Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S11 SIGNALS AND SYSTEMS

Midterm exam, 13 December 2023, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has four questions (16 points).

Question 1 (5 points)

Given the two signals $h(t) = u(t-1)$ and $x(t) = u(t-2)$, where $u(t)$ is the Heaviside unit step function.

(a) Determine the convolution $y(t) = h(t) * x(t)$ of the signals h and x by directly using the convolution integral.

(b) Determine the convolution $y(t) = h(t) * x(t)$ using the Laplace transform.

Suppose that this $x(t)$ is the input signal of a Linear and Time-Invariant (LTI) system and suppose that $h(t)$ is the impulse response of this system.

(c) Is the input signal $x(t)$ causal? Motivate your answer.

(d) Is the LTI system causal? Motivate your answer.

(e) Is the LTI system BIBO stable? Motivate your answer.

Question 2 (4 points)

(a) Given the signal $f(t) = e^{-t}u(t)$. Determine the two-sided Laplace transform of $\frac{\mathrm{d}f}{\mathrm{d}t}$ and give its ROC.

(b) Determine the two-sided Laplace transform of the signal $g(t) = \delta(2t+4)$, where $\delta(t)$ is the Dirac distribution, and give its ROC.

(c) Determine the Laplace transform of the signal $w(t) = (t^2 - 2t + 5)u(t-1)$ and give its ROC.

Question 3 (3 points)

Determine the inverse Laplace transforms of

(a) $F(s) = \dfrac{s}{s^2 - a^2}, \quad a > 0, \quad \mathrm{Re}(s) > a.$

(b) $G(s) = \frac{3s-1}{s(s-1)}, \quad \text{Re}(s) > 1.$

(c) $W(s) = \frac{6}{s^2 - 6s + 13}, \quad \text{Re}(s) > 3.$

Question 4 (4 points)

Given the periodic signal $x(t)$ with fundamental period $T_0 = 2\pi$ and

$$x(t) = e^t, \quad -\pi < t < \pi.$$

(a) Determine the power P_x of this periodic signal.

(b) Determine the Fourier coefficients X_k of this periodic signal.

(c) Show that

$$\sum_{k=-\infty}^{\infty} \frac{1}{k^2+1} = \frac{\pi}{\tanh(\pi)}.$$