Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S1 SIGNALS AND SYSTEMS

Midterm exam, 29 September 2025, 9:00–11:00

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has four questions (12 points).

Question 1 (4 points)

Evaluate the following integrals. Motivate your answer.

(a)
$$\int_{t=-4}^{4} t^2 \left[\delta(t+2) + \delta(t) + \delta(t-5)\right] \mathrm{d}t$$

(b)
$$\int_{t=-4}^{4} (t^2 + 2) \left[\delta(t) + 3\delta(t-2)\right] \mathrm{d}t$$

A signal $y(t)$ satisfies the differential equation

$$4y'(t) + 8y(t) = 8\delta(t)$$

and $y(0^-) = 0$.

(c) Determine $y(0^+)$. Motivate your answer.

(d) Find a signal $z(t)$ such that $z(t) = 0$ for $t < 0$ and $z'(t) = 5\delta(t) - 10e^{-2t}u(t)$.

Answer

Note: only answers are provided, with no or very little explanation. Your solutions at the exam should always be completely worked out and well motivated.

(a) 4

(b) 20

(c) 2

(d) $z(t) = 5e^{-2t}u(t)$

Question 2 (2 points)

Given the signal $x(t) = te^{-4t}u(t)$, where $u(t)$ is the Heaviside unit step function. Consider the signal $y(t) = x(t) * x(t)$.

(a) Is the signal $y(t)$ causal? Motivate your answer without calculations.

(b) Determine the signal $y(t)$ using the definition of the convolution integral.

Answer

(a) Yes, a causal signal convolved with a causal signal gives a causal signal.

(b) $y(t) = 0$ for $t < 0$, since y is causal. For $t > 0$ we obtain

$$
\begin{aligned}
y(t) &= \int_{\tau=-\infty}^{\infty} x(t-\tau)x(\tau)\,\mathrm{d}\tau = \int_{\tau=0}^{t} (t-\tau)e^{-4(t-\tau)}\tau e^{-4\tau}\,\mathrm{d}\tau \\
&= e^{-4t}\int_{\tau=0}^{t} (t-\tau)\tau\,\mathrm{d}\tau = \frac{1}{6}t^3 e^{-4t}.
\end{aligned}
$$

Question 3 (3 points)

(a) Determine the Laplace transform of

$$
f(t) = \begin{cases} 0 & \text{for } t < 6, \\ 3 & \text{for } t > 6. \end{cases}
$$

(b) Determine the Laplace transform of

$$
g(t) = \begin{cases} 0 & \text{for } t < 0, \\ 5 & \text{for } 0 < t < 1, \\ t & \text{for } t > 1. \end{cases}
$$

Let $u(t)$ denote the Heaviside unit step function and consider the signal

$$
w(t) = (1 - e^{-2t})^2 u(t).
$$

The Laplace transform of this signal is of the form

$$
W(s) = \frac{C}{p(s)}, \qquad \mathrm{Re}(s) > 0,
$$

with C a constant and $p(s)$ a polynomial in s.

(c) Determine C and the polynomial $p(s)$. Motive you answer.

Answer

(a) $f(t) = 3u(t-6)$. Using the table, we find $F(s) = 3e^{-6s}/s$, $\mathrm{Re}(s) > 0$.

(b) The signal is given by

$$
g(t) = 5[u(t) - u(t-1)] + tu(t-1) = 5u(t) - 4u(t-1) + (t-1)u(t-1).
$$

Using the Table we find

$$
F(s) = \frac{5}{s} - 4\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}, \quad \mathrm{Re}(s) > 0
$$

(c) $C = 8$ and $p(s) = s(s+2)(s+4)$.

Question 4 (3 points)

A periodic signal $x(t)$ with fundamental period $T_0 = 2\pi$ has a trigonometric Fourier series

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(kt) + d_k \sin(kt) \qquad \text{for } -\pi < t < \pi$$

with

$$c_0 = \pi/2, \quad d_k = 0, \quad \text{and} \quad c_k = \frac{1}{\pi k^2}\left[(-1)^k - 1\right] \qquad \text{for } k \geq 1.$$

(a) Is the signal $x(t)$ even, odd, or neither? Explain your answer.

(b) What is the DC component of $x(t)$? Motivate your answer.

We are given that $x(0) = 0$, that is, the signal $x(t)$ vanishes for $t = 0$.

(c) Show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$

Answer

(a) Even.

(b) The dc-component is $c_0 = \pi/2$.

(c) Set $t = 0$ in the Fourier series to obtain

$$c_0 + 2\sum_{k=1}^{\infty} c_k = 0.$$

Substitute c_0 and c_k and the result follows. Note that

$$c_k = \begin{cases} 0 & \text{for } k \text{ even} \\ -\frac{2}{\pi k^2} & \text{for } k \text{ odd.} \end{cases}$$