Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

EE2S11 SIGNALS AND SYSTEMS

Final exam, 29 January 2021, 13:30–15:50
Block 1 (13:30-14:30)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 14:25–14:40

This block consists of three questions (20 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (9 points)

(a) Let $x[n] = [\cdots, 0, \boxed{1}, 3, 2, 0, \cdots]$, where the 'box' denotes the value for $n = 0$. Determine the convolution $y[n] = x[n] * x[-n]$.

(b) Let $x[n] = 2^{-n-2}u[-n-2]$. Determine the z-transform, also specify the ROC.

Hint: you could first make a plot of $x[n]$.

(c) Let

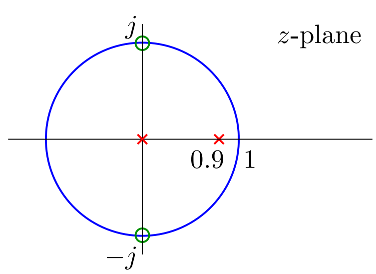
$$H(z) = \frac{1 - \frac{3}{2}z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 - 3z^{-1})}\,.$$

Draw a pole-zero plot, and determine $h[n]$ for (c1) ROC: $|z| < \frac{1}{2}$; (c2) ROC: $\frac{1}{2} < |z| < 3$; (c3) ROC: $|z| > 3$.

(d) Let $x[n] = [\cdots, 0, \boxed{1}, 3, 1, 0, \cdots]$. Determine the DTFT $X(e^{j\omega})$, also determine and give plots of the amplitude spectrum and the phase spectrum.

Question 2 (5 points)

Consider the pole-zero plot of a discrete-time causal filter with transfer function $H(z)$:



(a) Determine $H(z)$, up to an amplitude scale factor A.

(b) Suppose that we know that $h[0] = 2$. Determine A.

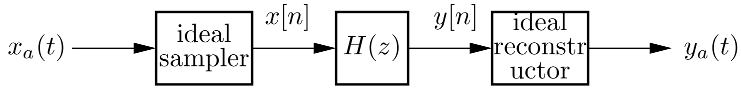
(c) Based on the pole-zero locations, contruct a sketch of the magnitude spectrum $|H(e^{j\omega})|$. Clearly indicate relevant values on the ω-axis.

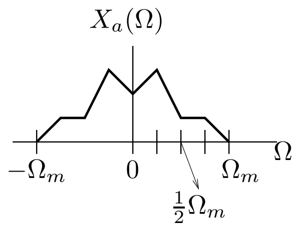
(d) Specify the ROC. Is this a stable filter?

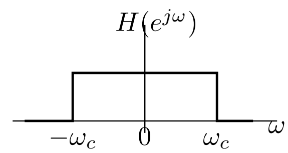
Question 3 (6 points)

A continuous-time signal $x_a(t)$ has a spectrum $X_a(\Omega)$ as indicated below. It is sampled at the Nyquist rate (resulting in $x[n]$), passed through a lowpass filter with frequency response $H(e^{j\omega})$ (resulting in $y[n]$), and reconstructed using an ideal DAC (which includes an ideal interpolation filter). The output signal is $y_a(t)$.

The cut-off frequency of the lowpass filter is $\omega_c = \Omega_m T/2$, where T is the sample period.







(a) Relate T to Ω_m.

(b) Draw the spectra $X(\omega)$, $Y(\omega)$, and $Y_a(\Omega)$. Clearly mark the relevant values on the frequency axis.

(c) Suppose now that we sample at twice the Nyquist rate. Again draw the spectra $X(\omega)$, $Y(\omega)$, and $Y_a(\Omega)$.

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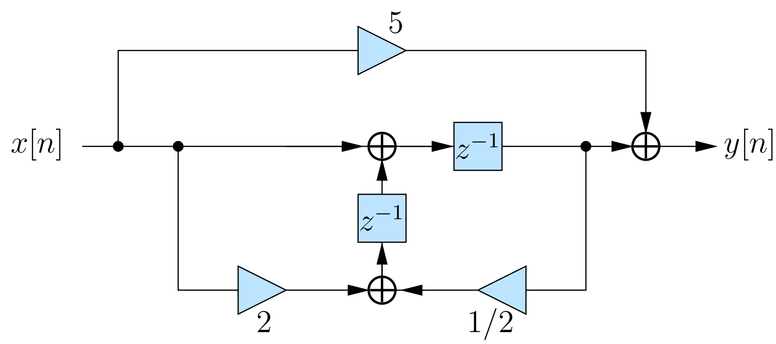
Block 2 (14:50-15:50)

Open book, strictly timed take-home exam. (Electronic) copies of the book and the course slides allowed. No other tools except a basic pocket calculator permitted.
Upload answers during 15:45–16:00

This block consists of three questions (20 points); more than usual, and this will be taken into account during grading. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 4 (6 points)

Consider the following realization of a causal system:



(a) Determine the transfer function $H(z)$.

(b) What is the difference equation implemented by this realization?

(c) Is this a stable realization? (motivate)

(d) Is this a minimal realization? (motivate)

(e) Draw the "Direct form no. II" realization of the filter and also specify the coefficients.

Question 5 (9 points)

A normalized second-order analog low-pass filter (Butterworth filter) is given by

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}\,.$$

The 3-dB cutoff frequency for this template is $\Omega_c = 1$ rad/s.

We are asked to design a second-order digital *high-pass* filter $G(z)$ with

$$\begin{array}{ll} \text{Passband frequency:} & \omega_p = 1 \text{ rad} \\ \text{Passband damping:} & 1 \text{ dB} \\ \text{Stopband frequency:} & \omega_s = 0.5 \text{ rad} \end{array}$$

We will first design an analog 2nd order high-pass filter $G_a(s)$ and then apply the bilinear transform.

(a) From the given specifications, what are the passband and stopband frequencies for the analog high-pass filter?

(b) Based on $H_a(s)$, what is the corresponding power spectrum $|H_a(j\Omega)|^2$?

(c) What frequency transformation is needed to transform $|H_a(j\Omega)|^2$ to a template $|G_a(j\Omega)|^2$ for the analog 2nd order high-pass filter, which involves design parameters ϵ and Ω_p ?

(d) What is the corresponding template high-pass filter $G_a(s)$?

(e) Compute the unknown parameters:

What is $|G_a(j\Omega)|^2$ and $G_a(s)$ that satisfies the specifications?

(f) What is the resulting digital high-pass filter $G(z)$ that satisfies the specifications?

(g) How much damping in the stopband is achieved? (specify in dB)

Question 6 (5 points)

(a) Determine the Fourier transform of

$$x(t) = \cos(\Omega_0 t)\sin(\Omega_1 t)\,.$$

(b) A periodic signal $x(t)$ has a Fourier series

$$x(t) = \sum_{k=1}^{\infty} \frac{2}{k^2} \cos(3kt/2)\,.$$

Compute the Fourier transform, $X(\Omega)$.

(c) Use the duality theorem to prove the following Fourier transform result:

$$x(t) = \frac{1}{t^2 + a^2}\,, \quad a > 0 \qquad \leftrightarrow \qquad X(\Omega) = \frac{\pi}{a} e^{-a|\Omega|}$$