Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S1 SIGNALS AND SYSTEMS

Part 2 exam, 5 November 2025, 9:00–11:00

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of five questions (23 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (7 points)

(a) Given the signal $x[n] = [\cdots, 0, \boxed{1}, 3, 0, 0, \cdots]$.

Determine $y[n] = x[n] * x[-n]$ using the convolution sum (in time-domain).

(b) Also determine $y[n]$ in (a) via the z-transform (do you obtain the same result?).

(c) Given $x[n] = (n-1)\,u[n]$. Determine $X(z)$, also specify the ROC.

(d) Given $X(z) = \dfrac{4z}{(z-1)(z+0.25)}$, $\quad$ ROC $= \{0.25 < |z| < 1\}$.

Determine $x[n]$ using the inverse z-transform.

(e) Let $x[n] = u[n+2] - u[n-3]$. Determine the DTFT $X(e^{j\omega})$.

(f) Suppose the DTFT of a signal $x[n]$ is $X(e^{j\omega})$. What is the DTFT of $\cos(3n) \cdot x[n]$?

Question 2 (3 points)

A causal system is specified by the transfer function

$$H(z) = \frac{z^2+1}{(z-0.9)(z+0.9)}.$$

(a) Determine all poles and zeros of the system and draw a pole-zero plot.

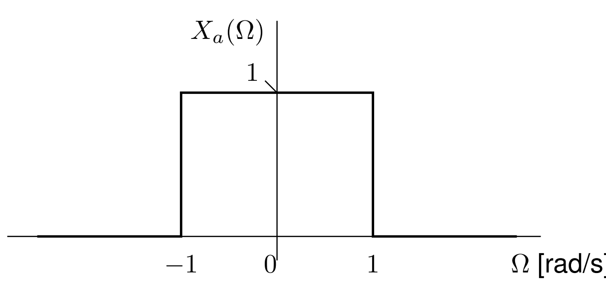
(b) What is the ROC?

(c) Is this a stable system? (Motivate.)

(d) Sketch the amplitude spectrum $|H(e^{j\omega})|$, also indicate values on the frequency axis.

Question 3 (5 points)

An analog signal $x_a(t)$ has Fourier Transform $X_a(\Omega) = u(\Omega+1) - u(\Omega-1)$, as shown in the figure.



(a) Determine $x_a(t)$.

(b) What is the largest value of the sample period T_s that would not cause aliasing when sampling $x_a(t)$?

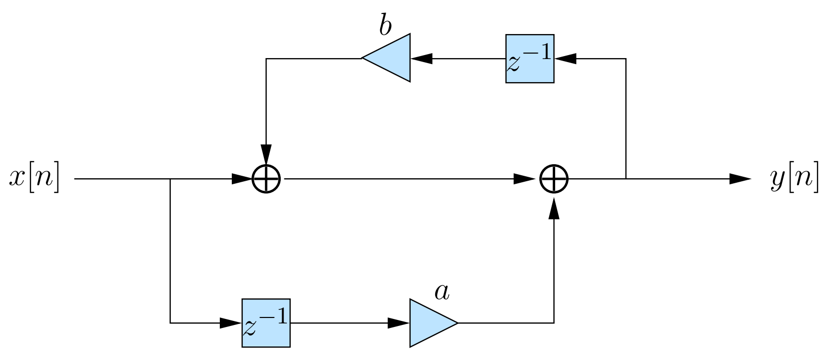
(c) For that T_s, let the resulting sampled signal be $x[n]$. Sketch the spectrum $X(e^{j\omega})$ (carefully label the axes).

(d) Let $y_a(t) = (x_a(t))^2$. Give an expression for $Y_a(\Omega)$ in terms of $X_a(\Omega)$.

(e) If we sample $y_a(t)$ at the same period T_s as before in (c), would aliasing occur? (Motivate your answer using a sketch.)

Question 4 (3 points)

(a) Determine the transfer function $H(z)$ of the following realization:



(b) Is this a minimal realization? (Why?)

(c) Draw the "Direct form no. II" realization and also specify the coefficients.

Question 5 (5 points)

A "template" second order analog lowpass filter (normalized Butterworth filter) is given by

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1} .$$

We will design a second-order digital *high*pass filter $G(z)$ with 3 dB cut-off frequency $\omega_c = \frac{3}{4}\pi$. For this, we first design a suitable highpass filter $G_a(s)$ in the analog domain, and then apply the bilinear transformation.

(a) For the template $H_a(s)$, demonstrate that the corresponding magnitude squared frequency response is

$$|H_a(j\Omega)|^2 = \frac{1}{1+\Omega^4} ,$$

showing that $H_a(s)$ is, indeed, a second-order Butterworth lowpass filter.

(b) For our design, what has to be the cut-off frequency in the analog frequency domain?

(c) Which frequency transformation do you apply to the template to arrive at an analog highpass filter with the desired cut-off frequency? What is the resulting $|G_a(j\Omega)|^2$?

(d) For this filter, at what frequency do we have a damping of 10 dB? What will be the corresponding frequency in digital domain?

(e) What is the resulting analog-domain highpass filter transfer function $G_a(s)$?

(f) What is the resulting $G(z)$?