Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S1 SIGNALS AND SYSTEMS

Midterm exam, 29 September 2025, 9:00–11:00

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has four questions (12 points).

Question 1 (4 points)

Evaluate the following integrals. Motivate your answer.

(a)
$$\int_{t=-4}^{4} t^2 \left[\delta(t+2) + \delta(t) + \delta(t-5)\right] \, \mathrm{d}t$$

(b)
$$\int_{t=-4}^{4} (t^2+2) \left[\delta(t) + 3\delta(t-2)\right] \, \mathrm{d}t$$

A signal $y(t)$ satisfies the differential equation

$$4y'(t) + 8y(t) = 8\delta(t)$$

and $y(0^-) = 0$.

(c) Determine $y(0^+)$. Motivate your answer.

(d) Find a signal $z(t)$ such that $z(t) = 0$ for $t < 0$ and $z'(t) = 5\delta(t) - 10e^{-2t}u(t)$.

Question 2 (2 points)

Given the signal $x(t) = te^{-4t}u(t)$, where $u(t)$ is the Heaviside unit step function. Consider the signal $y(t) = x(t) * x(t)$.

(a) Is the signal $y(t)$ causal? Motivate your answer without calculations.

(b) Determine the signal $y(t)$ using the definition of the convolution integral.

Question 3 (3 points)

(a) Determine the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{for } t < 6, \\ 3 & \text{for } t > 6. \end{cases}$$

(b) Determine the Laplace transform of

$$g(t) = \begin{cases} 0 & \text{for } t < 0, \\ 5 & \text{for } 0 < t < 1, \\ t & \text{for } t > 1. \end{cases}$$

Let $u(t)$ denote the Heaviside unit step function and consider the signal

$$w(t) = (1 - e^{-2t})^2 u(t).$$

The Laplace transform of this signal is of the form

$$W(s) = \frac{C}{p(s)}, \qquad \text{Re}(s) > 0,$$

with C a constant and $p(s)$ a polynomial in s.

(c) Determine C and the polynomial $p(s)$. Motive you answer.

Question 4 (3 points)

A periodic signal $x(t)$ with fundamental period $T_0 = 2\pi$ has a trigonometric Fourier series

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} c_k \cos(kt) + d_k \sin(kt) \qquad \text{for } -\pi < t < \pi$$

with

$$c_0 = \pi/2, \quad d_k = 0, \quad \text{and} \quad c_k = \frac{1}{\pi k^2}\left[(-1)^k - 1\right] \qquad \text{for } k \geq 1.$$

(a) Is the signal $x(t)$ even, odd, or neither? Explain your answer.

(b) What is the DC component of $x(t)$? Motivate your answer.

We are given that $x(0) = 0$, that is, the signal $x(t)$ vanishes for $t = 0$.

(c) Show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$