Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Signal Processing Systems

EE2S11 SIGNALS AND SYSTEMS

Part 2 exam, 2 February 2023, 13:30–15:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of six questions (34 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important.

Question 1 (4 points)

a) Let $a(t)$ and $b(t)$ be two (possibly complex) signals. Let $A(\Omega)$ be the Fourier transform of $a(t)$, and similar for $B(\Omega)$. Show that

$$\int_{-\infty}^{\infty} a(t)b^*(t)\mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\Omega)B^*(\Omega)\mathrm{d}\Omega\,.$$

We call $\int a(t)b^*(t)\,\mathrm{d}t$ the *inner product* of the two signals. The two signals are called *orthogonal* if their inner product is zero.

b) Let $s(t)$ be a real-valued signal and let $s'(t)$ be its derivative. Show that $s(t)$ is orthogonal to $s'(t)$.

Question 2 (7 points)

a) Given the signals $x[n] = \begin{cases} n, & 0 \le n \le 4, \\ 0, & \text{elsewhere} \end{cases}$ and $h[n] = [\cdots, 0, \boxed{2}, -1, 0, 0, \cdots]$.

Determine $y[n] = x[n] * h[n]$.

b) Given $X(z) = \dfrac{3\,z^{-1}}{(1-\frac{1}{2}\,z^{-1})(1-2z^{-1})}\,, \qquad z \in \text{ROC}.$

Determine $x[n]$ using the inverse z-transform if the ROC is $\frac{1}{2} < |z| < 2$.

c) Given the DTFT $X(e^{j\omega}) = e^{-j2\omega}\cos^2(\omega)$. Determine $x[n]$.

Hint: you could try to first determine the z-transform.

Question 3 (6 points)

The transfer function of a causal LTI system is given by $H(z) = \dfrac{z^2+2}{z^2+\frac{1}{2}}\,.$

a) Determine all poles and zeros of the system and draw a pole-zero diagram.

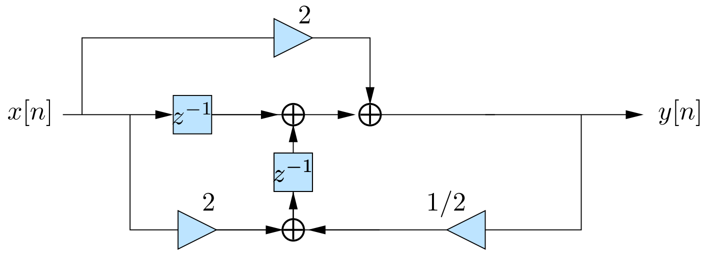
b) Specify the ROC.

c) Is the system BIBO stable? (Why?)

d) Determine the amplitude response $|H(e^{j\omega})|^2$ of the system.

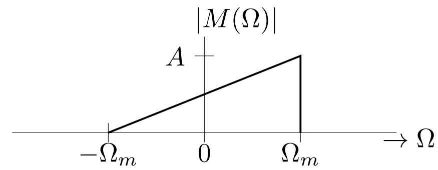
Question 4 (5 points)

a) Determine the transfer function $H(z)$ of the following system:



b) Is this a minimal realization?

c) Draw the “Direct Form II” realization.



Question 5 (6 points)

Given the continuous-time signal $x_a(t) = m(t)\cos(1000t)$.

Here, $m(t)$ is a signal with spectrum $M(\Omega)$ as shown in the figure above, where $\Omega_m = 200$ rad/s and A is a certain amplitude.

The signal $x_a(t)$ is sampled at $\Omega_s = 1500$ rad/s, resulting in a discrete-time signal $x[n]$.

a) Compute the Fourier Transform $X_a(\Omega)$ of $x_a(t)$. You don’t need to expand $M(\Omega)$.

b) Draw the amplitude spectrum $|X_a(\Omega)|$. Accurately indicate the frequencies and amplitudes. Also mark the sample frequency.

c) Do we satisfy the Nyquist criterion?

d) Let $X(\omega)$ denote the DTFT of $x[n]$. What is the relation between ω and Ω?

e) Make a drawing of the amplitude spectrum $|X(\omega)|$. Accurately indicate the frequencies (also draw the corresponding Ω-axis).

Question 6 (6 points)

We design an analog 3rd order *high-pass* Chebyshev filter with a stop-band frequency of 5 rad/s, a pass-band frequency of 10 rad/s and maximal damping in the pass-band of 2 dB.

The amplitude response of a prototype n-th order low-pass Chebyshev filter is given by

$$|H(\Omega)|^2 = \frac{1}{1 + \epsilon^2\, T_n^2(\Omega)} \,.$$

Also, the 3rd order Chebyshev polynomial is given by $T_3(\Omega) = 4\Omega^3 - 3\Omega$.

a) We first design a 3rd order Chebyshev low-pass filter with a pass-band frequency of 1 rad/s and a maximal damping in the pass-band of 2 dB. Determine all unknown parameters.

b) Which frequency transformation is needed to transform this lowpass filter into the desired high-pass filter?

c) Give the expression for the amplitude response $|G(\Omega)|^2$ of the resulting high-pass filter $G(\Omega)$.

d) What is the minimal damping in the stop-band of this high-pass filter?

e) Carefully draw the amplitude response of the high-pass filter; indicate the stop-band and pass-band frequencies, and also denote the corresponding dampings.