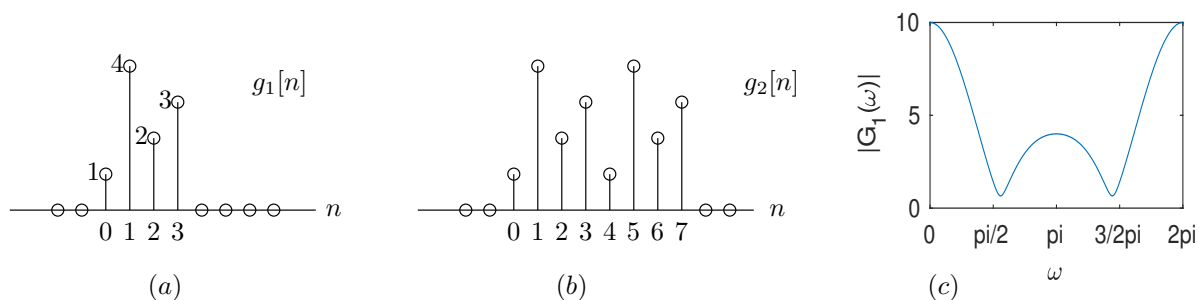
a) Derive that the DTFT of $g_2[n]$ is given by $G_2(\omega) = G_1(\omega) + e^{-4j\omega}G_1(\omega)$.

b) Take $N = 8$. What is the DFT of $g_1[n]$ $(n = 0, \cdots, N-1)$, expressed in terms of $G_1(\omega)$?

c) What changes if we take $N = 16$ or $N = 4$? Explain using a scetch of the spectrum.

d) What changes if we take the DFT of $g_2[n]$ $(n = 0, \cdots, N-1$, with $N = 8)$? Explain using a scetch.

e) Based on your result in item d), relate the DFT of $g_2[n]$ for $N = 8$ to the DFT of $g_1[n]$ for $N = 4$.

Question 4 (6 points)

We are sampling an analog signal that has a bandwidth of $B = 5$ kHz. We can take samples during 2 seconds. We want to show the spectrum using the DFT.

a) What is the minimal sample rate F_s needed to avoid aliasing?

b) At that rate, what is the resolution (in Hz) of the spectrum that we obtain? (Here, resolution is defined as the distance, in Hz, between two consecutive points in the spectrum.)

c) How can we improve the resolution by a factor 2? Give two options.