Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S11 SIGNALS AND SYSTEMS

Part 2 exam, 30 January 2024, 13:30–15:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of five questions (36 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (10 points)

a) Given the signals $x[n] = u[n+2] - u[n-2]$, $\quad h[n] = [\cdots, 0, \boxed{1}, -2, 0, 0, \cdots]$.
Determine $y[n] = x[n] * h[n]$ using the convolution sum (in time-domain).

b) Given $x[n] = u[n] - (\frac{1}{2})^n u[n-4]$. Determine $X(z)$ and also specify the ROC.

c) Given $x[n] = a^n u[n]$ with $|a| < 1$. Determine $y[n] = x[n] * x[-n]$. (Use the z-transform.)

d) Determine, if it exists, the frequency response $H(e^{j\omega})$ for the system defined by the difference equation

$$y[n] = 1.6y[n-1] - 0.64y[n-2] + x[n] - x[n-2]$$

e) Given an LTI system with transfer function $H(z) = 1 - 2z^{-1}$.
Determine a (bounded) input signal $x[n]$ for which the output signal is equal to $y[n] = \delta[n] + \frac{1}{2}\delta[n-1]$.

Question 2 (6 points)

The transfer function of a causal LTI system is given by $H(z) = \dfrac{z-1}{z(z+0.9)}$

a) Determine all poles and zeros of the system and make a drawing in the complex z-plane.

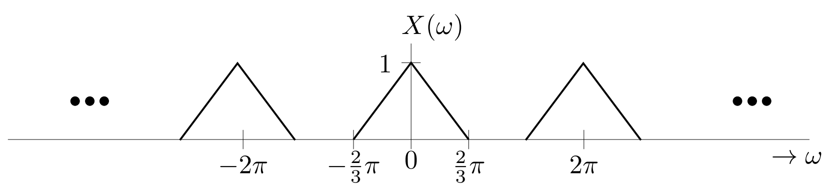
b) Specify the ROC.

c) Is the system BIBO stable? (Why?)

d) Draw, based on the poles and zeros of $H(z)$, the amplitude response. Is this a low-pass, high-pass or other kind of filter?

Question 3 (7 points)

An analog signal $x_a(t)$ with Fourier transform $X_a(\Omega)$ is band-limited at 10 kHz. The signal is sampled without aliasing at a sampling frequency F_s, resulting in the discrete-time signal $x[n]$. The spectrum $X(\omega)$ of $x[n]$ is shown below:



a) What is the relation between Ω and ω?

b) Which sampling frequency was used?

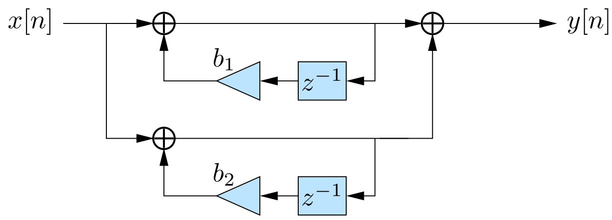
c) What is the smallest frequency at which we can sample $x_a(t)$ without aliasing?

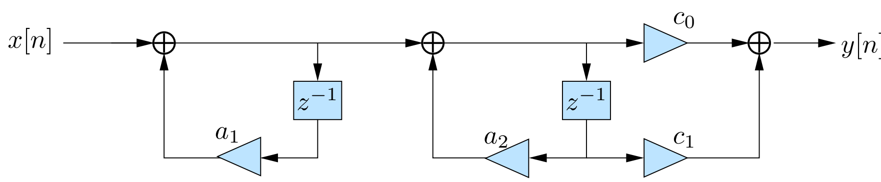
For this case, draw the resulting spectrum (also clearly mark the frequencies).

d) Consider the initial sampling rate. After sampling, $x_a(t)$ is reconstructed from $x[n]$ by means of an ideal D/A convertor and a low-pass filter. Specify the pass-band and stop-band frequencies of the filter.

Question 4 (6 points)

Given the realizations:





a) Determine a_1, a_2 and c_0, c_1 in terms of b_1, b_2 such that both systems are equivalent.

b) Are these minimal realizations?

c) Draw the "direct form no. II" realization and also specify the coefficients.

Question 5 (7 points)

A "template" third-order Butterworth filter has the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

The corresponding frequency response is $|H(j\Omega)|^2 = \dfrac{1}{1+\Omega^6}$.

a) Which frequency transform should we apply to the template to construct a low-pass Butterworth filter with a 3dB cut-off frequency of Ω_c?

b) What is the corresponding transfer function $G(s)$?

We now design an analog 3rd order low-pass Butterworth filter with a pass-band frequency of 3 rad/s, a stop-band frequency of 6 rad/s and a maximal damping in the pass-band of 0.5 dB.

c) Give a suitable expression for the frequency response (squared-amplitude) of this filter and determine its parameters.

d) For this filter, what is the minimal damping in the stop-band ?

e) Which transform should be applied to $|H(j\Omega)|^2$ to obtain this filter?

Determine the corresponding transfer function.