

EE2S11 SIGNALS AND SYSTEMS

Midterm exam, 8 December 2021, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has three questions (30 points).

Question 1 (8 points)

- (a) Given the signal $f(t) = u(t^2 - 4t)$, where u is the Heaviside unit step function. The derivative of f is of the form

$$\frac{df}{dt} = A\delta(t - \alpha) + B\delta(t - \beta),$$

where A , α , B , and β are constants with $\alpha > \beta$. Determine the constants A , α , B , and β .

- (b) Given the signals $v(t) = u(-t)$ and $w(t) = p(t)$, where u is the Heaviside unit step function and p the standard rectangular pulse:

$$p(t) = \begin{cases} 1 & \text{for } 0 < t < 1, \\ 0 & \text{for } t < 0 \text{ and } t > 1. \end{cases}$$

Determine the signal $z(t) = v(t) * w(t)$ by directly evaluating the convolution integral.

Question 2 (11 points)

- (a) The one-sided Laplace transform of a causal signal $f(t)$ is given by

$$F(s) = \frac{s^3 + 3s^2 + s + 8}{s^2 + 4s}, \quad \operatorname{Re}(s) > 0.$$

Determine $f(t)$.

- (b) The one-sided Laplace transform of the signal

$$f(t) = (1 - e^{-t})^3 u(t),$$

is of the form

$$F(s) = \frac{C}{p(s)}, \quad \operatorname{Re}(s) > 0,$$

where C is a constant and $p(s)$ a polynomial in s . Determine C and $p(s)$.

(c) The two-sided Laplace transform of a noncausal signal $y(t)$ is given by

$$Y(s) = \frac{1}{s^2} (e^{-s} - 1), \quad \operatorname{Re}(s) < 0.$$

Plot $y(t)$.

(d) Determine the two-sided Laplace transform of $g(t) = t^2$, $-\infty < t < \infty$.

Question 3 (11 points)

Given the periodic signal $x(t)$ with fundamental period $T_0 = \pi$ and

$$x(t) = \cos(t), \quad 0 < t < \pi.$$

- (a) Determine the average value of this signal.
- (b) Expand $x(t)$ in a Fourier sine series.
- (c) The Fourier coefficients of $x(t)$ decay as $1/k$ for $k \rightarrow \infty$. Explain why.
- (d) The signal $y(t)$ has a fundamental period $T_0 = 2\pi$ and is given by

$$y(t) = \operatorname{sign}(t),$$

on the interval $(-\pi, \pi)$. Use the Fourier series of $y(t)$ to show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$